General Certificate of Education
Advanced Subsidiary/Advanced

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MARKING SCHEMES

SUMMER 2007

MATHEMATICS

**WJEC
CBAC**

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2007 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

MATHEMATICS C1

1. (*a*) Gradient of *AB* = $\dfrac{\text{increase in } y}{\text{increase in } x}$ M1

     Gradient of *AB* = 2   (or equivalent)   A1

    (*b*) Gradient of *CD* = $\dfrac{k - (-1)}{5 - 2}$ B1

     Use of gradient *CD* = gradient *AB*   M1

     $\dfrac{k - (-1)}{5 - 2} = 2 \Rightarrow k = 5$ (convincing)   A1

    (*c*) Use of gradient of *L* = $\dfrac{-1}{\text{gradient of } CD}$ M1

     Equation of *L* : $y - 3 = -\frac{1}{2}\,[x - (-1)]$
     (f.t. candidate's gradient for *CD*)   A1

     Equation of *L* : $x + 2y - 5 = 0$ (convincing)   A1

    (*d*) A correct method for finding the equation of *CD*   M1

     Equation of *CD* : $2x - y - 5 = 0$
     (f.t. candidate's gradient for *CD*)   A1

     An attempt to solve equations of *L* and *CD* simultaneously   M1

     $x = 3, y = 1$ (c.a.o.)   A1

2. (*a*) $2\sqrt{8} = 2 \times 2 \times \sqrt{2}$ B1

     $\sqrt{18} = 3\sqrt{2}$ B1

     $\dfrac{12}{\sqrt{2}} = 3 \times 2 \times \sqrt{2}$ B1

     $2\sqrt{8} + \sqrt{18} - \dfrac{12}{\sqrt{2}} = \sqrt{2}$ (c.a.o.)   B1

    (*b*) $\dfrac{5 + \sqrt{15}}{5 - \sqrt{15}} = \dfrac{(5 + \sqrt{15})(5 + \sqrt{15})}{(5 - \sqrt{15})(5 + \sqrt{15})}$ M1

     Numerator: $25 + 15 + 5\sqrt{15} + 5\sqrt{15}$ A1

     Denominator: $25 - 15$ A1

     $\dfrac{5 + \sqrt{15}}{5 - \sqrt{15}} = 4 + \sqrt{15}$ (c.a.o.)   A1

     Special case
     If M1 not gained, allow B1 for correctly simplified numerator or
     denominator following multiplication of top and bottom by $5 - \sqrt{15}$

3. (a) Either: use of $f(3) = 0$
    Or: division by $(x - 3)$ M1
    A convincing argument that $p = 24$ A1
    Special case
    Candidates who assume $p = 24$ and verify the result using
    either method are awarded B1

   (b) $f(x) = (x - 3)(x^2 + ax + b)$ with one of a, b correct   M1
    $f(x) = (x - 3)(x^2 - 2x - 8)$ A1
    $f(x) = (x - 3)(x - 4)(x + 2)$ (f.t. one slip)   A1
    Roots are $x = 3$, $x = 4$, $x = -2$ (f.t. one slip)   A1

   (c)   Either: evaluation of $f(2)$
    Or: division by $(x - 2)$ M1
    Remainder $= 8$ A1

4. (a) An attempt to differentiate, at least one non-zero term correct   M1
    $\dfrac{dy}{dx} = -16x^{-2} + 3$ A1
    An attempt to substitute $x = 4$ in expression for $\dfrac{dy}{dx}$ m1
    When $x = 4$, $\dfrac{dy}{dx} = -1 + 3 = 2$ A1
    Equation of tangent is $y - 18 = 2(x - 4)$
        (f.t. if M1 and m1 both awarded)   A1

   (b) $4x + 7 = x^2 + 2x + 4$ M1
    An attempt to collect terms, form and solve quadratic equation   m1
    $x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = 3, x = -1$ (both values)   A1
    When $x = 3$, $y = 19$, when $x = -1$, $y = 3$ (both values)
        (f.t. one slip)   A1
    The line $y = 4x + 7$ intersects the curve $y = x^2 + 2x + 4$ at the points
    $(-1, 3)$ and $(3, 19$ (f.t. candidate's points)   E1

5. (a) $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
        (–1 for each error)
    (–1 for any subsequent 'simplification')   B2
    Substituting x for a and $\dfrac{1}{2x}$ for b in the $10a^3b^2$ term   M1
    Identifying $\dfrac{5}{2}$ (o.e.) as the coefficient of x A1

   (b)   Coefficient of $x^2 = \dfrac{n(n - 1)}{2}$ B1
    An attempt to solve $\dfrac{n(n - 1)}{2} = 36$ M1
    $n = 9$ (c.a.o.)   A1

6. $y = x^2 - 12x + 10$
  $y + \delta y = (x + \delta x)^2 - 12(x + \delta x) + 10$ B1
  Subtracting y from above to find δy M1
  $\delta y = 2x\delta x + (\delta x)^2 - 12\delta x$ A1
  Dividing by δx and letting $\delta x \to 0$ M1
  $\dfrac{dy}{dx} = 2x - 12$ (c.a.o.)   A1

7. (*a*) $a = 2$ B1
  $b = 1$ B1
  $c = 3$ B1

  (*b*) $\dfrac{1}{c + 4}$ on its own or maximum value = $\dfrac{1}{c + 4}$,
  with correct explanation or no explanation   B2

  $\dfrac{1}{c + 4}$ on its own or maximum value = $\dfrac{1}{c + 4}$,
  with incorrect explanation   B1

  minimum value = $\dfrac{1}{c + 4}$ with no explanation   B1

  minimum value = $\dfrac{1}{c + 4}$ with incorrect explanation   B0

  Special case
  Candidates who give maximum value = $\dfrac{1}{3}$ are awarded B1

8. (*a*) An expression for $b^2 - 4ac$, with $b = (2k + 1)$, and at least one of a or c
  correct   M1
  $b^2 - 4ac = (2k + 1)^2 - 4(1)(k^2 + k + 1)$ A1
  $b^2 - 4ac = -3$ (or < 0, convincing)   A1
  $b^2 - 4ac < 0 \Rightarrow$ no real roots   (f.t. one slip)   A1

  (*b*) Finding critical points $x = -3$, $x = -\frac{1}{2}$ B1
  $-3 < x < -\frac{1}{2}$ **or** $-\frac{1}{2} > x > -3$ **or** $(-3, -\frac{1}{2})$ **or** $-3 < x$ and $x < -\frac{1}{2}$
  or a correctly worded statement to the effect that x lies strictly between -3 and $-\frac{1}{2}$
  (f.t. candidate's critical points)   B2
  Note: $-3 \leq x \leq -\frac{1}{2}$,
  $-3 < x, x < -\frac{1}{2}$
  $-3 < x \ \ x < -\frac{1}{2}$
  $-3 < x$ or $x < -\frac{1}{2}$
  all earn B1

9. (*a*)
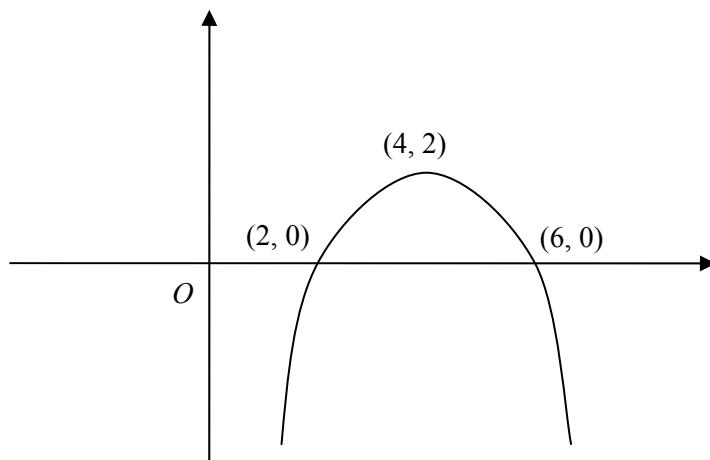
Concave down curve with stationary point at $(a, 2)$, $a \neq 1$ B1
$a = 4$ B1
Points of intersection with *x*-axis (2, 0), (6, 0)   B1

(*b*)
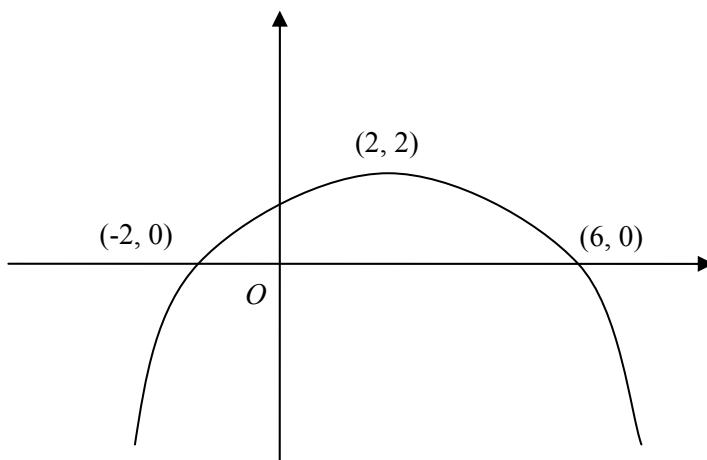
Concave down curve with positive intercept on *y*-axis   B1
Stationary point (2, 2)   B1
Points of intersection with *x*-axis (–2, 0), (6, 0)   B1

10. $\dfrac{dy}{dx} = 3x^2 - 2x - 1$ B1

Putting derived $\dfrac{dy}{dx} = 0$ M1

$(3x + 1)(x - 1) = 0 \Rightarrow x = -1/3, x = 1$ (both roots required)

                 (f.t. candidate's $\dfrac{dy}{dx}$)   A1

Stationary points are (–1/3, 59/27) and (1, 1)   (both correct)   (c.a.o.)   A1
A correct method for finding nature of stationary points   M1
(–1/3, 59/27) is a maximum point   (f.t. candidate's derived values)   A1

(1, 1) is a minimum point   (f.t. candidate's derived values)   A1

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MATHEMATICS C2

1.

0	1		
$\pi/8$	1·175875602		
$\pi/4$	1·306562965		
$3\pi/8$	1·387039845	(3 values correct)	B1
$\pi/2$	1·414213562	(5 values correct)	B1

Correct formula with $h = \pi/8$ M1

$I \approx \dfrac{\pi/8}{2} \times \{1 + 1·414213562 + 2(1·175875602 + 1·306562965 + 1·387039845)\}$

$I \approx 1·994$ (f.t. one slip)   A1

Special case for candidates who put $h = \pi/10$

0	1		
$\pi/10$	1·144122806		
$\pi/5$	1·260073511		
$3\pi/10$	1·344997024		
$2\pi/5$	1·396802247		
$\pi/2$	1·414213562	(all values correct)	B1

Correct formula with $h = \pi/10$ M1

$I \approx \dfrac{\pi/10}{2} \times \{1 + 1·414213562 + 2(1·144122806 + 1·260073511 + 1·344997024$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + 1·396802247)\}$

$I \approx 1·9$ (f.t. one slip)   A1

2.   (*a*) $3x = 60°, 240°, 420°, 600°$ (one value)   B1

             $x = 20°, 80°, 140°$ B1, B1, B1

             Note: Subtract 1 mark for each additional root in range,
                     ignore roots outside range.

