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GENERAL CERTIFICATE OF EDUCATION
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MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2008

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2008 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics C1 May 2008

Solutions and Mark Scheme

1. (*a*) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^1/_2$ (or equivalent) A1

(*b*) A correct method for finding the equation of AB using the candidate's value for the gradient of AB. M1

Equation of AB : $y - 4 = -{}^1/_2\,[x - (-7)]$ (or equivalent) A1

(f.t. the candidate's value for the gradient of AB)

Equation of AB : $x + 2y - 1 = 0$

(f.t. one error if both M1's are awarded) A1

(*c*) A correct method for finding the length of AB M1

$AB = \sqrt{125}$ A1

(*d*) A correct method for finding E M1

$E(-2, 1{\cdot}5)$ A1

(*e*) **Either:**

An attempt to find the gradient of a line perpendicular to AB using the fact that the product of the gradients of perpendicular lines = –1. M1

An attempt to find the gradient of the line passing through C and D M1

$2 = \dfrac{1 - (-15)}{6 - k}$ (Equating expressions for gradient) M1

$k = -2$ (f.t. candidate's gradient of AB) A1

Or:

An attempt to find the gradient of a line perpendicular to AB using the fact that the product of the gradients of perpendicular lines = –1. M1

An attempt to find the equation of line perpendicular to AB passing through C(or D) M1

$-15 - 1 = 2(k - 6)$

(substituting coordinates of unused point in the equation) M1

$k = -2$ (f.t. candidate's gradient of AB) A1

2. (*a*) **Either:**

$\sqrt{75} = 5\sqrt{3}$ B1

$\dfrac{9}{\sqrt{3}} = 3\sqrt{3}$ B1

$\sqrt{6} \times \sqrt{2} = 2\sqrt{3}$ B1

$\sqrt{75} - \dfrac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2}) = 4\sqrt{3}$ (c.a.o.) B1

Or:

$\sqrt{75} = \dfrac{15}{\sqrt{3}}$ B1

$\sqrt{6} \times \sqrt{2} = \dfrac{6}{\sqrt{3}}$ B1

$\sqrt{75} - \dfrac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2}) = \dfrac{12}{\sqrt{3}}$ B1

$\sqrt{75} - \dfrac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2}) = 4\sqrt{3}$ (c.a.o.) B1

(*b*) $\dfrac{5\sqrt{5} - 2}{4 + \sqrt{5}} = \dfrac{(5\sqrt{5} - 2)(4 - \sqrt{5})}{(4 + \sqrt{5})(4 - \sqrt{5})}$ M1

Numerator: $20\sqrt{5} - 25 - 8 + 2\sqrt{5}$ A1

Denominator: $16 - 5$ A1

$\dfrac{5\sqrt{5} - 2}{4 + \sqrt{5}} = 2\sqrt{5} - 3$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $4 + \sqrt{5}$

3. $\dfrac{dy}{dx} = 6x - 8$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\dfrac{dy}{dx}$ m1

Value of $\dfrac{dy}{dx}$ at $P = 4$ (c.a.o.) A1

Gradient of normal = $\dfrac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

y-coordinate of $P = 3$ B1

Equation of normal to C at P: $y - 3 = -\frac{1}{4}(x - 2)$ (or equivalent) A1

(f.t. **one** slip in **either** candidate's value for $\dfrac{dy}{dx}$ **or** candidate's value for the y-coordinate at P provided M1and both m1's awarded)

4. (*a*) $y + \delta y = 5(x + \delta x)^2 + 3(x + \delta x) - 4$ B1

Subtracting y from above to find δy M1

$\delta y = 10x\delta x + 5(\delta x)^2 + 3\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 10x + 3$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = -8 \times x^{-2} + 3 \times \frac{1}{2} \times x^{-1/2}$ B1, B1

Either $4^{-2} = \frac{1}{16}$ **or** $4^{-1/2} = \frac{1}{2}$ B1

$\frac{dy}{dx} = \frac{1}{4}$ (c.a.o) B1

5. (*a*) $a = 3$ B1

$b = -13$ B1

(*b*) $2b$ on its own or least (minimum) value = $2b$, with correct explanation or no explanation B1

$x = -a$ B1

Note: Candidates who use calculus are awarded B0, B0

6. $(5 + 2x)^3 = 125 + 150x + 60x^2 + 8x^3$

Two terms correct B1

Three terms correct B1

All 4 terms correct B1

7. (*a*) Use of $f(2) = 0$ M1

$32 + 4p - 22 + q = 0$ A1

Use of $f(-1) = 9$ M1

$-4 + p + 11 + q = 9$ A1

Solving simultaneous equations for p and q M1

$p = -4$, $q = 6$ (convincing) A1

Note:

Candidates who assume $p = -4$, $q = 6$ and then verify that $x - 2$ is a factor and that dividing the polynomial by $x + 1$ gives a remainder of 9 may be awarded M1 A1 M1 A1 M0 A0

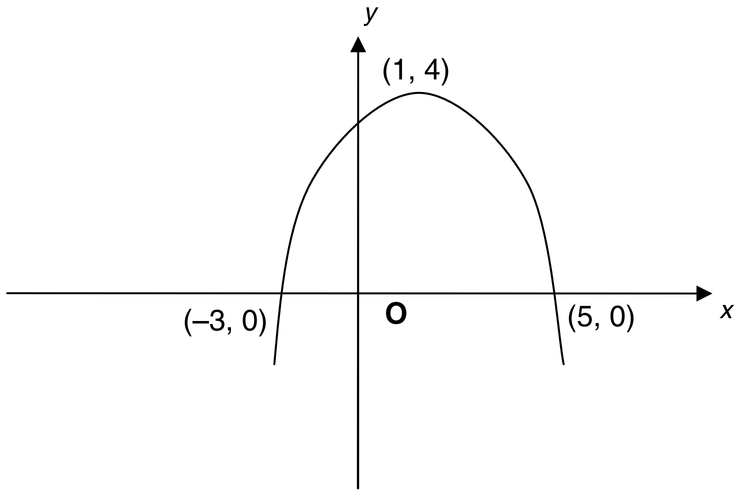
(*b*) $f(x) = (x - 2)(4x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x - 2)(4x^2 + 4x - 3)$ A1

$f(x) = (x - 2)(2x - 1)(2x + 3)$ A1

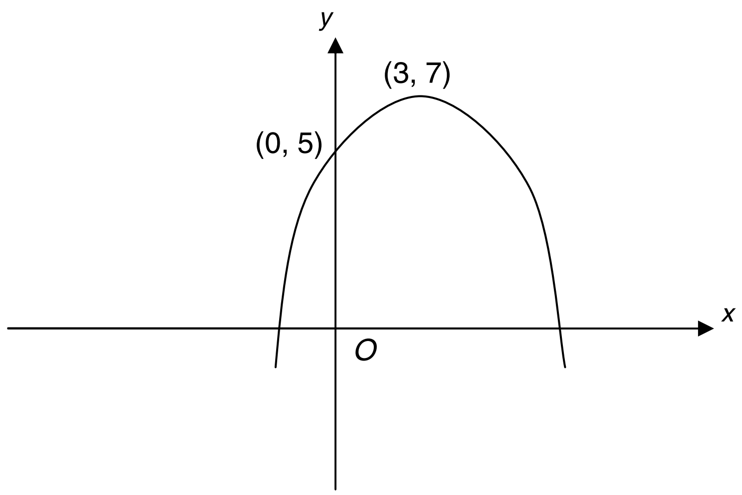
(f.t. only for $f(x) = (x - 2)(2x + 1)(2x - 3)$ from $4x^2 - 4x - 3$)