   (*b*) $4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$ (correct use of $\sin^2\theta = 1 - \cos^2\theta$)   M1

             An attempt to collect terms, form and solve quadratic equation
             in $\cos\theta$, either by using the quadratic formula or by getting the
             expression into the form $(a\cos\theta + b)(c\cos\theta + d)$,
             with $a \times c$ = coefficient of $\cos^2\theta$ and $b \times d$ = constant   m1

             $6\cos^2\theta - \cos\theta - 2 = 0 \Rightarrow (3\cos\theta - 2)(2\cos\theta + 1) = 0$

             $\Rightarrow \cos\theta = \underline{\dfrac{2}{3}}, \underline{-\dfrac{1}{2}}$ A1

             $\theta = 48·19°, 311·81°, 120°, 240°$ (48·19°, 311·81°)   B1
                                          (120°)   B1
                                          (240°)   B1

             Note: Subtract 1 mark for each additional root in range for each
                   branch, ignore roots outside range.
                   $\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks
                   $\cos\theta = +, +$, f.t. for 1 mark

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3.   (a) $7^2 = x^2 + (3x)^2 - 2 \times x \times 3x \times \cos 60°$ (correct use of cos rule)   M1
$7^2 = x^2 + 9x^2 - 3x^2$ A1
$7x^2 = 7^2 \Rightarrow x = \sqrt 7$ (convincing)   A1

     (b)   **Either:**

$\dfrac{7}{\sin 60°} = \dfrac{\sqrt 7}{\sin ACB}$ (correct use of sin rule)   M1

$ACB = 19 \cdot 11°$ A1

       Or:
$(\sqrt 7)^2 = 7^2 + (3\sqrt 7)^2 - 2 \times 7 \times (3\sqrt 7) \times \cos ACB$

                             (correct use of cos rule)   M1
$ACB = 19 \cdot 11°$ A1

4.   (a) $a + 2d = k(a + 5d)$ $(k = 4, \ ^1/_4)$ M1
$a + 2d = 4(a + 5d)$ A1
$\dfrac{20}{2}[2a + 19d] = 350$ B1

An attempt to solve simultaneous equations   M1
$d = 5$ $(a = -30)$ (c.a.o.)   A1
$a = -30$ $(d = 5)$ (f.t. one error)   A1

     (b) $-30 + (n - 1) \times 5 = 125$ (equation for n'th term and an
                                    attempt to solve, f.t. candidate's values for a, d)   M1
$n = 32$ (c.a.o.)   A1

5.   (a) $S_n = a + ar + \ldots + ar^{n-2} + ar^{n-1}$ (at least 3 terms, one at each end)   B1
$rS_n = ar + ar^2 + \ldots ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract)   M1
$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing)   A1

$S_\infty = \dfrac{a}{1 - r}$ B1

     (b)   (i) $\dfrac{a}{1 - r} = 10$ B1

                $\dfrac{a}{1 - 2r} = 15$ B1

                An attempt to eliminate a M1
                $r = 0 \cdot 25$ (c.a.o.)   A1

         (ii) $a = 7 \cdot 5$ B1
                $S_4 = \dfrac{7 \cdot 5[1 - 0 \cdot 25^4]}{1 - 0 \cdot 25}$

                         (award even if sum calculated for 2nd series)   M1
                $S_4 \approx 9 \cdot 96$ (f.t. candidate's derived values of a, r)   A1

6.   (*a*) $\dfrac{2x^{5/2}}{5/2} + \dfrac{9x^{-3}}{-3}$ $(+ c)$ B1,B1

      (*b*)   (i) $x^2 + 2 = 3x$ M1
                       An attempt to rewrite and solve quadratic equation
                       in *x*, either by using the quadratic formula or by getting the
                       expression into the form $(x + a)(x + b)$, with $a \times b = 2$ m1
                       $(x - 2)(x - 1) = 0 \Rightarrow x = 1, x = 2$ (both values, c.a.o.)   A1

                     $A(1, 3), B(2, 6)$ (both values, f.t. candidate's *x* values)   A1

          (ii)

                 Either:

                 Total area $= \displaystyle\int_0^1 3x\,\mathrm{d}x + \int_1^2 (x^2 + 2)\,\mathrm{d}x$ (use of integration)   M1

                                   (addition of integrals)   m1

                     $= \left[(3/2)x^2\right]_0^1 + \left[(1/3)x^3 + 2x\right]_1^2$ (correct integration)   B3

                     $= [3/2 - 0] + [(8/3 + 4) - (1/3 + 2)]$
                                 (use of candidate's 0, x_A, x_A, x_B as limits)   M1
                     $= 35/6$ (c.a.o.)   A1
                Or:
                Area of triangle $= 3/2$ (f.t. candidate's coordinates for *A*)   B1

                Area under curve $= \displaystyle\int_1^2 (x^2 + 2)\,\mathrm{d}x$ (use of integration)   M1

                     $= \left[(1/3)x^3 + 2x\right]_1^2$ (correct integration)   B2

                     $= (8/3 + 4) - (1/3 + 2)$
                               (use of candidate's x_A, x_B as limits)   M1
                    $= 13/3$

               Finding total area by adding values   m1
                     Total area $= 3/2 + 13/3 = 35/6$ (c.a.o.)   A1

7. (*a*) (i) Let $x = \log_a p$, $y = \log_a q$

Then $p = a^x$, $q = a^y$ (relationship between log and power) B1

$pq = a^x \times a^y = a^{x+y}$ (the laws of indicies) B1

$\log_a pq = x + y = \log_a p + \log_a q$ (convincing) B1

(ii) $\log_a x + \log_a (3x + 4) = \log_a x(3x + 4)$ (addition law) B1

$2 \log_a (3x - 4) = \log_a (3x - 4)^2$ (power law) B1

$x(3x + 4) = (3x - 4)^2$ (removing logs) M1

An attempt to rearrange and solve quadratic m1

$3x^2 - 14x + 8 = 0 \Rightarrow x = 2/3, x = 4$ (c.a.o.) A1

(*b*) **Either:**

$x \log_{10} 3 = \log_{10} 11$ B1

$x = \dfrac{\log_{10} 11}{\log_{10} 3} \Rightarrow x \approx 2{\cdot}183$ B1

Or:

$x = \log_3 11$ B1

$x \approx 2{\cdot}183$ B1

8. (*a*) $A(-2, 8)$ B1

A correct method for finding the radius M1

Radius = $\sqrt{50}$ A1

(*b*) An attempt to substitute $(x + 2)$ for y in the equation of the circle M1

$x^2 - 4x - 5 = 0$ (or $2x^2 - 8x - 10 = 0$) B1

$x = -1, x = 5$ (correctly solving candidate's quadratic, both values) A1

Points of intersection are $(-1, 1), (5, 7)$ (c.a.o.) A1

9. (*a*) Length of arc = 6θ B1

Circumference of circle = $2 \times \pi \times 6$ B1

$2 \times \pi \times 6 = 2 \times 6\theta + 24$

(expression of the given information as an equation using the candidate's
expressions for length of arc and circumference) M1

$\theta = \pi - 2$ (convincing) A1

(*b*) **Either:**

Area of unshaded sector = $(1/2) \times 6^2 \times (\pi - 2)$ [= 20·55] B1

Area of shaded sector = $36\pi - (1/2) \times 6^2 \times (\pi - 2)$

(f.t. candidate's expression/value for area of unshaded sector) M1

Area of shaded sector = 92·55 cm^2 (c.a.o.) A1

Or:

Angle in shaded sector = $2\pi - (\pi - 2)$ [= 5·14] B1

Area of shaded sector = $(1/2) \times 6^2 \times [2\pi - (\pi - 2)]$ M1

Area of shaded sector = 92·52 cm^2 (c.a.o.) A1

Accept answers in the interval [92·5, 92·6]

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MATHEMATICS C3

1. $h = 0·1$ M1 ($h = 0·1$ correct
                                                                                                              formula

Integral $\approx \dfrac{0·1}{3}$ [0·5 + 0·4279957 + 4(0·4772563 + 0·4420154)   B1 (3 values)

                       + 2(0·4582276)]   B1 (2 values)

             $\approx$ 0·184   A1 (F.T. one slip)

                                                                                                                                    4

2.   *(a)* $\theta = 0$ for example   B1 (choice of values
                   $\cos \theta = 1$ and attempt to use)

                   $1 - 2 \cos^2\theta = -1$ B1 (for correct
                                                                                                              demonstration)

                   $(\therefore \cos 2\theta \neq 1 - 2\cos^2\theta)$

         (b) $\text{cosec}^2\theta - 1 = 7 - 2 \, \text{cosec} \, \theta$ M1 (substitution of $\cot^2\theta =$
                                                                                                              $\text{cosec}^2\theta - 1$)

                   $\text{cosec}^2\theta + 2 \, \text{cosec} \, \theta - 8 = 0$ M1 (attempt to solve)
                   $(\text{cosec} \, \theta + 4)(\text{cosec} \, \theta - 2) = 0$

                   $\text{cosec} \, \theta = -4, 2$

                   $\sin \theta = -\dfrac{1}{4}, \dfrac{1}{2}$ A1

                   $\theta = 194·5, 345·5, \ 30°, 150°$ B1 (194.5), B1 (345.5)
                                                                                                              B1 (30°, 150°)

                                                                                                                                    8

3.   *(a)*   (i) $\dfrac{dy}{dx} = \dfrac{5t^4 + 20t^2}{10t}$ M1 (attempt to use $\dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}}$)

                                                                                                              A1 A1

                   (ii) $\dfrac{5t^4 + 20t^2}{10t} = 1$ M1 (use of equation and attempt

                                                                                                              to simplify)

                           $\dfrac{t^3 + 4t}{2} = 1$

                           $t^3 + 4t - 2 = 0$ A1 (convincing)

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(b)

t	$t^3 + 4t - 2$
0	-2
1	3

Change of sign indicates presence of root between 0 and 1

M1 (attempt to find signs or values)

A1 (correct values or signs and conclusion)

$t_0 = 0\cdot5$, $t_1 = 0\cdot46875$

B1 (t_1)

$t_2 = 0\cdot4742508$, $t_3 = 0\cdot4733336$, $t_4 = 0\cdot4734880$ (0·4735)

B1 (t_4 to 4 decimal places)

Try 0·47345, 0·47355

t	$t^3 + 4t - 2$
0·47345	$-0\cdot00007$
0·47355	$0\cdot0004$

M1 (attempt to find signs or values)

M1 (correct values or signs)

Change of sign indicates root is 0·4735
(correct to 4 decimal places)

A1

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4. (a) Graph

B1

Graph

M1 (shape)
A1 ((0, 4))
A1 (($\pm$ 2, 0))

(b)
$$5x - 3 > 4$$
$$x > \frac{7}{5}$$

B1

or $\qquad 5x - 3 < -4$
$$x < -\frac{1}{5}$$

M1

A1 (must have 'or' in either part)

(o.e.)

<u>Alternatively</u>
$(5x - 3)^2 > 16$

M1 (forming quadratic and attempting to solve)

$25x^2 - 30x - 7 > 0$
$(5x + 1)(5x - 7) > 0$
$$-\frac{1}{5}, \frac{7}{5}$$

A1 (fixed points)

$x < -\frac{1}{5}$ or $x > \frac{7}{5}$ $\qquad$ (o.e.)

A1 (F.T. fixed points)

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5. $6y\dfrac{dy}{dx} + 2xy^3 + 3x^2y^2\dfrac{dy}{dx} + 4x^3 - 2x = 0$

B1 $(6y\dfrac{dy}{dx})$

B1 $(2xy^3 + 3x^2y^2\dfrac{dy}{dx})$

B1 (correct differentiation of x^4, x^2)

$\dfrac{dy}{dx} = -4$

B1 (F.T. one slip)

4

6.   *(a)*   (i) $x^2\cos x + 2x\sin x$

M1 $(x^2f(x) + \sin x g(x))$
A1 $(f(x) = \cos x,\ g(x) = 2x)$

(ii) $\dfrac{2x}{x^2 + 3}$

M1 $(\dfrac{f(x)}{x^2 + 3}, f(x) \neq 1)$, A1

(iii) $-2e^{9-2x}$

M1 (ke^{9-2x}) A1 $(k = -2)$

(iv) $-\dfrac{24}{(3x + 7)^3}$

M1 $(\dfrac{k}{(3x + 7)^3}$, allow unsimplified)

A1 (simplified answer)

(v) $-\dfrac{24}{\sqrt{1 - (3x)^2}}$ (o.e.)

M1 $(\dfrac{k}{\sqrt{1 - (3x)^2}})$ A1 $(k = 3)$

(S. Case allow B1 for $\dfrac{3}{\sqrt{1 - 3x^2}}$)

(b) $\dfrac{dy}{dx} = \dfrac{(1 - \tan x)(\sec^2 x) - (1 + \tan x)(-\sec^2 x)}{(1 - \tan x)^2}$

M1$((1 - \tan x)f(x) - \dfrac{(1 + \tan x)g(x)}{(1 + \tan x)^2}$

$= \dfrac{2\sec^2 x}{(1 - \tan x)^2}$

A1 $(f(x) = \sec^2 x\ g(x) = -\sec^2 x)$

A1 (simplified)

which is positive since $\sec^2 > 0,\ (1 - \tan x)^2 > 0$

B1 (convincing)

14

7.   *(a)*   (i) $-\dfrac{1}{2}\ln|5 - 2x|$ $(+ C)$

M1 A1

(ii) $\dfrac{(3x+2)^{21}}{63}$ $(+ C)$

M1 $(k\,(3x + 2)^{21})$

A2$(k=\dfrac{1}{63})$ or A1 $(k=\dfrac{1}{21}$ or $\dfrac{1}{3})$

(iii) $\dfrac{1}{7}e^{7x}$ $(+ C)$

M1 (ke^{7x}) A1 $(k = \dfrac{1}{7})$

(b) $\displaystyle\int_{0}^{\frac{\pi}{3}} \cos(3x + \frac{\pi}{3})dx$

$= \left[\dfrac{1}{3}\sin(3x + \dfrac{\pi}{3})\right]_{0}^{\frac{\pi}{3}}$

M1 $[k \sin (3x + \dfrac{\pi}{3})$, allow

$k = 1, \dfrac{1}{3}, -\dfrac{1}{3}, 3]$

A1 $(k = \dfrac{1}{3})$

$= \dfrac{1}{3}\sin\left(\dfrac{4\pi}{3}\right) - \dfrac{1}{3}\sin\dfrac{\pi}{3}$

m1 (F.T. $k(\sin \dfrac{4\pi}{3} - \sin\dfrac{\pi}{3})$)

$= -\dfrac{2}{3}\sin\dfrac{\pi}{3}$

$= -\dfrac{\sqrt{3}}{3}$ or $-0{\cdot}577$ correct to 3 decimal places $-(0{\cdot}578)$

A1 (C.A.O.)

11

8. (a) Range of $f\,[1, \infty)$, Range of $g\,[1, \infty)$

B1, B1

(b) $gf(x)$ $=$ $(e^x)^2 + 1$

M1 (correct order of composition)

$=$ $e^{2x} + 1$

A1

(c) domain of gf $[0, \infty)$

B1

range of gf $[2, \infty)$

B1

(d) Graph $y = e^x$

B1 (full curve with (0, 1)
B1 (truncated)

$y = e^{2x} + 1$

B1 (0, 2)
B1 (steeper curve)
B1 (truncated)

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9. *(a)* Let $y = \dfrac{8}{x+2}$

$(x+2)\,y = 8$ M1 (attempt to isolate x)

$x + 2 = \dfrac{8}{y}$

$x = \dfrac{8}{y} - 2$ A1

$f^{-1}(x) = \dfrac{8}{x} - 2$ A1 (F.T.)

(b) domain $(0, 4]$ B1

4

MATHEMATICS C4

1. *(a)* Let $\dfrac{x+3}{x^2(x-1)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1}$

M1 (correct form)

$\therefore x + 3 \equiv Ax(x-1) + B(x-1) + Cx^2$

M1 (clearing fractions and attempt to solve)

$\underline{x = 1}$ $4 = C,$ $C = 4$

A1 (2 constants)

$\underline{x = 0}$ $3 = -B,$ $B = -3$

A1 (other constant

Equate coefficients of x^2 $0 = A + C,$ $A = -4$

(F.T. if 2 Ms scored)

No need for display

(b) $\displaystyle\int -\frac{4}{x}dx - \int \frac{3}{x^2}dx + \int \frac{4}{x-1}dx$

$= -4\,ln(x) + \dfrac{3}{x} + 4\,ln\,|x-1|$ (o.e.)

B1, B1 (two logs)

$(+ C)$

6

2. $5x^4 + y^2 + 2xy\,\dfrac{dy}{dx} + 3y^2\,\dfrac{dy}{dx} = 0$

B1 $(y^2 + 2xy\,\dfrac{dy}{dx})$

B1 $(3y^2\,\dfrac{dy}{dx})$

$\dfrac{dy}{dx} = -\dfrac{2}{3}$ (o.e.)

B1 (C.A.O.)

Equation is $y - 3 = -\dfrac{2}{3}(x + 1)$

B1 (F.T. candidate's $\dfrac{dy}{dx}$)

4

3. $4\cos x + 2\sin x = R\cos(x - \alpha)$
$R\cos\alpha = 4,\ R\sin\alpha = 2$

M1 (for R cos $(x \pm \alpha)$)
B1 $\left(\sqrt{20}\right)$

$R = \sqrt{20},\ \alpha = \tan^{-1}\left(\dfrac{1}{2}\right) = 26.6°$

A1 (correct α for given

presentation)

$\cos(x - 26.6°) = \dfrac{3}{\sqrt{20}}$

M1 $(\cos(x \pm 26.6°) = \dfrac{3}{\sqrt{20}}))$

$x - 26.6° = 47.9°,\ 312.1°$

A1 (for one value) (C.A.O.)

$x = 74.4°,\ 338.7°$ (C.A.O.)

(accept 74, 75, 338°, 339°)
A1, A1

7

14

4. $(1+4x)^{\frac{1}{2}} - \dfrac{1}{1+3x}$

$$= 1 + \left(\frac{1}{2}\right)(4x) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\frac{1}{2!}(4x)^2 + ... \qquad\qquad \text{B2 (−1) each error}$$

$$- \left(1 - 3x + \frac{(-1)(-2)}{2}(3x)^2 + ...\right) \qquad\qquad \text{B2 (−1) each error}$$

(correct expansion of $(1+3x)^{-n}$)

$= 1 + 2x - 2x^2 + ...$

$\quad - 1 + 3x - 9x^2 + ... \qquad\qquad$ B2

$= 5x - 11\,x^2 + ... \qquad\qquad$ (−1 each error)

Expansion valid for $|x| < \dfrac{1}{4}$ \qquad\qquad (o.e.) \qquad B1

7

5. Volume $= \pi \displaystyle\int_0^1 (e^{2x} + 1)\ dx$ \qquad\qquad B1 (with or without limits,

after squaring $\sqrt{}$)

$$= \pi\left[\frac{e^{2x}}{2} + x\right]_0^1 \qquad\qquad \text{B1 (correct integration)}$$

$$= \pi\left[\frac{e^{2x}}{2} + 1 - \frac{1}{2}\right] \qquad\qquad \text{M1 (correct use of limits after}$$

attempted integration)

$\approx\ 13{\cdot}177$ \qquad\qquad A1 (C.A.O.)

4

6. *(a)* $\dfrac{dy}{dx} = \dfrac{2t}{2} = t$ M1 (correct attempt to find gradient)

Gradient of normal $= -\dfrac{1}{t}$ A1

Equation is

$$y - p^2 = -\dfrac{1}{p}(x - 2p)$$ M1 $(y - y_1 = m(x - x_1))$ o.e.

$$py - p^3 = -x + 2p$$

$$x + py = p^3 + 2p$$ A1 (convincing)

(b) <u>A</u> $y = 0,\ x = p^3 + 2p$ B1 (must be correct for A and B)

<u>B</u> $x = 0,\ y = p^2 + 2$ B1

$$p^3 + 2p\ =\ 2(p^2 + 2)$$ M1 (candidate's OA $= k \times$ candidate's
$$p\,(p^2 + 2) = 2(p^2 + 2)$$ OB, $k = \dfrac{1}{2}$ or 2)

$$p = 2$$ A1 (C.A.O.)

8

7. *(a)* $\displaystyle\int x^2 \ln x\,dx = \dfrac{x^3 \ln x}{3} - \int \dfrac{x^3}{3}\cdot\dfrac{1}{x}\,dx$ M1 (Parts and correct choice of *u,v*)

$$= \dfrac{x^3 \ln x}{3} - \int \dfrac{x^2}{3}\,dx$$ A1

M1 (division)

$$= \dfrac{x^3 \ln x}{3} - \dfrac{x^3}{9}\qquad (+\,C)$$ A1 (C.A.O.)

(b) $dx = 2\cos\theta\,d\theta$

When $x = 0,\ \theta = 0$

$$x = \sqrt{2},\ \sin\theta = \dfrac{1}{\sqrt{2}},\ \theta = \dfrac{\pi}{4}$$ B1

$$\int_0^{\frac{\pi}{4}} \dfrac{4\sin^2\theta}{\sqrt{4 - 4\sin^2\theta}}.2\cos\theta\,d\theta$$ M1 (substitution for *dx* and *x*)

A1 (any limits)

$$= \int_0^{\frac{\pi}{4}} \dfrac{4\sin^2\theta}{2\sqrt{1 - \sin^2\theta}}.2\cos\theta\,d\theta$$

16

$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^2\theta\, 2\cos\theta\, d\theta}{2\cos\theta}$$

$$= \int_0^{\frac{\pi}{4}} 4\sin^2\theta\, d\theta$$

A1 (convincing, proof of $k = 4$,

any limits)

$$\int = \int_0^{\frac{\pi}{4}} 2(1 - \cos 2\theta)\, d\theta$$

M1 ($a + b\cos 2\theta$)

A1 ($a = \dfrac{k}{2}$, $b = -\dfrac{k}{2}$)

$$= \left[2\theta - \sin 2\theta\right]_0^{\frac{\pi}{4}}$$

A1 (correct integration of two terms)

$$= \frac{\pi}{2} - 1(\approx 0\cdot 571)$$

A1 (either answer, C.A.O.)

12

8.   *(a)* $\dfrac{dP}{dt} = kP$

B1

(b) $\displaystyle\int \frac{dP}{P} = \int k\,dt$

M1 (Separation of variables and $\int$)

ln P = *kt* + C
t = 0, *P* = 50
ln 50 = C
ln P – *ln* 50 = *kt*

A1
F.T. right hand side)
M1 (attempt to find *C*)

$ln\, \dfrac{P}{50} = kt$

M1 (combination of logs and

$\dfrac{P}{50} = e^{kt}$

attempt to exponentiate)

$P = 50e^{kt}$

A1

(c) $65 = 50\, e^{7k}$

M1 (taking logs correctly)

$ln\, \dfrac{65}{50} = 7k$ $k = 0.03748$

A1

After sixteen years, $P = 50 \exp (\cdot03748 \times 16)$
            $\approx$ £91 (nearest pound)

M1 (use of formula)
A1 (C.A.O.)

10

17

9. (a) (i) $\mathbf{AB} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} - (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ M1 (**b** – **a**)

           $= 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ A1

      (ii) Equation of AB is   M1 (**a** + λ (**b** – **a**))

           $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda\,(2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$ (o.e.)   A1 (must involve **r** or **OP** = ...
         F.T. **AB**)

      (iii) (The point lies on both lines)

           $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda\,(2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$

             $= 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu\,(\mathbf{i} + \mathbf{j} + 4\mathbf{k})$

           $1 + 2\,\lambda = 2 + \mu$ M1 (attempt to equate components,
         one correct)

           $3 + 3\,\lambda = 3 + \mu$ A1 (other correct)
         M1 (correct attempt to solve)
           $\lambda = -1,\ \mu = -3$ A1 (F.T. candidate's equations)

           Position vector of point of intersection is $-\mathbf{i} - 5\mathbf{k}$ A1 (C.A.O.)

    (b) $\cos\theta = \dfrac{(\mathbf{i} + 2\mathbf{j} - \mathbf{k}).(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})}{|\,\mathbf{i} + 2\mathbf{j} - \mathbf{k}\,\|\,3\mathbf{i} - \mathbf{j} + 2\mathbf{k}\,|}$ M1 (attempt to use correct

         formula)

         $= \dfrac{1 \times 3 - 2 \times 1 - 1 \times 2}{\sqrt{1^2 + 2^2 + (-1)^2}\ \sqrt{3^2 + 1^2 + 2^2}}$ M1 (correct attempt to find scalar

         product)
         A1 (scalar product)

         $= \dfrac{-1}{\sqrt{6}\,\sqrt{14}}$ B1 (one correct modulus)

         B1 (F.T. arithmetic slip
         in scalar product)

         $\theta = 96 \cdot 3°$ (accept nearest degree)   B1 (C.A.O.)