8. (*a*)



Concave down curve with maximum (1, 4) or (5, 4)	B1
Two of the three points correct	B1
All three points correct	B1

(*b*)



Concave down curve with maximum (3, 7) or (3, 1)	B1
Maximum point (3, 7)	B1
Point of intersection with *y*-axis (0, 5)	B1

9. $\frac{dy}{dx} = -6x^2 + 6x + 12$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = 2, -1$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are (2, 15) and (–1, –12) (both correct) (c.a.o) A1

A correct method for finding nature of stationary points M1

(2, 15) is a maximum point (f.t. candidate's derived values) A1

(–1, –12) is a minimum point (f.t. candidate's derived values) A1

10. (*a*) Finding critical points $x = -1{\cdot}5$, $x = 3$ B1

A statement (mathematical or otherwise) to the effect that

$x \le -1{\cdot}5$ or $3 \le x$ (or equivalent)

(f.t. candidate's critical points) B2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

(*b*) (i) An expression for $b^2 - 4ac$, with $b = (-)6$ and at least one of a or c correct M1

$b^2 - 4ac = [(-)6]^2 - 4 \times 3 \times m$ A1

$b^2 - 4ac < 0$ m1

$m > 3$ (c.a.o.) A1

(ii) $3x^2 - 4x + 7 = 2x + k$ M1

No points of intersection $\Leftrightarrow 3x^2 - 6x + (7 - k) = 0$ has no real roots (allow one slip in quadratic) m1

$4 > k$ A1

(if candidate uses (*b*)(i), f.t. candidate's inequality for m)

Mathematics C2 May 2008

Solutions and Mark Scheme

1.	0	1		
	0·2	1·060596059		
	0·4	1·249358235	(2 values correct)	B1
	0·6	1·586018915	(4 values correct)	B1

Correct formula with $h = 0{\cdot}2$ M1

$I \approx \frac{0{\cdot}2}{2} \times \{1 + 1{\cdot}586018915 + 2(1{\cdot}060596059 + 1{\cdot}249358235)\}$

$I \approx 0{\cdot}72059275$

$I \approx 0{\cdot}721$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}15$

0	1		
0·15	1·033939138		
0·3	1·137993409		
0·45	1·318644196		
0·6	1·586018915	(all values correct)	B1

Correct formula with $h = 0{\cdot}15$ M1

$I \approx \frac{0{\cdot}15}{2} \times \{1 + 1{\cdot}586018915 + 2(1{\cdot}033939138 + 1{\cdot}137993409 + 1{\cdot}318644196)\}$

$I \approx 0{\cdot}71753793$

$I \approx 0{\cdot}718$ (f.t. one slip) A1

2. (*a*) $\tan\theta = 1{\cdot}5$ (c.a.o.) B1

$\theta = 56{\cdot}31°$ B1

$\theta = 236{\cdot}31°$ B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*b*) $3x = 25{\cdot}84°, 334{\cdot}16°, 385{\cdot}84°, 694{\cdot}16°$ (one value) B1

$x = 8{\cdot}61°$ (f.t candidate's value for $3x$) B1

$x = 111{\cdot}39°, 128{\cdot}61°$ (f.t candidate's value for $3x$) B1, B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) $\sin^2\theta - 4(1 - \sin^2\theta) = 8\sin\theta$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = coefficient of $\sin^2\theta$ and $b \times d$ = constant m1

$5\sin^2\theta - 8\sin\theta - 4 = 0 \Rightarrow (5\sin\theta + 2)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{2}{5}$, ($\sin\theta = 2$) (c.a.o.) A1

$\theta = 203{\cdot}58°, 336{\cdot}42°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $5\sin\theta + 2 = 0$, ignore roots outside range.
$\sin\theta = -$, f.t. for 2 marks, $\sin\theta = +$, f.t. for 1 mark

3. (*a*) $15 = \frac{1}{2} \times x \times (x + 4) \times \sin 150°$ (correct use of area formula) M1

Either $x(x + 4) = 60$ or expressing the equation correctly in the form $ax^2 + bx + c = 0$ A1

$x = 6$ (negative value must be rejected) (c.a.o.) A1

(*b*) $BC^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 150°$

(correct substitution of candidate's derived values in cos rule) M1

$BC = 15{\cdot}5$ cm (f.t. candidate's derived value for x) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-2)d] + [a + (n-1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + [a + d] + a$

$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$

(reverse and add) M1

$2S_n = n[2a + (n-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $\frac{10}{2} \times [2a + 9d] = 320$ B1

$2a + 9d = 64$

$[a + 11(12)d] + [a + 15(16)d] = 166$ M1

$[a + 11d] + [a + 15d] = 166$ A1

$2a + 26d = 166$

An attempt to solve the candidate's two equations simultaneously by eliminating one unknown M1

$d = 6$, $a = 5$ (both values) (c.a.o.) A1

5. $a + ar = 7{\cdot}2$ B1

$\frac{a}{1 - r} = 20$ B1

A valid attempt to eliminate a M1

$20(1 - r) + 20(1 - r)r = 7{\cdot}2$ (a correct quadratic in r) A1

$r = 0{\cdot}8$ ($r = -0{\cdot}8$) (c.a.o.) A1

$r = 0{\cdot}8$ and $a = 4$ (f.t. candidate's positive value for r) A1

6. (a) $5 \times \dfrac{x^{3/2}}{3/2} - 4 \times \dfrac{x^{1/3}}{1/3}$ $(+c)$ B1,B1

(b) (i) $4 - x^2 = 3x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = -4$ m1

$(x - 1)(x + 4) = 0 \Rightarrow x = 1 \quad (-4)$ (c.a.o.) A1

$A(1, 3)$ (f.t. candidate's x-value, dependent on M1 only) A1

$B(2, 0)$ B1

(ii) Area of triangle = 3/2

(f.t. candidate's coordinates for A) B1

Area under curve $= \displaystyle\int_1^2 (4 - x^2)\, dx$

(use of integration) M1

$= [4x - (1/3)x^3]_1^2$

(correct integration) B2

$= [(8 - 8/3) - (4 - 1/3)]$

(substitution of candidate's limits) m1

$= 5/3$

Use of candidate's, x_A, x_B as limits and trying to find total area by adding area of triangle and area under curve m1

Total area = 3/2 + 5/3 = 19/6 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indicies) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) $\log_a(3x + 4) - \log_a x = 3 \log_a 2$.