                                                                                  15

10. $3n + 2n^{3} = 3(2k) + 2(2k)^3$

                $= 6k + 16k^3$

                $= 2(3k + 8k^3)$ B1 (either 2x ( )
         or even + even = even)
         which is even   B1
         Contradiction
         (Thus n is odd)

                                                                                     2

MATHEMATICS FP1

1. $f(x + h) - f(x) = (x + h)^4 - x^4$ B1

    $= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4$ M1A1

    $= 4x^3h + 6x^2h^2 + 4xh^3 + h^4$ A1

    $\dfrac{f(x + h) - f(x)}{h} = 4x^3 + 6x^2h + ...$ M1

    $f'(x) = \displaystyle\lim_{h \to 0} \; 4x^3 + 6x^2h + ...$ A1

    $= 4x^3$ AK

2. $\dfrac{(1 + 7i)}{(3 + i)} = \dfrac{(1 + 7i)(3 - i)}{(3 + i)(3 - i)} = 1 + 2i$ M1A1

    $2z + \bar{z} = 2(x + iy) + x - iy = 3x + iy$ M1A1

    Equating real and imaginary parts,   M1
    $\qquad\qquad\qquad x = 1/3, y = 2$ A1

3. *(a)* One of the other roots is $2 - 3i$.   B1
    Let the 3rd root be r.
    Then, $(2 + 3i)(2 - 3i)r = 13r = -26$,   M1A1
    $\qquad\qquad\qquad r = -2$ A1

    (b) $p = -$ sum of roots $= -2$ M1A1
    $q = (2 + 3i)(2 - 3i) - 2(2 + 3i + 2 - 3i) = 5$ M1A1

4. *(a)* $T_n = 3n^2 + 2n - 3(n - 1)^2 - 2(n - 1)$ M1

    $= 3n^2 + 2n - 3n^2 + 6n - 3 - 2n + 2$ A1

    $= 6n - 1$ AG

    [Do not accept a solution which 'goes backwards']

    (b) $\text{Sum} = \displaystyle\sum_{r=1}^{n}(6r - 1)^2 = 36\sum_{r=1}^{n}r^2 - 12\sum_{r=1}^{n}r + \sum_{r=1}^{n}1$ M1A1

    $= \dfrac{36n(n + 1)(2n + 1)}{6} - \dfrac{12n(n + 1)}{2} + n$ A1

    $= 12n^3 + 18n^2 + 6n - 6n^2 - 6n + n$ A1

    $= 12n^3 + 12n^2 + n$ A1

5. The statement is true for $n = 1$ since the formula gives the correct answer 1/2.   B1
Let the statement be true for $n = k$, ie

$$\sum_{r=1}^{k} r \times \left(\frac{1}{2}\right)^r = 2 - (k+2)\left(\frac{1}{2}\right)^k \quad \text{M1}$$

Consider $\displaystyle\sum_{r=1}^{k+1} r \times \left(\frac{1}{2}\right)^r = 2 - (k+2)\left(\frac{1}{2}\right)^k + (k+1)\times\left(\frac{1}{2}\right)^{k+1}$ M1A1

$$= 2 - \left(\frac{1}{2}\right)^{k+1}(2k+4-k-1)$$ A1

$$= 2 - (k+1+2)\left(\frac{1}{2}\right)^{k+1}$$ A1

Thus, true for $n = k \Rightarrow$ True for $n = k + 1$.   A1
Since the statement is true for $n = 1$ and true for k implies true for $k + 1$,
the statement is proved to be true by mathematical induction.   A1

6. $\ln y = x \ln x$ M1

$\dfrac{1}{y}\dfrac{dy}{dx} = \ln x + 1$ m1A1

$\dfrac{dy}{dx} = x^x(\ln x + 1)$ A1

$\dfrac{d^2 y}{dx^2} = x^x(\ln x + 1)(\ln x + 1) + x^x.\dfrac{1}{x}$ M1A1A1

whence the printed result.   AG

7.   *(a)*   Det $= 2(-20 - 8) + 1(1 + 15) + 2(24 - 4)$ M1A1
      $= 0$ therefore singular   A1

(b)   (i)   Using reduction to echelon form,
            $2x + y + 2z = 3$ M1
            $5y - 4z = -7$ A1
            $15y - 12z = 2k - 3$ A1
      For consistency, we require
            $2k - 3 = -21$ M1
         $k = -9$ A1

(ii)   Put $z = \alpha$.   M1
      $y = \dfrac{4\alpha - 7}{5}$ A1

      $x = \dfrac{11 - 7\alpha}{5}$ M1A1

20

8. (a) Under T_1, $(1, 0) \to (a, c)$ and $(0, 1) \to (b, d)$.   B1B1

We are now given that $(1,0) \to (0, -1)$ and $(0, 1) \to (-1,0)$ M1

So $a = 0$, $c = -1$, $b = -1$, $d = 0$.   A1

(b) (i) $T_2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ M1

$$= \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ A1

(ii)   Fixed points satisfy

$$\begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ M1

giving

$-y + 2 = x$

$-x + 2 = y$ A1

These equations are consistent confirming that fixed points lie on the line   A1

$x + y = 2$ A1

(iii)   A reflection in the line $x + y = 2$.   B1

9. $u + iv = (x + iy)^2 = x^2 - y^2 + 2xyi$ M1

$u = x^2 - y^2 ; v = 2xy$ A1A1

Substituting for y,   M1

$u = x^2 - (2x^2 - 1) = 1 - x^2$ A1

$v^2 = 4x^2(2x^2 - 1)$ A1

Eliminating x,   M1

$$v^2 = 4(1 - u)(2 - 2u - 1)$$ A1

$= 4(1 - u)(1 - 2u)$ A1

MATHEMATICS FP2

1. $x = y^2 \Rightarrow dx = 2y\,dy$ and $[1, 4] \to [1, 2]$ B1B1

$$I = \int_1^2 \frac{2y\,dy}{\sqrt{y^2(9 - y^2)}}$$ M1

$$= 2\int_1^2 \frac{dy}{\sqrt{9 - y^2}}$$ A1

$$= 2\left[\sin^{-1}\left(\frac{y}{3}\right) \right]_1^2$$ A1

$$= 0.78$$ A1

2. EITHER $1 + \sqrt{3}i = 2(\cos\pi/3 + i\sin\pi/3)$ M1A1

Square root $= \sqrt{2}(\cos\pi/6 + i\sin\pi/6)$ M1A1

$\qquad\qquad\qquad = 1.22 + 0.71i$ A1

Other root $= -1.22 - 0.71i$. $(\pm\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i)$ A1

OR   Let $\sqrt{1 + \sqrt{3}i} = x + iy$ M1

$x^2 - y^2 = 1;\ 2xy = 3$ A1

$x^4 - x^2 - 3/4 = 0$ M1

$\qquad\qquad x^2 = 3/2$ A1

$\qquad\qquad x = \pm\sqrt{3/2}$ A1

$\qquad\qquad y = \pm 1/\sqrt{2}$ A1

3. (a)   Let $\dfrac{(x+1)(x+2)}{(x-1)(x^2+1)} \equiv \dfrac{A}{x-1} + \dfrac{Bx+C}{(x^2+1)}$ M1

$(x + 1)(x + 2) \equiv A(x^2 + 1) + (x - 1)(Bx + C)$ A1

$x = 1$ gives $A = 3$ A1

Coeff of x^2 gives $B = -2$ A1

Constant term gives $C = 1$ A1

(b) $\int f(x)\,dx = \int \left[\dfrac{3}{x-1} - \dfrac{2x}{x^2+1} + \dfrac{1}{x^2+1} \right] dx$ M1

$= 3\ln(x - 1) - \ln(x^2 + 1) + \tan^{-1} x\ \ (+C)$ A1A1A1

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4. $2\sin3\theta\cos\theta = \cos\theta$ — M1A1

$\cos\theta(2\sin3\theta - 1) = 0$ — M1A1

Either $\cos\theta = 0$ whence $\theta = (2n+1)\dfrac{\pi}{2}$ oe — M1A1

Or $\sin3\theta = \dfrac{1}{2}$ whence $3\theta = \dfrac{\pi}{6} + 2n\pi$ or $\dfrac{5\pi}{6} + 2n\pi$ — M1A1

So $\theta = \dfrac{\pi}{18} + \dfrac{2n\pi}{3}$ or $\dfrac{5\pi}{18} + \dfrac{2n\pi}{3}$ oe — A1

[Accept degrees and other alternative forms]

5. *(a)* The equation in standard form is

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$ — M1

so $a = 5$ and $b = 4$. — A1

The coordinates of the foci are $(\pm\sqrt{a^2 - b^2},0)$ ie $(\pm 3, 0)$ — M1A1

(b) $\dfrac{(5\cos\theta)^2}{25} + \dfrac{(4\sin\theta)^2}{16} = 1$ so the point lies on the ellipse. — B1

(c) (i) $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = -\dfrac{4\cos\theta}{5\sin\theta}$ — M1A1

Gradient of normal $= \dfrac{5\sin\theta}{4\cos\theta}$ — A1

Equation of normal is

$$y - 4\sin\theta = \frac{5\sin\theta}{4\cos\theta}(x - 5\cos\theta)$$ — M1

$4y\cos\theta - 5x\sin\theta + 25\sin\theta\cos\theta - 16\sin\theta\cos\theta$ — A1

Whence printed result.

(ii) Putting $y = 0$, cords of Q are $\left(\dfrac{9}{5}\cos\theta,0\right)$ — B1

Putting $x = 0$, cords of R are $\left(0,-\dfrac{9}{4}\sin\theta\right)$ — B1

Coords of M are $(x,y) = \left(\dfrac{9}{10}\cos\theta,-\dfrac{9}{8}\sin\theta\right)$ — B1

Eliminating θ,

$\dfrac{x^2}{(9/10)^2} + \dfrac{y^2}{(9/8)^2} = 1$, ie an ellipse — M1A1

[These 2 marks can be gained by noting that the above are the parametric equations of an ellipse]

6. (a) $\quad f'(x) = \dfrac{2x.x - (x^2 + 4)}{x^2} = \dfrac{x^2 - 4}{x^2}$ 　　　　　　　M1A1

The stationary points are $x = \pm 2$, $y = \pm 4$ 　　　　　　　A1A1

(b) The asymptotes are $x = 0$, $y = x$. B1B1

(c) Graph 　　　　　　　　　　　　　　　　　　　　　　　　G2

(d) $f(1) = 5, f(5) = 5.8$ 　　　　　　　　　　　　　　　　　M1
so upper end of $f(A) = 5.8$ 　　　　　　　　　　　　　　　A1
Since there is a minimum at (2,4), 　　　　　　　　　　　M1
Lower end of $f(A) = 4$ 　　　　　　　　　　　　　　　　　A1
ie $f(A) = [4,5.8]$

7. (a) $\quad z^n = \cos n\theta + i \sin n\theta$ 　　　　　　　　　　　　　B1

$$\dfrac{1}{z^n} = \dfrac{1}{\cos n\theta + i \sin n\theta} = \cos n\theta - i \sin n\theta$$ 　　　M1A1

[Accept $z^{-n} = \cos(-n\theta) + i\sin(-n\theta) = \cos(n\theta) - i\sin(n\theta)$]
Adding,

$$z^n + \dfrac{1}{z^n} = 2\cos n\theta$$ 　　　　　　　　　　　　　　AG

(b) $\quad \left(z + \dfrac{1}{z}\right)^5 = z^5 + 5z^4.\left(\dfrac{1}{z}\right) + 10z^3.\left(\dfrac{1}{z}\right)^2 + 10z^2.\left(\dfrac{1}{z}\right)^3 + 5z.\left(\dfrac{1}{z}\right)^4 + \left(\dfrac{1}{z}\right)^5$

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　M1A1

$$= z^5 + \dfrac{1}{z^5} + 5\left(z^3 + \dfrac{1}{z^3}\right) + 10\left(z + \dfrac{1}{z}\right)$$ 　　　A1

$$(2\cos\theta)^5 = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$$ 　　　M1

$$\cos^5\theta = \dfrac{1}{16}\cos 5\theta + \dfrac{5}{16}\cos 3\theta + \dfrac{5}{8}\cos\theta$$ 　　　A1

8. (a) For $x = 1$, the values given by the two forms of $f(x)$ are
$4.1^2 = 4$ and $(1+1)^2 = 4$ M1
These are equal so the function is continuous.   A1

(b) For $0 < x < 1$,
$f'(x) = 8x > 0$ for all x B1
For $1 \leq x < 2$,
$f'(x) = 2(x+1) > 0$ for all x B1

(c) For $0 < x < 1$ and therefore $0 < y < 4$, put
$y = x^2$ so $x = \sqrt{y}$ M1A1
So for $0 < x < 4$,
$f^{-1}(x) = \sqrt{x/4}$ A1
[correct domain required]
For $1 \leq x < 2$ and therefore $4 \leq y < 9$, put
$y = (x+1)^2$ so $x = \sqrt{y}$ - 1   M1A1
So for $4 \leq x < 9$,
$f^{-1}(x) = \sqrt{x} - 1$ A1
[correct domain required]

MATHEMATICS FP3

1. $dx = 2\cosh\theta d\theta$; $[0,1] \to [\sinh^{-1}(1/2), \sinh^{-1}1]$ B1B1

$$I = \int_{\sinh^{-1}0.5}^{\sinh^{-1}1} \frac{2\cosh\theta d\theta}{\sqrt{4\sinh^2\theta - 4\sinh\theta + 1 + 4\sinh\theta - 2 + 5}}$$ M1A1

$$= \int_{\sinh^{-1}0.5}^{\sinh^{-1}1} \frac{2\cosh\theta d\theta}{2\cosh\theta}$$ A1

$$= [\theta]_{\sinh^{-1}0.5}^{\sinh^{-1}1}$$ A1

$$= \sinh^{-1}1 - \sinh^{-1}0.5$$ A1
$$= 0.400$$ A1

2. (a) $f'(x) = 3x^2 + 6x + 6$ B1

$= 3[(x+1)^2 + 1]$ M1
> 0 for all x.   A1
The equation has 1 real root.   B1

(b)(i) $f(0) = -5, f(1) = 5$, sign change means root in $(0,1)$ B1

(ii) The Newton-Raphson iteration is
$$x \to x - \frac{x^3 + 3x^2 + 6x - 5}{3x^2 + 6x + 6}$$ M1A1

Taking $x_0 = 0.5$, we obtain the following values   M1
0.6153846154   A1
0.6097099622
0.6096954941   A1
Required value = 0.6097   A1

3 (a) $2x + 2y\dfrac{dy}{dx} = 0$ M1

$$\frac{dy}{dx} = -\frac{x}{y}$$ A1

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{y^2}$$ M1

$$= \frac{x^2 + y^2}{y^2} = \frac{a^2}{y^2}$$ A1

(b) $\text{CSA} = 2\pi \int_{-a}^{a} y \cdot \sqrt{\frac{a^2}{y^2}} dx$ M1A1

$= 2\pi a [x]_{-a}^{a}$ A1
$= 2\pi a . 2a$ A1
$= 4\pi a^2$ AG

4. (a) At the point of intersection,

$e^{\theta} = 2e^{-\theta}$ — M1

$e^{2\theta} = 2$ — A1

$\theta = \dfrac{1}{2}\ln 2$ — A1

$r = \sqrt{2}$ — A1

(b) C2 Area $= \dfrac{1}{2}\displaystyle\int_{0}^{0.5\ln 2} 4e^{-2\theta}\,d\theta = \left[-e^{-2\theta}\right]_{0}^{0.5\ln 2}$ (1/2) — M1A1

C1 area $= \dfrac{1}{2}\displaystyle\int_{0}^{0.5\ln 2} e^{2\theta}\,d\theta = \dfrac{1}{4}\left[e^{2\theta}\right]_{0}^{0.5\ln 2}$ (1/4) — M1A1

Reqd area = difference = 1/4 — A1

5. (a) $a\cosh x + b\sinh x = r\cosh x\cosh\alpha + r\sinh x\sinh\alpha$ — M1

so $r\cosh\alpha = a$ and $r\sinh\alpha = b$ — A1

Dividing,

$\tanh\alpha = \dfrac{b}{a}$ — m1

$\alpha = \tanh^{-1}(b/a)$ — A1

$= \dfrac{1}{2}\ln\left(\dfrac{1+b/a}{1-b/a}\right)$ — A1

= printed result

Squaring and subtracting,

$r^{2}(\cosh^{2}\alpha - \sinh^{2}\alpha) = a^{2} - b^{2}$ — M1A1

$r = \sqrt{a^{2}-b^{2}}$ — A1

(b) In this case, $\alpha = \ln 2$ and $r = 4$ — B1

so $4\cosh(x + \ln 2) = 4$ — B1

$x + \ln 2 = 0$ so $x = -\ln 2$ — M1A1

[Accept solution using first principles giving M1A1 for reaching $4e^{2x} - 4e^{x} + 1 = 0$]

6. (a) $f'(x) = \dfrac{1}{\tan(\pi/4 + x)}.\sec^{2}(\pi/4 + x)$ — M1A1

$= \dfrac{1}{\sin(\pi/4 + x)\cos(\pi/4 + x)}$ — A1

$= 2\mathrm{cosec}(\pi/2 + 2x)$ — A1

$= 2\sec 2x$ — AG

(b) $f(0) = 0$ B1
$f'(0) = 2$ B1
$f''(x) = 4\sec 2x\tan 2x ,(\, f''(0) = 0)$ B1
$f'''(x) = 8\sec^3 2x + 8\sec 2x\tan^2 2x$ B1
$f'''(0) = 8$ B1

The series expansion is

$$f(x) = 2x + \frac{4}{3}x^3 + ...$$ M1A1

[FT on values of derivatives]

(c) $2x + \frac{4}{3}x^3 = 10x^3$ M1

$x^2 = \frac{3}{13}$ A1

$x = 0.48$ A1

7.   (a) $I_n = -\int_0^1 x^n \, d\left(-\frac{2}{5}(1-x)^{5/2}\right)$ M1

$= \left[-\frac{2}{5}x^n(1-x)^{5/2}\right]_0^1 + \int_0^1 \frac{2n}{5}x^{n-1}(1-x)^{5/2}\,dx$ A1A1

$= \frac{2n}{5}\int_0^1 x^{n-1}(1-x)(1-x)^{3/2}\,dx$ A1A1

$= \frac{2n}{5}I_{n-1} - \frac{2n}{5}I_n$ A1

$\left(1 + \frac{2n}{5}\right)I_n = \frac{2n}{5}I_{n-1}$ A1

whence the printed result

(b) $I_0 = \int_0^1 (1-x)^{3/2}\,dx$ M1

$= \left[-\frac{2}{5}(1-x)^{5/2}\right]_0^1$ A1

$= \frac{2}{5}$ A1

$I_2 = \frac{4}{9}I_1$ B1

$= \frac{4}{9}\cdot\frac{2}{7}I_0$ B1

$= \frac{16}{315}(0.05)$ B1

MATHEMATICS M1

1. (a)   Using $v = u + at$ with $u = 5$, $a = 0.6$, $t = 25$ M1
           $v = 5 + 0.6 \times 25$ A1
           $v = \underline{20 \text{ ms}^{-1}}$ A1

    (b)
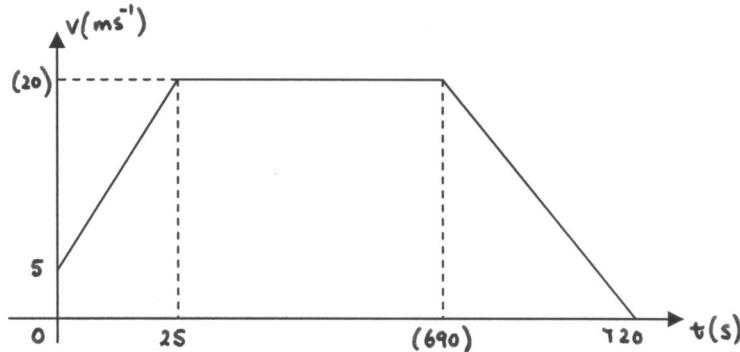
                                                                                           M1 A1 A1

    (c)   Using $v = u + at$ with $u = 20$, $v = 0$, $t = 30$ M1
           $a = -\dfrac{20}{30}$

           magnification of deceleration $= -\dfrac{2}{3}$ ft(a)   A1

    (d)   Distance = Area under graph   used   M1
           Distance $= 0.5 (5 + 20) 25 + 20(690 - 25) + 0.5 (20)(30)$ B1(ft)
           Distance $= \underline{13912.5\text{m}}$ A1(ft)

2. (a)   Using $s = ut + 0.5at^2$ with $s = (-)1.75$, $a = (-)9.8$, $t = 2.5$ M1
           $-1.75 = 25u - 4.9 \times 2.5^2$ A1
           $u = \underline{11.55\text{ms}^{-1}}$

    (b)   Using $v^2 = u^2 + 2as$ with $v = 0$, $u = 11.55$(c), $a = (-)9.8$ M1
           $0 = 11.55^2 - 2 \times 9.8s$ A1
           $s = 6.80625$
           Therefore greatest height above ground $= 6.80625 + 1.75$
                                                  $= \underline{8.55625 \text{ m}}$ cao   A1

    (c)   Using $v = u + at$ with $u = 11.55$(c), $a = (-)9.8$, $t = 2.5$ M1
           $v = 11.55 - 9.8 \times 2.5$ A1
           $v = \underline{-12.95 \text{ ms}^{-1}}$
           Therefore speed $= \underline{12.95 \text{ ms}^{-1}}$ cao   A1

    (d)   Speed after bounce $= 0.8 \times 12.95$ M1
                             $= \underline{10.36 \text{ ms}^{-1}}$ ft(c)   A1

1

3.
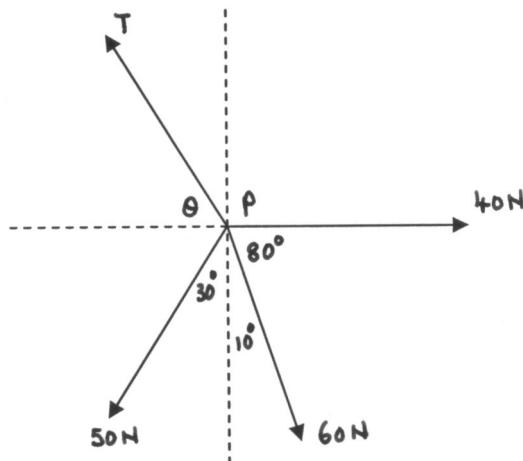
Resolve in any direction.	M1
$T\cos\theta + 50\sin 30^\circ = 40 + 60\cos 80^\circ$	B1 A1
$\qquad T\cos\theta = 40 - 25 + 60\cos 80^\circ$	
$\qquad\qquad = 15 + 60\cos 80^\circ$	
Resolve in a direction to obtain independent equation	M1
$T\sin\theta = 50\cos 30^\circ + 60\cos 10^\circ$	A1 A1
sensible attempt to eliminate variable.	M1

$$\tan\theta = \frac{50\cos 30^\circ + 60\cos 10^\circ}{15 + 60\cos 80^\circ}$$

$\theta = \underline{76.06^\circ}$	cao	A1

$$T = \sqrt{(15 + 60\cos 80^\circ)^2 + (50\cos 30^\circ + 60\cos 10^\circ)^2}$$

$T = \underline{105.5\ N}$	cao	A1

4.
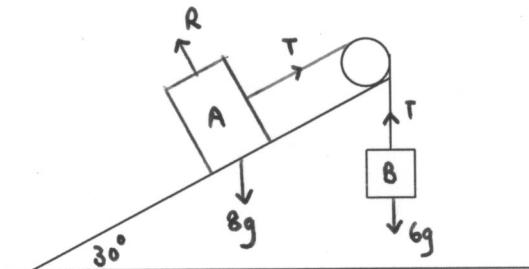
(a)	Apply N2L to *B*		M1
	$6g - T = 6a$		A1
	Apply N2L to *A*		M1
	$T - 8g\sin 25^\circ = 8a$		A1
	Adding $6g - 8g\sin 25^\circ = 14a$		m1
	$\qquad a = 1.4\ \text{ms}^{-2}$	cao	A1
	$\qquad T = 6(g - a)$		
	$\qquad T = \underline{50.4\ N}$	cao	A1
(b)	Magnitude of acceleration of *A* and *B* are equal.		B1