$\log_a \left(\frac{3x + 4}{x}\right) = \log_a 2^3$ (use of subtraction law) B1

(use of power law) B1

$\frac{3x + 4}{x} = 2^3$ (removing logs) (c.a.o) B1

$x = 0{\cdot}8$ (f.t. for $2^3 = 6, 9$ only) B1

(*c*) **Either:**

$(3y + 2) \log_{10} 4 = \log_{10} 7$

(taking logs on both sides and using the power law) M1

$y = \frac{\log_{10} 7 \pm 2 \log_{10} 4}{3 \log_{10} 4}$ m1

$y = -0{\cdot}199$ (c.a.o.) A1

Or:

$3y + 2 = \log_4 7$ (rewriting as a log equation) M1

$y = \frac{\log_4 7 \pm 2}{3}$ m1

$y = -0{\cdot}199$ (c.a.o.) A1

8. (*a*) (i) $A(5, 3)$ B1

(ii) A correct method for finding radius M1

Radius = $\sqrt{65}$ A1

(iii) Equation of C: $(x - 5)^2 + (y - 3)^2 = (\sqrt{65})^2$

(f.t. candidate's coordinates of A.) B1

(*b*) **Either:**

An attempt to substitute the coordinates of R in the equation of C M1

Verification that L.H.S. of equation of $C = 65 \Rightarrow R$ lies on C A1

Or:

An attempt to find AR^2 M1

$AR^2 = 65 \Rightarrow R$ lies on C A1

(*c*) **Either:**

$RQ = \sqrt{26}$ ($RP = \sqrt{234}$) B1

$\sin QPR = \frac{\sqrt{26}}{2\sqrt{65}}$ $\left[\cos QPR = \frac{\sqrt{234}}{2\sqrt{65}}\right]$ M1

$QPR = 18{\cdot}43°$ A1

Or:

$RQ = \sqrt{26}$ or $RP = \sqrt{234}$ B1

$(\sqrt{26})^2 = (\sqrt{234})^2 + (2\sqrt{65})^2 - 2 \times (\sqrt{234}) \times (2\sqrt{65}) \times \cos QPR$

(correct use of cos rule) M1

$QPR = 18{\cdot}43°$ A1

9. Let $A\hat{O}B = \theta$ radians

(*a*) $6 \times \theta = 5{\cdot}4$ M1

$\theta = 0{\cdot}9$ A1

Area of sector $AOB = \frac{1}{2} \times 6^2 \times \theta$ M1

Area of sector $AOB = 16{\cdot}2\text{ cm}^2$ (convincing) A1

(*b*) Area of triangle $AOB = \frac{1}{2} \times 6^2 \times \sin\theta$ M1

Area of triangle $AOB = 14{\cdot}1\text{ cm}^2$ (f.t. candidate's value for θ) A1

Shaded area $= 2{\cdot}1\text{ cm}^2$ (f.t. candidate's value for θ) A1

A LEVEL MATHEMATICS

PAPER C3

Summer 2008

Marking Scheme

1.	$h = 0{\cdot}25$	M1 ($h = 0{\cdot}25$, correct formula)
	Integral $\approx \frac{0.25}{3}$ [0·4142136 + 1·9282847 +	B1 (3 correct values)
	4 (1·5112992 + 1·7655028)	B1 (2 correct values)
	+ 2(1·6274893)]	
	$\approx 1{\cdot}642$	A1 (F.T. one slip)
		4
2. *(a)*	$\theta = \frac{\pi}{6}$, for example	B1 (choice of θ and one correct evalution)
	$\tan 2\theta = \tan\frac{\pi}{3} = \sqrt{3} \approx 1{\cdot}73.$	
	$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} \approx 0{\cdot}866$	B1 (2 correct evaluations)
	(Statement is false)	
(b)	$2(1 + \tan^2\theta) = 8 - \tan\theta$	M1 (use of correct formula $\sec^2\theta = 1 + \tan^2\theta$)
	$2\tan^2\theta + \tan\theta - 6 = 0$	M1 (attempt to solve quadratic)
	$(2\tan\theta - 3)(\tan\theta + 2) = 0$	
	$\tan\theta = \frac{3}{2}, -2$	A1
	θ = 56·3°, 236·3°, 116·6°, 296·6° (allow to nearest degree)	B1 (56·3°, 236·3°) B1 (116·6°) B1 (296·6°)
		8

3.	$2x + \sin y + x\cos y\,\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$	B1 ($\sin y + x\cos y\,\frac{dy}{dx}$)
		B1 ($3y^2\frac{dy}{dx}$)
	$2 + 0 - \frac{dy}{dx} + 3\pi^2\frac{dy}{dx} = 0$	B1 (correct differentiation of x^2, π^3, 1)
	$\frac{dy}{dx} = \frac{-2}{3\pi^2 - 1} \approx 0\cdot0699$ (allow $-0\cdot07(0)$)	B1 (F.T. $\frac{d}{dy}(\sin y) = -\cos y$)
		4
4.	*(a)* $\frac{dy}{dt} = 2e^{2t}$	M1 (ke^{2t}, $k = \frac{1}{2}$, 1, 2) A1 ($k = 2$)
	$\frac{dy}{dt} = \frac{1}{t}$	B1
	$\frac{dy}{dx} = 2te^{2t}$ (o.e.)	A1 (C.A.O)
	(b) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 2te^{2t} + 4te^{2t}$	M1 ($f(t)\,e^{2t} + 2t\,g(t)$) A1 ($f(t) = 2$, $g(t) = 2e^{2t}$)
	$\frac{d^2y}{dx^2} = 2te^{2t}(1 + 2t)$ (o.e.)	M1 (use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$)
		A1 (F. T candidates $\frac{d}{dt}\left(\frac{dy}{dx}\right)$)
		8

5.	*(a)*	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \quad - \quad 3x^{\frac{1}{2}}$	B1, B1
		$\left(\frac{dy}{dx} = 0\right) \quad 1 = 3\sqrt{x}\ \sqrt{1-x^2}$	M1 (move one term to other side, and attempt to square both sides)
		$1 = 9x(1-x^2)$	A1
		$9x^3 - 9x + 1 = 0$	

(b)

x	$9x^3 - 9x + 1$
0	1
$0 \cdot 2$	$-0 \cdot 728$

M1 (attempt to check values or signs)

Change of sign indicates α is between 0 and 0·2 — A1 (correct values or signs and conclusion)

$x_0 = 0{\cdot}1$, $x_1 = 1{\cdot}1121111$, $x_2 = 0{\cdot}11252022$, $x_3 = 0{\cdot}1125357$ — B1 (x_1)

$x_3 \approx 0{\cdot}11254$ — B1 (x_3)

Check 0·112535, 0·112545

x	$9x^3 - 9x + 1$
0·112535	0·000011
0·112545	– 0·00008

M1 (attempt to check values or signs)
A1 (correct values or signs, F.T. x_3)

Change of sign indicates α is 0·11254 correct to 5 dec places — A1

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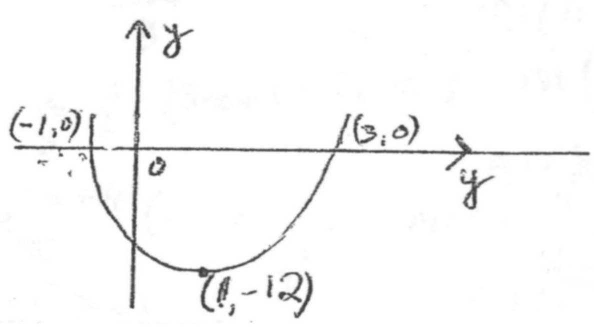
6. *(a)*



B1 (2 correct x values)

B1 ($y = -12$ for st pt)

B1 (all correct, correct shape)

(b) $4\,|\,x\,| = 1$ B1 ($a\,|\,x\,| = b$, $a = 4$, $b = 1$)

$x = \pm\dfrac{1}{4}$ B1 (both values, F.T. a, b)

(c) $2x - 9 > 3$

$x > 6$ B1

or

$2x - 9 < -3$ M1

$x > 3$ A1

$x > 6$ or $x > 3$ (o.e.) e.g. $(-\infty, 3) \cup (6, \infty)$ A1 (answer must contain 'or') (o.e.)

alternatively

$(2x - 9)^2 > 9$

$4x^2 - 36x + 72 > 0$

$x^2 - 9x + 18 > 0$ B1 (for $x > 6$)

$(x - 3)(x - 6) > 0$ M1 A1 for $x < 3$

$x > 6$ or $x < 3$ A1 for union of intervals

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7.	*(a)*	(i)	$-\frac{\cos 3x}{3}\ (+C)$	M1 ($k\cos 3x$, $k = \pm\frac{1}{3}, -1, -3$) A1 ($k = -\frac{1}{3}$)
		(ii)	$\frac{2}{3}\ ln\ \lvert 3x+5 \rvert\ (+C)$	M1 ($k\ ln \lvert 3x+5 \rvert$) A1 ($k = \frac{2}{3}$)
		(iii)	$\frac{e^{3x+4}}{3}(+C)$	M1 (ke^{3x+4}) A1 ($k = \frac{1}{3}$)
	(b)		$\left[-\frac{1}{6(2x+1)^3}\right]_0^1$	M1 ($a(2x+1)^{-3}$, $a \pm \frac{1}{6}, -\frac{1}{3}, -\frac{2}{3}\ -\frac{1}{8}$) A1 ($a = -\frac{1}{6}$)
	=		$-\frac{1}{6\times 3^3}+\frac{1}{6}$	A1
	=		$\frac{13}{81}\approx 0\cdot 160$	A1 (C.A.O, either answer)

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8.	*(a)*	$-2\operatorname{cosec}^2 2x$	M1 ($k \operatorname{cosec}^2 2x$) A1 ($k = -2$)
	(b)	$x^2 \frac{1}{x} + 2x \ln x = x + 2x \ln x$	M1 ($x^2 f(x) + \ln x g(x)$)
			A1 ($f(x) = \frac{1}{x}$, $g(x) = 2x$ and simplification)
	(c)	$\frac{(x^2-2)(2x)-(x^2+1)(2x)}{(x^2-2)^2}$	M1 $\left(\frac{(x^2-2)f(x)-(x^2+1)g(x)}{(x^2-2)^2}\right)$
			A1 ($f(x) = 2x$, $g(x) = 2x$)
		$= \frac{-6x}{(x^2-2)^2}$	A1 (simplified form, C.A.O.)
			7
9.	*(a)*	Range is $[-2, \infty)$,	B1
	(b)	Let $y - (x+1)^2 - 2$ $y + 2 = (x+1)^2$	M1 ($y \pm 2 = (x+1)^2$) and attempt to isolate x
		$x = -1 \pm \sqrt{y+2}$	A2 (A1 for $-1 + \sqrt{y+2}$
		Take – ve square root since domain is $x \le -1$	A1 (correct choice of sign)
		$x = -1 - \sqrt{x+2}$	
		$f^{-1}(x) = -1 - \sqrt{x+2}$	B1 (F.T expression for x)
		Domain is $[-2, \infty)$, Range is $(-\infty, -1]$ (or $x \ge -2$) (or $f^{-1}(x) \le -1$)	B1 (both F.T. from (*a*))
			7

10. *(a)* $f(-1) = 2\,e{-1} \approx 0{\cdot}736$ B1

$gf{-1}$ cannot be formed since domain

of g is $x \geq 1$ B1

(b) $fg(x) = 2e^{\,3lnx}$ B1

$= 2\text{e}^{\,lnx3}$ B1

$= 2x^3$ B1

Domain is $[1, \infty)$, range is $[2, \infty)$ (o.e.) B1, B1

7

MATHEMATICS C4

1.	*(a)*	Let $\frac{1}{x^2(2x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$	M1 (Correct form)	
		$1 \equiv Ax(2x-1) + B(2x-1) + Cx^2$	M1 (correct clearing and attempt to substitute)	
		$\underline{x=0}$ $\quad 1 = B(-1) \quad \therefore B = -1$		
		$\underline{x=\frac{1}{2}}$ $\quad 1 = C\frac{1}{4} \quad \therefore C = 4$	A1 (2 constants C.A.O.)	
		$\underline{x^2}$ $\quad 0 = 2A + C \quad \therefore A = -2$	A1 (third constant, F.T. one slip)	
		(no need for display)		
	(b)	$\int \left(\frac{-2}{x} - \frac{1}{x^2} + \frac{4}{2x-1}\right)dx$		
		$= -2\ln\lvert x\rvert + \frac{1}{x} + 2\ln\lvert 2x-1\rvert$ $(+C)$	B1,B1.B1	
				7
2.		$2x + x\frac{dy}{dx} + y + 4y\frac{dy}{dx} = 0$	B1 $(x\frac{dy}{dx} + y)$ B1 $(4y\frac{dy}{dx})$	
		$\frac{dy}{dx} = 5$	B1 (C.A.O.)	
		Gradient of normal $= -\frac{1}{5}$	M1 ($\frac{-1}{candidate's\,\frac{dy}{dx}}$, *numerical value*)	
		Equation of normal is $y - 1 = -\frac{1}{5}(x+3)$	A1 (F.T. candidate's value)	
				5
3.	*(a)*	$R\sin\alpha = 2$, $R\cos\alpha = 3$ $R = \sqrt{13}$, $\alpha = \tan^{-1}\left(\frac{2}{3}\right) = 33.7° or 34°$	B1 ($R = \sqrt{13}$) M1 (correct method for α) A1 ($\alpha = 34°$)	
	(b)	$\cos(x - 33.7°) = \frac{1}{\sqrt{13}}$		
		$x - 33.7° = 73.9°, 286.1°$ $x = 107.6°, 319.8°$	B1 (one value) B1, B1	**6**

4.	Volume $= \pi \int_1^4 \left(x + \frac{3}{\sqrt{x}} \right)^2 dx$	B1
	$= \pi \int_1^4 \left(x^2 + 6\sqrt{x} + \frac{9}{x} \right) dx$	M1 (attempt to square, at least 2 correct terms) A1 (all correct)
	$= \pi \left[\frac{x^3}{3} + 4x^{\frac{3}{2}} + 9 \ln x \right]_1^4$	A3 (integration of 3 terms, F.T. similar work $Ax^2 + B\sqrt{x} + \frac{c}{x}$)
	$= \pi [49 + 9 \ln x] \simeq 193.1$ or 61.48π	A1 (C.A.O.)
		7
5. *(a)*	$\frac{dy}{dx} = \frac{-2\sin 2t}{4\cos t}$	M1 ($\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$) B1 ($4\cos t$) M1 ($k \sin 2t, k = -1, \pm 2, -\frac{1}{2}$) A1 ($k = -2$)
	$= \frac{-4 \sin t \cos t}{4 \cos t} = -\sin t$	M1 (correct use of formula) A1 (C.A.O.)
(b)	Equation of tangent is $y - \cos 2p = -\sin p (x - 4\sin p)$	M1 ($y - y_1 = m(x - x_1)$)
	$x \sin p + y = \cos 2p + 4\sin^2 p$	
	$= 1 - 2\sin^2 p + 4\sin^2 p$	M1 (attempt to use correct formula)
	$= 1 + 2\sin^2 p$	A1
		9

6. *(a)* $\int (3x+1)e^{2x}dx = (3x+1)\frac{e^{2x}}{2} - \int \frac{3}{2}e^{2x}dx$ M1 ($f(x)(3x+1) - \int 3f(x)dx$)

A1 ($f(x) = ke^{2x}, k = 1, \frac{1}{2}, 2$)

A1 ($k = \frac{1}{2}$)

$= (3x+1)\frac{e^{2x}}{2} - \frac{3e^{2x}}{4}(+C)$ A1 (F.T. one slip)

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{9 - 9\sin^2\theta}.3\cos\theta d\theta$ B1 (for first line, unsimplified)

B1 (simplified without limits)

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9\cos^2\theta d\theta$ B1 (limits)

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$ M1 ($\cos^2\theta = a + b\cos 2\theta$)

A1 ($a = b = \frac{1}{2}$)

$= \left[\frac{9\theta}{2} + \frac{9\sin 2\theta}{4}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ M1 ($k\sin 2\theta, k = \pm\frac{b}{2}, 2b, b$)

A1 ($k = \frac{b}{2}$)

$= \frac{3\pi}{2} - \frac{9\sqrt{3}}{8} \quad or \quad 2.764$ A1 (C.A.O.)