5.
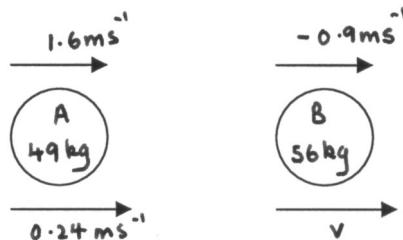
(a)   Conservation of momentum   M1

$49 \times 1.6 - 56 \times 0.9 = 49 \times 0.24 + 56v$ A1

                $v = \underline{0.29}$ convincing   A1

(b)   Restitution   M1

$0.29 - 0.24 = -e(-0.9 - 1.6)$ A1

         $e = \dfrac{0.05}{2.5} = 0.02$ cao   A1

(c) $|I| = 56(0.29 + 0.9)$ M1

        $= \underline{66.64 \text{ Ns}}$ A1 B1

(d)   Objects are modelled as particles.   B1

6.
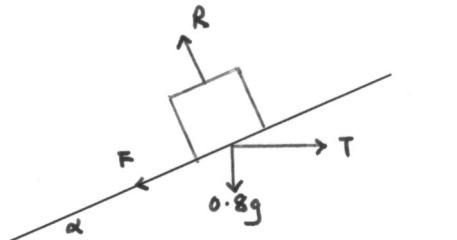
Resolve vertically   all forces, dim. correct   M1

     $R \cos \alpha = 0.8g + F \sin \alpha$ A1

       $F = 0R$ M1

    $0.8R = 0.8g + 0.4R \times 0.6$

       $R = 14 \text{ N}$ substitution of F A1

Resolve horizontally   all forces, dim. Correct   M1

     $R \sin \alpha + F \cos \alpha = T$ A1

    $14 \times 0.6 + 0.4 \times 14 \times 0.8 = T$ elimination of variable   m1

    $T = \underline{12.88 \text{ N}}$ cao   A1

3

7.
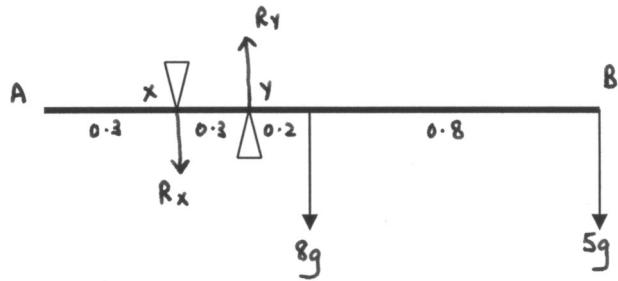
Moments about Y	all forces	M1
$0.3\,R_X = 0.2 \times 8g + 1 \times 5g$		A1 B1
$R_X = 22g$		
$R_X = \underline{215.6\ N}$	cao	A1
Resolve vertically		M1
$R_Y = R_X + 8g + 5g$		A1
$R_Y = 35g = \underline{343\ N}$	ft R_X only	A1

8. *(a)*

Particle	mass	from AC	from AB	
P	2m	7.5	0	B1
Q	3m	2.4	4.2	B1
R	5m	0	3.5	B1

(i) Moments about AC equation   M1

$10\,\bar{x} = 7.5 \times 2 + 2.4 \times 3 + 0 \times 5$ ft   A1

$10\,\bar{x} = 22.2$

$\bar{x} = \underline{2.22\ cm}$ cao   A1

(ii) Moments about AB M1

$10\,\bar{y} = 0 \times 2 + 4.2 \times 3 + 3.5 \times 5$ ft   A1

$10\,\bar{y} = 30.1$

$\bar{y} = \underline{3.01\ cm}$ cao   A1

(b) $\theta = \tan^{-1}\left(\dfrac{3.01}{8-2.22}\right)$ ft   M1 A1

$\theta = \underline{27.51^{\circ}}$ ft   A1

4

MATHEMATICS M2

1. *(a)* When P is at rest, $v = 0$.   used   M1

$$3t^2 - 24t + 45 = 0$$
$$3(t - 3)(t - 5) = 0$$

Therefore P first comes to rest when $t = 3$.   A1

(b) $a = \dfrac{dv}{dt}$ used   M1

     $a = \underline{6t - 24}$ A1

(c) Displacement $s = \displaystyle\int v \, dt$ used   M1

         $s = t^3 - 12t^2 + 45t \; (+ \, C)$ A1

When $t = 0$, $s = 0$, therefore $C = 0$

         $s = t^3 - 12t^2 + 45t$ A1

(d) Distance travelled in the first 3 s $= 3^3 - 12(3)^2 + 45(3)$ M1

                              $= 27 - 108 + 135$

                              $= \underline{54 \text{ m}}$ ft   A1

(e) Displacement after 4s $= 4^3 - 12(4)^2 + 45(4)$

                        $= \underline{52 \text{ m}}$ B1

Distance travelled in 4 s $= 54 + (54 - 52)$

                        $= \underline{56 \text{ m}}$ B1

2.
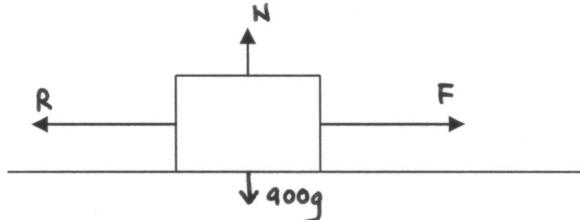
(a) At maximum speed $F = R$ used   M1

                 $F = \dfrac{P}{V}$ used   M1

Therefore $1800 = \dfrac{45 \times 1000}{V}$

                 $V = \underline{25 \text{ ms}^{-1}}$ cao   A1

(b) N2L   all forces, dim. cor.   M1

$F - R - mg \sin 4° = ma$ A1

$F = \dfrac{45000}{15} \; (= 3000)$ B1

$3000 - 1800 - 900 \times 9.8 \sin 4° = 900 \times a$ A1

                     $a = \underline{0.65 \text{ ms}^{-2}}$ cao   A1

(c)   W.D. $= F.d$ used   M1

     $= 1800 \times 800 = \underline{1440000 \text{ J}}$ cao   A1

3. (a)   PE at start of motion $= 3 \times 9.8 \times (0.8 + 0.4)$ M1 A1
                                  $= 35.28$ J

          EE at end of motion $= \dfrac{1}{2} \lambda \dfrac{(0.4)^2}{0.8}$ M1 A1

                              $= 0.1\, \lambda$

          Energy consideration $=$ M1
                     $0.1\, \lambda = 35.28$ A1
                     $\lambda = \underline{352.8 \text{ N}}$ convincing   A1

    (b)   Hooke's Law $T = \dfrac{\lambda x}{l}$ used   M1

                              $= \dfrac{352.8 \times 0.4}{0.8}$

                              $= \underline{176.4 \text{ N}}$ cao   A1

          N2L   dim cor.   M1
                     $T - 3g = 3a$ A1
               $176.4 - 3 \times 9.8 = 3a$
                     $a = \underline{49 \text{ ms}^{-2}}$ A1

4.
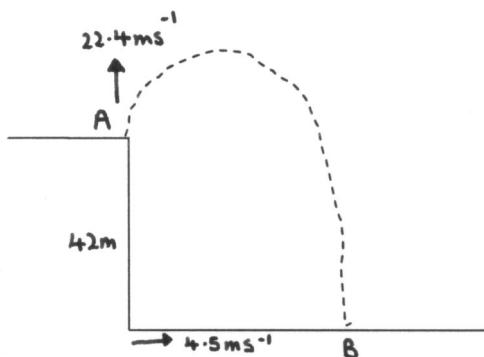
    (a)   Consider vertical motion
           Using $v = u + at$ with $u = 22.4$, $a = (-)9.8$, $t = 2$.   M1
                     $v = 22.4 - 9.8 \times 2$ A1
           $v = 2.8 \text{ms}^{-1}$ cao   A1

           Speed $= \sqrt{4.5^2 + 2.8^2}$ M1
                     $= \underline{5.3 \text{ ms}^{-1}}$ ft v A1

    (b)   Using $s = ut + 0.5at^2$ with $s = (-)42$, $u = 22.4$, $a = (-)9.8$ M1
                     $-42 = 22.4t - 4.9t^2$ A1
           $0.7t^2 - 3.2t - 6 = 0$

           $t = \dfrac{3.2 \pm \sqrt{3.2^2 + 4 \times 0.7 \times 6}}{2 \times 0.7}$ m1

           $T = \underline{6 \text{ s}}$ (other solution negative)   A1

   (c)   Horizontal distance between A and B $= 4.5 \times 6$ M1
                                                 $= \underline{27 \text{ m}}$ ft (b)   A1

5. *(a)* **a**, **b** perpendicular $\Rightarrow$ $\quad$ **a.b** = 0 $\qquad$ M1

$\qquad\qquad\qquad\qquad\qquad$ **a.b** = $-2 + 13y - 50$ $\qquad$ M1 A1

$\qquad\qquad\qquad$ $-2 + 13y - 50 = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $y = \underline{4}$ $\qquad$ A1

(b) **a**, **b** parallel $\Rightarrow$ $\quad$ **a** = -2**b** $\qquad$ M1

$\qquad\qquad\qquad\qquad$ $-2y = 13$

$\qquad\qquad\qquad\qquad\quad$ $y = \underline{-6.5}$ $\qquad$ A1

6. *(a)* Angular velocity $\omega = \dfrac{v}{r}$ $\qquad$ used $\qquad$ M1

$\qquad\qquad\qquad$ $\omega = \dfrac{3}{0.4}$

$\qquad\qquad\qquad$ $\omega = \underline{7.5 \text{ rad s}^{-1}}$ $\qquad$ cao $\qquad$ A1

(b) Tension in the string $T = \dfrac{mv^2}{r}$ $\qquad$ oe $\qquad$ M1

$\qquad\qquad\qquad$ $T = \dfrac{0.8 \times 3^2}{0.4} = \underline{18 \text{ N}}$ $\qquad$ cao $\qquad$ A1

7.
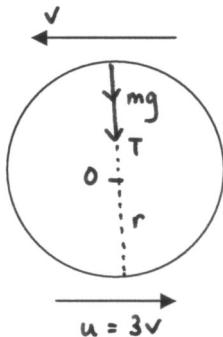
(a) Conservation of energy $\qquad$ M1

$\qquad\qquad$ $0.5mu^2 = 0.5mv^2 + mg \times 2r$ $\qquad$ A1 A1

$\qquad\qquad\quad$ $9v^2 = v^2 + 9.8 \times 2 \times 0.9 \times 2$ $\qquad u = 3v$ $\qquad$ B1

$\qquad\qquad\quad$ $8v^2 = 35.28$

$\qquad\qquad\qquad$ $v = 2.1$

$\quad$ and $\qquad$ $u = 6.3$ $\qquad$ convincing $\qquad$ A1

(b) N2L towards centre O $\qquad$ M1

$\qquad$ $T + mg = \dfrac{mv^2}{r}$ $\qquad$ A1

$\qquad$ $T = \dfrac{3 \times 2.1^2}{0.9} - 3 \times 9.8$ $\qquad$ A1

$\qquad$ $T = \underline{-14.7 \text{ N}}$ $\qquad$ cao $\qquad$ A1

(c) Object would not move in complete circles as T is negative,
i.e. rod exerted a thrust which a string cannot exert. $\qquad$ B1 E1

7

8. (a) $\mathbf{r_A} = (0\mathbf{i} + 3\mathbf{j} - 140\mathbf{k}) + t(3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ M1 A1
         $\mathbf{r_B} = (-9\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}) + t(-2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$ A1

   (b) $\mathbf{r_A} - \mathbf{r_B} = (9 + 5t)\mathbf{i} + (7 - 8t)\mathbf{j} + (2t - 134)\mathbf{k}$ si   M1 A1
         $AB^2 = (9 + 5t)^2 + (7 - 8t)^2 + (2t - 134)^2$ ft   B1
         $AB^2 = 93t^2 - 558t + 18086$