12

7.	*(a)*	$\frac{dW}{dt} = kW \qquad (k > 0)$	B1
	(b)	$\int \frac{dW}{W} = \int k dt$	M1 (attempt to separate variables)
		$\ln W = kt + C$	A1 (allow absence of *C*)
		$t = 0, \quad W = 0.1, \quad C = \ln 0.1$	B1 (value of *C*)
		$\ln \frac{W}{0.1} = kt$	M1 (use of logs or exponentials)
		$\frac{W}{0.1} = e^{kt}$	
		$k = 3.0007$	B1 (value of k)
		$W = 0.1e^{3t}$	A1
			7
8.	*(a)(i)*	**AB** = **i** + 2**j** – 2**k**	B1 (**AB**)
	(ii)	Equation of AB is **r** = 4**i** – **j** +**k** + λ (**i** + 2**j** – 2**k**)	M1 (reasonable attempt to write equations) A1 (must contain **r**, F.T. candidate's **AB**)
	(b)	(Point of intersection is on both lines) Equate coeffs of **i** and **j** (any two of **i,j,k**)	M1 (attempt to write equations using candidate's equation)
		$1 + \mu = 4 + \lambda$	A1 (2 correct equations, F.T. candidate's equations)
		$\lambda = \frac{1}{3} \quad \left(\mu = \frac{10}{3}\right)$	M1 (attempt to solve)
			A1 (C.A.O.)
		Position vector is $\frac{13}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$	A1 (F.T. value of λ *or* μ)
	(c)	angle between $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$ is required	B1 (coeffs of λ and μ)
		$\lvert \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} \rvert = 3, \ \lvert \mathbf{i} - \mathbf{j} + \mathbf{k} \rvert = \sqrt{3}$	B1 (for one modulus)
		$(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}).(\mathbf{i} - \mathbf{j} + \mathbf{k}) = \lvert \quad \rvert \times \lvert \quad \rvert \cos\theta$	M1 (use of correct formula)
		$1 - 2 - 2 = 3\sqrt{3}\cos\theta$	B1 (l.h.s. unsimplified)
		$\theta = 125.3°$	A1 (C.A.O.)
			13

9. $(1+3x)(1-2x)^{-\frac{1}{2}} = (1+3x)\left(1+\left(-\tfrac{1}{2}\right)(-2x)+\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\tfrac{1}{2!}1(-2x)^2+\cdots\right)$

B1, B1 (unsimplified)

$= (1+3x)\left(1+x+\tfrac{3}{2}x^2+\cdots\right)$

$= 1+4x+\dfrac{9x^2}{2}+\cdots$ B1 $(1+4x)$

B1 $\left(\dfrac{9x^2}{2}\right)$

Exparsim is valid for $|x|<\frac{1}{2}$ B1

5

10. $\left(x^2+49<14x\right)$

$x^2-14x+49<0$ B1

$(x-7)^2<0$ B1

$x-7$ is not real B1

contradiction B1 (accept impossibility)

$\left(\therefore x+\dfrac{49}{x}\geq 14 \quad \textit{for all real and positive } x\right)$

4

A/AS Level Mathematics - FP1 – June 2008 – Markscheme – Draft 4 (Post Stand)

1		$S_n = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	M1
		$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$	A1A1
		$= \frac{n(n+1)(3n^2+3n+4n+2)}{12}$	M1
		$= \frac{n(n+1)(3n^2+7n+2)}{12}$	A1
		$= \frac{n(n+1)(n+2)(3n+1)}{12}$	A1
2	(a)	det**A** = 2(2 – 2) + 4(2 – 1) +2(1 – 2) = 2	M1A1
		Cofactor matrix $= \begin{bmatrix} 0 & 1 & -1 \\ -2 & 0 & 2 \\ 4 & -2 & 0 \end{bmatrix}$	M1A1
		[Accept matrix of minors]	
		Adjugate matrix $= \begin{bmatrix} 0 & -2 & 4 \\ 1 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}$	A1
		[Award A0A0 if rule of signs not used]	
		Inverse matrix $= \frac{1}{2}\begin{bmatrix} 0 & -2 & 4 \\ 1 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}$	A1
	(b)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -2 & 4 \\ 1 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 8 \\ 8 \\ 5 \end{bmatrix}$	M1
		$= \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$	A1
		[FT from their inverse]	
3	(a)	$(2-\mathrm{i})^2 = 4 - 4\mathrm{i} + \mathrm{i}^2 = 3 - 4\mathrm{i}$	M1A1
		$\frac{7-4\mathrm{i}}{2+\mathrm{i}} = \frac{(7-4\mathrm{i})(2-\mathrm{i})}{(2+\mathrm{i})(2-\mathrm{i})} = 2 - 3\mathrm{i}$	M1A1
		$z = 3 - 4\mathrm{i} + 2 - 3\mathrm{i} - 8 = -3 - 7\mathrm{i}$ (cao)	A1
	(b)	Mod(z) = $\sqrt{58}$ (7.62)	B1
		$\tan^{-1}(7/3) = 1.17$ or 67°	B1
		Arg(z) = 4.31 or 247°	B1

[FT from (a) but only for the 2nd B1 in arg if their z is in the 2nd or 3rd quadrant]

4 (a)

$2x + y + 3z = 5$

$-5y + z = 7$ M1A1A1

$5y - z = k - 10$

[Or equivalent row operations]

$2x + y + 3z = 5$

$-5y + z = 7$ A1

$0 = k - 3$

[Award this A1 for -7 = k – 10]

$k = 3$ (cao) A1

(b) Let $z = \alpha$. M1

$y = \frac{\alpha - 7}{5}$ A1

$x = \frac{16 - 8\alpha}{5}$ A1

[FT from (a)]

Other possible solutions are

Let $x = \alpha$

$y = \frac{-8-\alpha}{8}; z = \frac{16-5\alpha}{8}$

Let $y = \alpha$

$z = 5\alpha + 7; x = -8\alpha - 8$

5 The statement is true for n = 1 since 7 + 5 is divisible by 6. B1

Let the statement be true for $n = k$, ie $7^k + 5$ is divisible by 6 (so that $7^k + 5 = 6N$). M1

Consider $7^{k+1} + 5 = 7 \times 7^k + 5$ M1A1

$= 7(6N - 5) + 5$ A1

$= 42N - 30$ A1

This is divisible by 6 so true for $n = k \Rightarrow$ true for $n = k + 1$, hence proved by induction. A1

6 (a) Let the roots be α, β, γ. Then

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\beta\gamma + \gamma\alpha + \alpha\beta = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$ B1

[Award this B1 if candidates go straight to what follows]

Now use the fact that the roots are α, 2α and 4α. M1

Then

$$(1 + 2 + 4)\alpha = -\frac{b}{a}$$

$$(2 + 4 + 8)\alpha^2 = \frac{c}{a}$$ A2

$$(1 \times 2 \times 4)\alpha^3 = -\frac{d}{a}$$

[Award -1 for each error]

It follows that

$$\frac{7\alpha \times 14\alpha^2}{8\alpha^3} = \frac{-b}{a} \times \frac{c}{a} \times \frac{-a}{d}$$ M1A1

$$\frac{98}{8} = \frac{bc}{ad}$$ A1

$$4bc = 49ad$$

(b) $7\alpha = \frac{42}{8}$ so $\alpha = \frac{3}{4}$ M1A1

The roots are $\frac{3}{4}, \frac{3}{2}, 3$ A1

7 (a) Rotation matrix $= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

Translation matrix $= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ B1

[Award this B1 for writing their matrices in the right order]

$$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Fixed points satisfy

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ M1

giving

$$-y+1 = x$$

$$x+2 = y$$ A1

The solution is $(x, y) = \left(-\frac{1}{2}, \frac{3}{2}\right)$. M1A1

(c) Under T,

$$x' = -y+1$$

$$y' = x+2$$ M1A1

or

$$x = y'-2$$

$$y = -x'+1$$ A1

So $y = 2x - 1$ becomes M1

$$-x'+1 = 2(y'-2)-1$$

$$x'+2y' = 6$$ A1

[FT their x,y]

Alternatively, they might show that the image of $(x, 2x-1)$ is $(2-2x, x+2)$ for which award M1A1A1, then M1A1 for eliminating x.

8 (a) $\ln f(x) = \cos x \ln x$ M1

$\dfrac{f'(x)}{f(x)} = -\sin x \ln x + \dfrac{\cos x}{x}$ A1A1

$f'(x) = x^{\cos x}\left(-\sin x \ln x + \dfrac{\cos x}{x}\right)$ A1

(b) At a stationary point,

$\sin x \ln x = \dfrac{\cos x}{x}$ M1

$\tan x \ln x = \dfrac{1}{x}$ A1

leading to the printed result.

(c) When $\alpha = 1.27$, $\alpha \ln\alpha \tan\alpha = 0.978..$ or $\alpha \ln\alpha \tan\alpha - 1 = -0.021..$

When $\alpha = 1.28$, $\alpha \ln\alpha \tan\alpha = 1.055..$ or $\alpha \ln\alpha \tan\alpha - 1 = 0.055..$ M1

Since these values are either side of 1 (or 0), the root lies between 1.27 and 1.28. A1

[The actual values and an appropriate comment are required for A1]

9 (a) $$x + \mathrm{i}y + 1 = \frac{1}{u + \mathrm{i}v} = \frac{u - \mathrm{i}v}{u^2 + v^2}$$ M1A1

Equating real and imaginary parts, M1

$$x + 1 = \frac{u}{u^2 + v^2}\ ;\ y = \frac{-v}{u^2 + v^2}$$ A1

(b) Substituting

$$\left(\frac{u}{u^2 + v^2}\right)^2 + \left(\frac{-v}{u^2 + v^2}\right)^2 = 4$$ M1A1

Simplifying, $u^2 + v^2 = \frac{1}{4}$ M1A1

A/AS Mathematics -FP2 – June 2008 – Markscheme – Final Draft

1 (a) Odd because $f(-x) = \dfrac{-x}{(-x)^2 + 1} = -f(x)$ B1B1

(b) Neither because $f(-x) = e^{-x} + 1$ which is neither $f(x)$ or $f(-x)$. B1B1

2 For $x < 2$, $f'(x) = 3ax^2$ B1

For $x \ge 2$, $f'(x) = 2bx$ B1

$1 + 8a = 4b - 3$, $12a = 4b$ B1B1

The solution is $a = 1$, $b = 3$ cao M1A1

3 (a) $u = x^2 \Rightarrow du = 2xdx$ B1

and $[0,\sqrt{3}] \to [0, 3]$ B1

$I = \dfrac{1}{2}\displaystyle\int_0^3 \frac{du}{9+u^2}$ M1

$= \dfrac{1}{6}\left[\arctan\left(\dfrac{u}{3}\right)\right]_0^3$ A1

$= \dfrac{\pi}{24}$ A1

(b) $\displaystyle\int_0^1 \frac{dx}{\sqrt{25-9x^2}} = \frac{1}{3}\int_0^1 \frac{dx}{\sqrt{25/9 - x^2}}$ M1

$= \dfrac{1}{3}\left[\sin^{-1}\left(\dfrac{3x}{5}\right)\right]_0^1$ A1

$= \dfrac{1}{3}\sin^{-1} 0.6$ A1

$= 0.2145$ A1

[FT on minor arithmetic error, do not FT the omission of the factor 1/3]

4 (a) Substituting, M1

$2 \times \dfrac{2t}{1+t^2} + 3 \times \dfrac{1-t^2}{1+t^2} = 1$ A1

$4t + 3 - 3t^2 = 1 + t^2$ A1

$2t^2 - 2t - 1 = 0$ AG

(b) $t = \dfrac{2 \pm \sqrt{12}}{4}$ (-0.366, 1.366) B1

Either

$\dfrac{\theta}{2} = -0.350879..$ or $\dfrac{\theta}{2} = 0.93888..$ B1

General solution is

$\dfrac{\theta}{2} = -0.350879.. + n\pi$ or $\dfrac{\theta}{2} = 0.93888.. + n\pi$ M1

$\theta = -0.702/5.58 + 2n\pi$, $\theta = 1.88 + 2n\pi$ A1A1

[FT from their particular solution]

5 (a) $\dfrac{dy}{dx} = \dfrac{dy/dp}{dx/dp} = \dfrac{2a}{2ap} = \dfrac{1}{p}$ M1A1

Gradient of normal = $-p$ A1

Equation of normal is

$y - 2ap = -p(x - ap^2)$ M1

whence the printed result.

(b)(i) Putting $y = 0$, M1

$x = a(2 + p^2)$ A1

Coords of R are $(x,y) = \left(\dfrac{1}{2}(ap^2 + a\{2 + p^2\}), \dfrac{1}{2}(0 + 2ap)\right)$ M1

$= \left(a(1 + p^2), ap\right)$ A1

(ii) Eliminating p,

$x - a = a \times \dfrac{y^2}{a^2}$ or $y^2 = a(x - a)$ M1A1

Coordinates of focus = $(a + \dfrac{a}{4}, 0)$, ie $(\dfrac{5a}{4}, 0)$ M1A1

6 (a) $z^{-n} = \cos(-n\theta) + i\sin(-n\theta)$ M1

$= \cos n\theta - i\sin n\theta$ A1

$z^n - z^{-n} = \cos n\theta + i\sin n\theta - (\cos n\theta - i\sin n\theta)$ M1

$= 2i\sin n\theta$

(b) $\left(z - \dfrac{1}{z}\right)^3 = z^3 - 3z^2.\left(\dfrac{1}{z}\right) + 3z.\left(\dfrac{1}{z}\right)^2 - \left(\dfrac{1}{z}\right)^3$ M1A1

$= z^3 - \dfrac{1}{z^3} - 3\left(z - \dfrac{1}{z}\right)$ m1

$(2i\sin\theta)^3 = 2i\sin 3\theta - 6i\sin\theta$ A1

$\sin^3\theta = \dfrac{3}{4}\sin\theta - \dfrac{1}{4}\sin 3\theta$ A1

7 (a) Let $\dfrac{5-3x}{(x-1)(x-3)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x-3} = \dfrac{A(x-3)+B(x-1)}{(x-1)(x-3)}$ M1