   (c)   At minimum distance $\dfrac{\mathrm{d}AB^2}{\mathrm{d}t^2} = 0$ M1

         $\dfrac{\mathrm{d}AB^2}{\mathrm{d}t^2} = 2(9 + 5t)(5) + 2(7 - 8t)(-8) + 2(2t - 134)(2)$ m1 A1

         $45 + 25t - 56 + 64t + 4t - 268 = 0$
         $93t - 279 = 0$
         $t = \underline{3\ \text{s}}$ cao   A1

8

MATHEMATICS M3

1. *(a)* (i)   N2L $F = ma$ used   M1

                     $400 - 16v^2 = 800a$ A1

Divide by 800 and using $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{400 - 16v^2}{800}$$

                                           convincing   A1

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{25 - v^2}{50}$$

(ii) $50\displaystyle\int \frac{\mathrm{d}v}{25 - v^2} = \int \mathrm{d}t$ sp.var.   M1

$$\frac{50}{2 \times 5} \ln\left|\frac{5+v}{5-v}\right| = t + C$$

                                                   A2

When $t = 0$, $x = 0$.   m1

        $C = 0$ ft   A1

$$t = 5\ln\left|\frac{5+v}{5-v}\right|$$

When $v = 2$,

$$t = 5\ln\left|\frac{7}{3}\right| = \underline{4.24\ \text{s}}$$ cao   A1

(b) $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{25 - v^2}{50}$ M1

$$\frac{50}{2}\int \frac{2v}{25 - v^2}\,\mathrm{d}v = \int \mathrm{d}x$$ m1

$$-25\ln\left|25 - v^2\right| = x + C$$ A2

When $x = 0$, $v = 0$.   m1

        $C = -25\ln\left|25\right|$ ft +/-   A1

When $v = 2$,

        $x = 25\ln 25 - 25\ln 21$

        $x = \underline{4.36\ \text{m}}$ cao   A1

2. (a) Period $= \dfrac{2\pi}{\omega} = 2 \times 4$ M1

$\omega = 0.25\pi$ A1

Max speed $= a\omega = 3\pi$ M1

    $a = 3\pi / 0.25\pi$

 $a = \underline{12 \text{ m}}$ convincing   A1

(b)
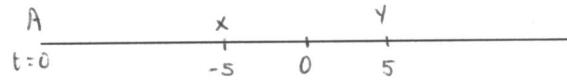
        $x = -12 \cos(0.25t)$ +/-   M1

When $t = \dfrac{2}{3}$ $x = -12\cos(0.25\pi \times \dfrac{2}{3})$ m1

        $x = -12 \times \dfrac{\sqrt{3}}{2} = -6\sqrt{3}$ +/-   A1

Distance of P from A when $t = \dfrac{2}{3} = 12 - 6\sqrt{3} = \underline{1.61 \text{ m}}$ cao   A1

(c) $v = \dfrac{dx}{dt}$ used   M1

    $v = -12 \cdot -\sin(0.25t) \cdot (0.25\pi)$

    $v = 3\pi \sin(0.25t)$ ft   A1

When $t = \dfrac{2}{3}$, $v = 3\pi \sin(0.25\pi \times \dfrac{2}{3}) = \underline{1.5\pi \text{ ms}^{-1}}$ cao   A1

(d)   At X, $x = -5$ M1

      $-5 = -12 \cos(0.25\pi)$

      $t_X = \underline{1.4528 \text{ s}}$ ft ω   A1

At Y, $x = 5$

      $5 = -12 \cos(0.25\pi)$

      $t_Y = \underline{5.5472 \text{ s}}$ ft ω   A1

Therefore required time $= t_Y - t_X = 1.0944 = \underline{1.09 \text{ s}}$ cao   A1

3.   *(a)*
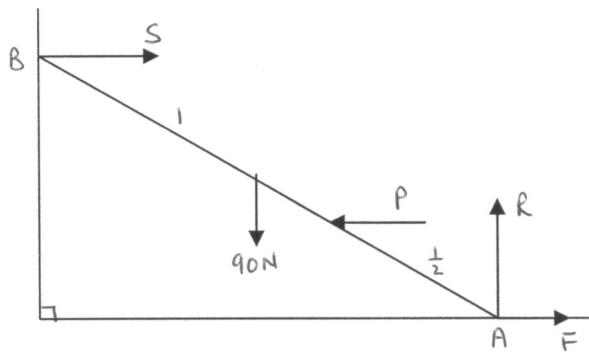
<div align="right">B1 B1</div>

(b)   Resolve vertically upwards   M1
$R = 90$ A1

Resolve horizontally to the right   M1
$S + F = P$ ft diagram   A1

Moments about A M1
$P \times 0.5 \cos \theta + 90 \sin \theta = S \times 2\cos \theta$ A1 A1
$P + 180 \tan \theta = 4S$
$P - 0.6 \times 90 = S$ $F = \mu R$ B1

Therefore $P + 180 \times 0.8 = 4(P - 54)$ m1
$P + 144 = 4P - 216$
$3P = 360$
$P = \underline{120 \text{ N}}$ A1

(c)   I have assumed that the ladder is a rigid rod.   B1

4.

<table>
<tr><td>cos θ = 0.8,</td><td>sin θ = 0.6</td><td>si</td><td>B1</td></tr>
<tr><td>$u = 6 \sin \theta$</td><td></td><td></td><td>M1</td></tr>
<tr><td>$u = 6 \times 0.6 = 3.6$ ms^{-1}</td><td></td><td>si</td><td>A1</td></tr>
</table>
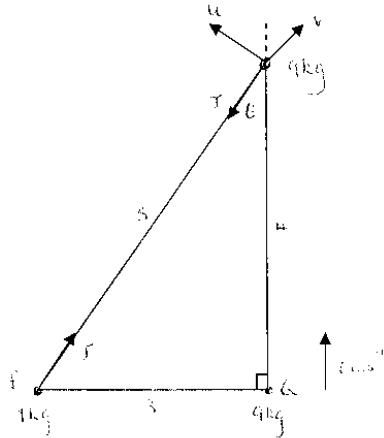
Impulse = change in momentum

For *P*   M1
$$J = 7v \hspace{6cm} \text{A1}$$

For *Q*   M1
$$9 \times 6 \cos \theta - J = 9v \hspace{4cm} \text{A1}$$

Adding $54 \times 0.8 = 16v$ m1
$$v = 2.7 \text{ ms}^{-1} \hspace{6cm} \text{A1}$$
Speed of *P* = <u>2.7 ms^{-1}</u>

$$\text{Speed of } Q = \sqrt{2.7^2 + 3.6^2} \hspace{4cm} \text{M1}$$
$$= \underline{4.5 \text{ ms}^{-1}} \hspace{3cm} \text{ft 2.7(c)} \hspace{1cm} \text{A1}$$

$$J = 7 \times 2.7$$
$$= \underline{18.9 \text{ Ns}} \hspace{3cm} \text{ft 2.7(c)} \hspace{1cm} \text{A1}$$

$$\text{Required angle } \alpha = \tan^{-1}\left(\frac{u}{v}\right) \hspace{3cm} \text{M1}$$
$$= \tan^{-1}\left(\frac{3.6}{2.7}\right)$$
$$= \underline{53.13^{\circ}} \hspace{3cm} \text{ft} \hspace{1.5cm} \text{A1}$$

5. *(a)*   N2L   M1

$$(6120 - 80t) - (120 + 40v) = 800a$$ A1

$$6000 - 40v - 80t = 800a$$ A1

Divide by 40

$$150 - \frac{dx}{dt} - 2t = 20\frac{d^2x}{dt^2}$$ B1

$$20\frac{d^2x}{dt^2} + \frac{dx}{dt} = 150 - 2t$$ a1

(b)   Auxiliary equation $20m^2 + m = 0$ M1

                                     $m = 0. -0.05$ both   A1

Complementary Function is $x = A + Be^{-0.05t}$ ft m   B1

Particular integral, try $x = at^2 + bt$ M1

$$\frac{dx}{dt} = 2at + b, \quad \frac{d^2x}{dt^2} = 2a$$

$$20(2a) + (2at + b) = 150 - 2t$$ A1

$$2a = -2$$ comp.coef.   m1

$$a = -1$$

$$-40 + b = 150$$

$$b = 190$$ both cao   A1

General solution is $x = A + B^{-0.05t} - t^2 + 190t$ B1

When $t = 0, x = 0 \Rightarrow$ $A + B = 0$ m1

$$\frac{dx}{dt} = -0.05Be^{-0.05t} - 2t + 190$$ ft   B1

When $t = 0, \dfrac{dx}{dt} = 0 \Rightarrow$ $0 = -0.05B + 190$ m1

$$B = 3800$$

$$A = -3800$$ both cao   A1

Therefore $\underline{x = 3800(e^{-0.05t} - 1) - t^2 + 190t}$

MATHEMATICS S1

1. (a) $P(A \cap B) = 0.6 \times 0.3$ B1

           $P(A \cup B) = 0.6 + 0.3 - 0.6 \times 0.3$ M1A1

                $= 0.72$ A1

      (b) $P(A' \cap B') = (1 - 0.6)(1 - 0.3)$ or $1 - 0.72$ M1A1

                $= 0.28$ A1

      (c) $P(A \mid A \cup B) = \dfrac{P(A)}{P(A \cup B)}$

        [FT their P(A∪B) from (a)]

                $= \dfrac{0.6}{0.72}$ B1B1

                $= \dfrac{5}{6}$ B1

        [Award final B1 only if previous 2 B1s are awarded]

2. (a)   (i) $P(X = 5) = e^{-4.5} \times \dfrac{4.5^5}{5!} = 0.171$ M1A1

          (ii) $P(X \le 2) = e^{-4.5}(1 + 4.5 + \dfrac{4.5^2}{2})$ M1A1

                    $= 0.17$ (cao)   A1

      (b) $P(3 \le X \le 7) = 0.9134 - 0.1736$ or $0.8264 - 0.0866$ B1B1

                $= 0.740$ (cao)   B1

3. $E(Y) = 5a - b = 0$ M1A1

        $\mathrm{Var}(Y) = 4a^2 = 1$ M1A1

        $a = \dfrac{1}{2}, b = \dfrac{5}{2}$ (cao)   A1A1

4. (a)   There are 36 possible pairs   B1

           $P(A) = \dfrac{15}{36} \left(\dfrac{5}{12} \right)$ M1A1

      (b) $P(B) = \dfrac{10}{36} \left(\dfrac{5}{18} \right)$ M1A1

      (c) $P(A \cap B) = \dfrac{5}{36}$ M1A1

        Attempting to compare P(A∩B) with P(A)P(B) or P(A│B) with P(A) or

        P(B│A) with P(B)   M1

        $P(A)P(B) = \dfrac{25}{216} \ne P(A \cap B)$ or $P(A \mid B) = 0.5 \ne P(A)$ or $P(B \mid A) = 1/3 \ne P(B)$ A1

        A and B are therefore not independent.   A1

        (FT from previous line)

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5. *(a)* (i) X is B(5, 0.4) (parameters si) B1

(ii) Mean = 2 B1
SD = $\sqrt{(5\times0.4\times0.6)} = 1.10$ M1A1

(iii) $P(X \geq 3) = 0.317$ M1A1

(b) Y is B(24, 0.05) and therefore approx Po(1.2). M1A1
$P(Y \leq 2) = 0.88$ M1A1

6. *(a)* Sum of probs = 1 so M1

$$k(1+4+9+16) = 1 \Rightarrow k = \frac{1}{30}$$ A1

(b) $E(X) = \dfrac{1}{30}(1+8+27+64)$ M1A1

$$= \frac{10}{3}$$ A1

$E(X^2) = \dfrac{1}{30}(1+16+81+256) \quad (=11.8)$ M1A1

$\mathrm{Var}(X) = 354/30 - (10/3)^2$ M1
$= 31/45 \ (0.69) \ \text{(cao)}$ A1

7. *(a)* $E\left(\dfrac{1}{X}\right) = \dfrac{6}{5}\displaystyle\int_{1}^{2} \frac{1}{x}.x(x\text{-}1)\mathrm{d}x$ M1A1

$$= \frac{6}{5}\left[\frac{x^2}{2} - x\right]_{1}^{2}$$ A1

$$= \frac{3}{5}$$ A1

(b) (i) $F(x) = \displaystyle\int_{1}^{x} \frac{6}{5}(y^2 - y)\mathrm{d}y$ M1A1

$$= \frac{6}{5}\left[\frac{y^3}{3} - \frac{y^2}{2}\right]_{1}^{x}$$ A1

$$= \frac{2}{5}x^3 - \frac{3}{5}x^2 + \frac{1}{5}$$ A1

(ii) $P(X \leq 1.75) = F(1.75) = 0.506$ (81/160) M1A1

(iii) The median is less than 1.75 B1
because the answer to (ii) > 0.5 (oe) B1
(FT on answer to (ii))