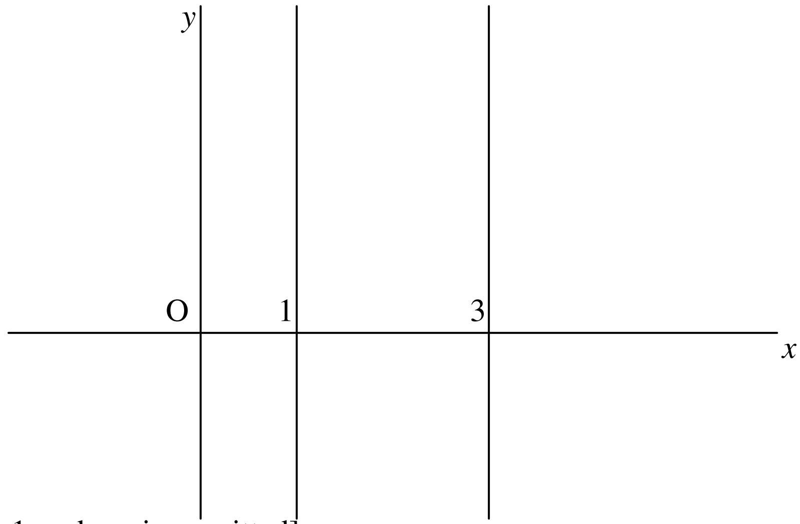
$x = 1$ gives $A = -1$; $x = 3$ gives $B = -2$ A1A1

(b) $f'(x) = \dfrac{1}{(x-1)^2} + \dfrac{2}{(x-3)^2}$ B1

This is the sum of 2 positive quantities which can never be zero so no stationary points. M1A1

[Accept a solution which obtains the derivative from the original form of $f(x)$ and then shows that the numerator has no real zeroes]

(c)



G2

[-1 each region omitted]

(i) (5/3, 0); (0,5/3) B1B1

(ii) $x = 1$, $x = 3$, $y = 0$ B1B1B1

(d) Consider

$$\frac{5-3x}{(x-1)(x-3)} = 1$$ M1

$$5 - 3x = x^2 - 4x + 3$$

$$x^2 - x - 2 = 0$$ A1

$$x = -1, 2$$ A1

$f^{-1}(A) = (-\infty, -1) \cup (5/3, 2)$ A1A1

8 (a) Modulus = 8, Argument = 90° B1B1

First cube root = $\sqrt[3]{8}, \angle 90/3$ M1A1

$= \sqrt{3} + \mathrm{i}$ A1

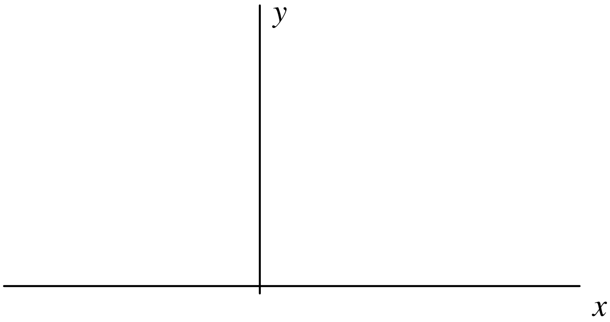
Second cube root = $\sqrt[3]{8}, \angle(90/3 + 120)$ M1A1

$= -\sqrt{3} + \mathrm{i}$ A1

Third cube root = $\sqrt[3]{8}, \angle(90/3 + 240)$ M1

$= -2\mathrm{i}$ A1

1 (a)



G2

[Award G1 for a graph of the difference]

The two graphs intersect at the two points shown, one being $x = 0$ and the other $x > 0$. B1

(b) The Newton-Raphson iteration is

$$x \to x - \frac{(\cosh x - 1 - \sin x)}{(\sinh x - \cos x)}$$ M1A1

Starting with $x = 1.5$, we obtain

1.5

1.327589340 M1A1

1.295884299

1.294832874

1.294831731 A1

So root = 1.2948 (correct to 4 dps) A1

2 $dx = \cosh\theta d\theta$; $[1,2] \to [0, \sinh^{-1} 1]$ (0.881) B1B1

$$I = \int_0^{\sinh^{-1} 1} \sqrt{1 + 2\sinh\theta + \sinh^2\theta - 2 - 2\sinh\theta + 2}\ \cosh d\theta$$ M1

$$= \int_0^{\sinh^{-1} 1} \cosh^2\theta\, d\theta$$ A1

$$= \frac{1}{2}\int_0^{\sinh^{-1} 1} (1 + \cosh 2\theta)\, d\theta$$ M1

$$= \frac{1}{2}\left[\theta + \frac{1}{2}\sinh 2\theta\right]_0^{\sinh^{-1} 1}$$ (FT on minor integration error] A1

$$= \frac{1}{2}\left[\sinh^{-1} 1 + \frac{1}{2}\sinh(2\sinh^{-1} 1)\right]$$ A1

= 1.15 (cao) A1

3	(a)	$f(1) = 1$	B1
		$f'(x) = -\frac{1}{2}x^{-3/2}$, $f'(1) = -\frac{1}{2}$	B1
		$f''(x) = \frac{3}{4}x^{-5/2}$, $f''(1) = \frac{3}{4}$	B1
		$\frac{1}{\sqrt{x}} = 1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 + \ldots$	B1
	(b)	$f\left(\frac{8}{9}\right) \approx 1 + \frac{1}{2} \times \frac{1}{9} + \frac{3}{8} \times \frac{1}{81}$	M1
		$\frac{3}{2\sqrt{2}} \approx \frac{687}{648}$ (or $\frac{3\sqrt{2}}{4} \approx \frac{687}{648}$)	M1A1
		$\sqrt{2} \approx \frac{324}{229}$ or ($\frac{229}{162}$) cao	A1

4	(a) Consider		
		$1 - \tanh^2 x = 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2}$	M1
		$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$	A1
		$= \frac{4}{(e^x + e^{-x})^2}$	M1A1
		$= \operatorname{sech}^2 x$	AG
	(b)	$5 - 5\tanh^2 x = 11 - 13\tanh x$	M1
		$5\tanh^2 x - 13\tanh x + 6 = 0$	A1
		$\tanh x = \frac{3}{5}$, 2	M1A1
		$\tanh x = 2$ has no solution.	B1
		$x = \frac{1}{2}\ln\left(\frac{1+3/5}{1-3/5}\right)$	M1A1
		$= \ln 2$	A1

Alternative solution:

Substitute for sech and tanh in terms of exponential functions,

$e^{4x} - e^{2x} - 12 = 0$	M1A1
$e^{2x} = 4$ or -3	M1A1
$e^{2x} = -3$ has no solution	B1
$e^{2x} = 4$ gives $2x = \ln 4$	M1A1
$x = \ln 2$	A1

5 (a) $I_n = \int_1^2 (\ln x)^n \mathrm{d}\left(\frac{1}{2}x^2\right)$ M1 A1

$= \left[\frac{1}{2}x^2(\ln x)^n\right]_1^2 - \int_1^2 \frac{1}{2}x^2 . n(\ln x)^{n-1} . \frac{1}{x}\mathrm{d}x$ A1A1

$= 2(\ln 2)^n - \frac{n}{2}I_{n-1}$ A1

(b) $I_2 = 2(\ln 2)^2 - I_1$ M1

$= 2(\ln 2)^2 - \left(2\ln 2 - \frac{1}{2}I_0\right)$ A1

$= 2(\ln 2)^2 - 2\ln 2 + \frac{1}{2}\times\left[\frac{x^2}{2}\right]_1^2$ A1

$= 2(\ln 2)^2 - 2\ln 2 + \frac{3}{4}$ A1

$= 0.325$ A1

6 (a) $\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2} = \sqrt{(-3\cos^2\theta\sin\theta)^2 + (3\sin^2\theta\cos\theta)^2}$ M1A1