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8. *(a)* $P(\text{Red}) = \dfrac{1}{3} \times \dfrac{1}{5} + \dfrac{1}{3} \times \dfrac{2}{5} + \dfrac{1}{3} \times \dfrac{3}{5}$ M1A1

$= \dfrac{2}{5}$ A1

(b) $P(\text{Box A} \mid \text{Red}) = \dfrac{1/15}{2/5}$ B1B1

$= \dfrac{1}{6}$ B1

(c) $P(\text{Box B} \mid \text{Red}) = \dfrac{1}{3}$ B1

$P(\text{Box C} \mid \text{Red}) = \dfrac{1}{2}$ B1

$P(\text{Red}) = \dfrac{1}{6} \times 0 + \dfrac{1}{3} \times \dfrac{1}{4} + \dfrac{1}{2} \times \dfrac{2}{4} = \dfrac{1}{3}$ M1A1

Alternative solution:

P(1st ball red and 2nd ball from chosen box red) $= \dfrac{1}{3} \times \dfrac{2}{5} \times \dfrac{1}{4} + \dfrac{1}{3} \times \dfrac{3}{5} \times \dfrac{1}{2}$ M1A1

$(= \dfrac{2}{15})$

P(2nd ball from chosen box red $\mid$ 1st ball red) $= \dfrac{2/15}{2/5}$ B1B1

$= 1/3$

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MATHEMATICS S2

1. *(a)* $\bar{x} = \dfrac{29.43}{9}$ (= 3.27)   B1

         SE of $\bar{X} = \dfrac{0.15}{\sqrt{9}}$ (= 0.05)   B1

         90% conf limits are

                   $3.27 \pm 1.645 \times 0.05$ M1A1

         [M1 correct form, A1 1.645, FT on their mean and SE]

         giving [3.19,3.35]   A1

    (b)   We solve

         $\dfrac{0.15}{\sqrt{n}} = 0.5 \times 0.05$ giving $n = 36$ (cao)   M1A1

2.   *(a)*   (i)   Using Var(X) = $E(X^2) - [E(X)]^2$ M1

                       $2 = E(X^2) - 4$ A1

               $E(X^2) = 6$

               Similarly,

               $E(Y^2) = 12$ B1

        (ii) $E(X^2 Y^2) = E(X^2)E(Y^2) = 72$ M1A1

    (b)   Var(U) = $E(X^2 Y^2) - [E(XY)]^2$ M1

                  $= 72 - (2 \times 3)^2$ A1

                  $= 36$ A1

             SD = 6   A1

3.   *(a)*   (i) $z = \dfrac{80 - 75}{5} = 1$ M1A1

             Prob = 0.8413 (cao)   A1

        (ii) $z = 0.674$ B1

            UQ = $75 + 0.674 \times 5$ M1

               $= 78.4$ A1

    (b)   Put $U = X_1 + X_2 - (Y_1 + Y_2 + Y_3)$

         $E(U) = -18$ (accept $\pm$)   B1

         Var(U) = $2 \times 5^2 + 3 \times 4^2 = 98$ M1A1

         $z = \dfrac{18}{\sqrt{98}} = (\pm)1.82$ M1A1

         [FT on mean and variance]   Prob = 0.9656   (cao)   A1

4. *(a)* $f(r) = 1/5$ $(0 \le r \le 5, = 0$ otherwise) B1
[Accept labelled sketch]

(b) $$E(A) = \int_0^5 \pi r^2 . \frac{1}{5} \, dr$$ M1A1

$$= \frac{\pi}{15} \left[r^3 \right]_0^5$$ A1

$$= \frac{25\pi}{3}$$ (cao) A1

Alternative solution
$$E(R^2) = \text{Var}(R) + [E(R)]^2$$ M1

$$= \frac{1}{12} \times 5^2 + \left(\frac{5}{2} \right)^2$$ A1A1

$$E(A) = \frac{25\pi}{3} \text{ (cao)}$$ A1

(c) $$P(\pi R^2 > 25) = P\left(R > \sqrt{\frac{25}{\pi}}\right)$$ M1A1

$$= \frac{5 - \sqrt{25/\pi}}{5}$$ m1

$$= 0.436$$ A1

5. *(a)* $H_0 : p = 0.75$ versus $H_1 : p < 0.75$ B1

(b) (i) Under H_0, X is B(20,0.75) (si) B1
and Y (No of misses) is B(20,0.25)(si) M1A1
Using tables, we find that $k = 12$ B2

(ii) We require
$P(X > 12 \,|\, p = 0.5)$ M1
$= 0.132$ A1
[Do not accept the use of a normal approximation in this question]

6. *(a)* X is Poi(14) (si) B1

(i) $$P(X = 10) = e^{-14} \times \frac{14^{10}}{10!} = 0.0663$$ M1A1

(ii) $$P(X > 12) = 0.6415$$ M1A1

(b) (i) $H_0 : \mu = 2$ versus $H_1 : \mu > 2$ (Accept 14) B1

(ii) p-value $= P(X \ge 20)$ M1
$= 0.0765$ A1
We cannot conclude that the mean has increased. B1
[Do not accept the use of a normal approximation in (a) or (b)(ii)]

(iii)   Under H_0, X is Po(200) $\approx$ N(200,200)   M1A1

$$z = \frac{229.5 - 200}{\sqrt{200}}$$ M1A1A1

     = 2.09   A1
     p-value = 0.0183   A1
 Strong evidence that the mean has increased.   B1
[No c/c gives $z = 2.12$, $p = 0.0170$; wrong c/c gives $z = 2.16$, $p = 0.0154$]

7.   *(a)*   (i) $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$ B1

     (ii) $\bar{x}_1 = \dfrac{31.71}{5}$ (6.342)   B1

     $\bar{x}_2 = \dfrac{31.53}{5}$ (6.306)   B1

SE of difference of means = $\sqrt{\dfrac{0.025^2 \times 2}{5}}$ M1

     = 0.0158   A1

$$z = \frac{6.342 - 6.306}{0.0158}$$ M1

     = 2.28   A1
Prob from tables = 0.0113   B1
     p-value = 0.0226   B1
Strong evidence of a difference in acidity levels.   B1

(b)   95% confidence limits are
     $6.342 - 6.306 \pm 1.96 \times 0.0158$ M1A1
giving [0.005, 0.067]   A1

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MATHEMATICS S3

1. (a) $P(X = 3) = P(1,2) + P(2,1) = \dfrac{1}{6} \times \dfrac{2}{5} \times 2 = \dfrac{2}{15}$ M1A1

   (b) $P(X = 4) = P(2,2) + P(1,3) + P(3,1)$ M1

   $$= \dfrac{2}{6} \times \dfrac{1}{5} + \dfrac{1}{6} \times \dfrac{3}{5} \times 2 = \dfrac{4}{15}$$ A1A1

   $P(X = 5) = P(2,3) + P(3,2) = \dfrac{2}{6} \times \dfrac{3}{5} \times 2 = \dfrac{2}{5}$ M1A1

   $P(X = 6) = P(3,3) = \dfrac{3}{6} \times \dfrac{2}{5} = \dfrac{1}{5}$ B1

2. (a) $\Sigma x = 246 \,; \Sigma x^2 = 6055.74$ si   B1B1

   UE of $\mu = 24.6$ B1

   UE of $\sigma^2 = \dfrac{6055.74}{9} - \dfrac{246^2}{9 \times 10}$ M1

   $= 0.46$ A1

   (b) Use of a t-value   M1

   95% confidence limits for μ are

   $\bar{x} \pm t \times SE$ m1

   $24.6 \pm 2.262 \sqrt{\dfrac{0.46}{10}}$ A1

   giving $[24.1, 25.1]$ cao   A1

3. (a) $\hat{p} = \dfrac{120}{200}$ B1

   $SE = \sqrt{\dfrac{0.6 \times 0.4}{200}}$ B1

   Approx 95% confidence limits are

   $\hat{p} \pm z \times SE$ M1

   giving $[[0.532, 0.668]$ cao   A1

   (b) Yes because the interval is entirely above 0.5   B1

   (c) We require

   $2 \times 1.96 \sqrt{\dfrac{0.6 \times 0.4}{n}} = 0.05$ M1A1

   $n = 0.24 \left(\dfrac{3.92}{0.05}\right)^2 = 1475/1476$ M1A1

   [Accept the use of $p = 0.5$ giving $n = 1537$]

4. *(a)* UE of $\mu = 2.56$ B1
UE of Var$(X) = 2.56$ B1

(b) ESE $= \sqrt{\dfrac{2.56}{100}} = 0.16$ M1A1

(c) Approx 90% confidence limits for μ are
$2.56 \pm 1.645 \times 0.16$ M1A1
giving $[2.30, 2.82]$ A1

(d) The 1.645 requires the normality of $\overline{X}$. B1

5. *(a)* $H_0 : \mu_x = \mu_y$ versus $H_1 : \mu_x < \mu_y$ B1

(b) $\overline{x} = 42.82; \overline{y} = 43.4$ B1B1

$s_x^2 = \dfrac{185855}{99} - \dfrac{4282^2}{100 \times 99} = 25.250..$ M1A1

$s_y^2 = \dfrac{230347}{119} - \dfrac{5208^2}{120 \times 119} = 36.300..$ A1
[Accept division by n]

Test stat $= \dfrac{43.4 - 42.82}{\sqrt{\dfrac{36.300..}{120} + \dfrac{25.250..}{100}}}$ M1A1A1

$= (\pm)0.78$ cao A1
p-value $= 0.2177$ A1
Insufficient evidence to support the psychologist's theory B1

6. *(a)* X_1 is B$(n, 3\theta)$, X_2 is B$(n, 1 - 4\theta)$ (si) B1B1

$E(U_1) = \dfrac{n \times 3\theta}{3n} = \theta$ M1A1

$E(U_2) = \dfrac{n - n(1 - 4\theta)}{4n} = \theta$ M1A1

(b) (i) Var$(U_1) = \dfrac{n.3\theta(1 - 3\theta)}{9n^2} = \dfrac{\theta(1 - 3\theta)}{3n}$ M1A1

Var$(U_2) = \dfrac{n.4\theta(1 - 4\theta)}{16n^2} = \dfrac{\theta(1 - 4\theta)}{4n}$ M1A1

$\dfrac{\text{Var}(U_1)}{\text{Var}(U_2)} = \dfrac{\theta(1 - 3\theta)}{3n} \cdot \dfrac{4n}{\theta(1 - 4\theta)} = \dfrac{4(1 - 3\theta)}{3(1 - 4\theta)}$ A1AG

(ii) U_2 is better because it has the smaller variance. M1A1

7. (a) $\Sigma x = 105$, $\Sigma y = 262$ si   B1

$S_{xx} = 700$ B1

$S_{xy} = 830$ B1

$b = \dfrac{700}{830}$ M1

    $= 1.1857$ A1

$a = \dfrac{262 - 105 \times 1.1857}{7}$ M1

    $= 19.6429$ A1

(b) (i) $H_0 : \beta = 1.2$ versus $H_1 : \beta \neq 1.2$ B1

    (ii)   SE of $b = \dfrac{0.25}{\sqrt{2275 - 105^2/7}} = 0.009449$ M1A1

Test statistic $= \dfrac{1.1857 - 1.2}{0.009449}$ M1A1

       $= -1.51$ A1

Prob from tables $= 0.0656$ A1

     p-value $= 0.1312$ B1

We conclude that $\beta = 1.2$.   B1

WJEC
CBAC