$= \sqrt{9\cos^4\theta\sin^2\theta + 9\sin^4\theta\cos^2\theta}$ A1

$= \sqrt{9\sin^2\theta\cos^2\theta(\cos^2\theta + \sin^2\theta)}$ A1

$= 3\sin\theta\cos\theta$ A1

$= \frac{3}{2}\sin 2\theta$ AG

(b)(i) Length $= \int_0^{\pi/2} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ M1

$= \int_0^{\pi/2} \frac{3}{2}\sin 2\theta\,\mathrm{d}\theta$ A1

$= \left[-\frac{3}{4}\cos 2\theta\right]_0^{\pi/2}$ A1

$= \frac{3}{2}$ A1

(ii) CSA $= \int_0^{\pi/2} y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ M1

$= 2\pi\int_0^{\pi/2} \sin^3\theta . 3\sin\theta\cos\theta\,\mathrm{d}\theta$ A1

= answer given

$= \frac{6\pi}{5}\left[\sin^5\theta\right]_0^{\pi/2}$ M1A1

$= \frac{6\pi}{5}$ (3.77) A1

7 (a)(i) $\frac{dy}{d\theta} = \frac{d}{d\theta}[(1-\theta)\sin\theta]$ M1

$= (1-\theta)\cos\theta - \sin\theta$ A1

$= 0$ when $(1 - \theta) = \tan\theta$ A1

leading to the printed answer.

(ii) Area = $\frac{1}{2}\int_0^1 (1-\theta)^2 d\theta$ M1A1

$= \frac{1}{2}\left[\frac{-(1-\theta)^3}{3}\right]_0^1$ A1

$= \frac{1}{6}$

(b)(i) At the point of intersection, $1-\theta = 2\theta^2$ M1

$2\theta^2 + \theta - 1 = 0$ A1

$(2\theta-1)(\theta+1) = 0$

$\theta = \frac{1}{2}$ A1

$r = \frac{1}{2}$ A1

(c) Area = $\frac{1}{2}\int_0^{1/2} (2\theta^2)^2 d\theta$ M1A1

$= \frac{2}{5}\left[\theta^5\right]_0^{1/2}$ A1

$= \frac{1}{80}$ A1

[Award M1A1 for evaluating this integral but then mis-using it subsequently]

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GENERAL CERTIFICATE OF EDUCATION
TYSTYSGRIF ADDYSG GYFFREDINOL

MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3
AS/Advanced

SUMMER 2008

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2008 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics M1(June 2008)

Markscheme

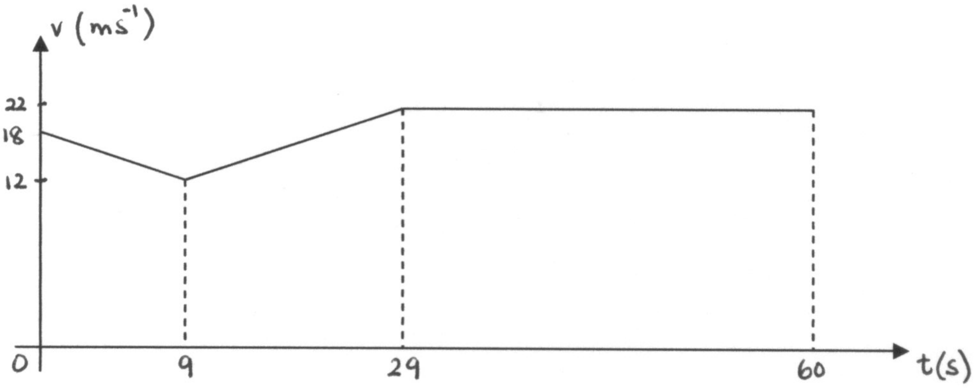
1.(a) Using $v = u + at$ with $v = 22$, $u = 12$, $a = 0.5$ M1

$22 = 12 + 0.5t$

$t = \dfrac{22 - 12}{0.5}$

$= \underline{20 \text{ s}}$ cao A1

1.(b)



attempt at v-t graph M1

Straight line joining (0, 18) to (9, 12) A1

Straight line joining c's (9, 12) to ((c's 29, 22) A1

Straight line joining (c's 29, 22) to (c's 29+31, 22) + units and labels A1

2.(a) Using $v = u + at$ with $v = 0$, $a = (-)9.8$, $t = 2.5$ M1

$0 = u + (-9.8) \times 2.5$

$u = \underline{24.5 \text{ ms}^{-1}}$ A1

2.(b) Using $s = ut + 0.5at^2$ with $u = 24.5$, $a = (-)9.8$, $t = 4$ M1

$s = 24.5 \times 4 + 0.5 \times (-9.8) \times 4^2$ A1

$= \underline{19.6 \text{ m}}$ cao A1

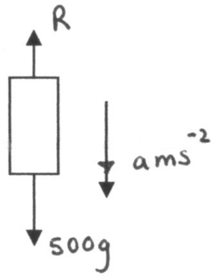
2.(c) Using $v^2 = u^2 + 2as$ with $u = 24.5$, $a = (-)9.8$, $s = (-)70$ M1

$v^2 = 24.5^2 + 2 \times (-9.8) \times (-70)$ A1

$= 1972.25$

$v = \underline{44.41 \text{ ms}^{-1}}$ cao A1

3.

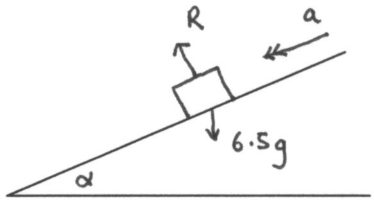


(a)	N2Lapplied to lift and man		M1
	$-R + 500g = 500a$		A1
	$500a = 500 \times 9.8 - 4800$		
	$a = \underline{0.2 \text{ ms}^{-2}}$	cao	A1
(b)	N2L applied to man		M1
	$70g - R = 70a$		A1
	$R = \underline{672 \text{ N}}$	ft a	A1

4.(a)	Apply N2L to *B*		M1
	$9g - T = 9a$		B1
	Apply N2l to *A*		
	$T - 5g = 5a$		A1
	Adding $4g = 14a$		m1
	$a = \dfrac{4 \times 9.8}{14}$		
	$= \underline{2.8 \text{ ms}^{-2}}$	cao	A1
4.(b)	$T = 5(g + a)$		
	$= 5(9.8 + 2.8)$		
	$= \underline{63 \text{ N}}$	cao	A1

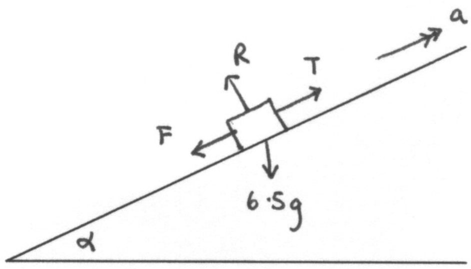
5.(a)	Impulse = change in momentum		
	$= 0.7(2 - 5)$		M1
	$= \underline{-2.1 \text{ Ns}}$		A1 B1
	Direction of impulse is opposite to motion.	oe	B1
5.(b)	Speed after impact $= 2 \times 0.6$		M1
	$= \underline{1.2 \text{ ms}^{-1}}$		A1

6.(a)



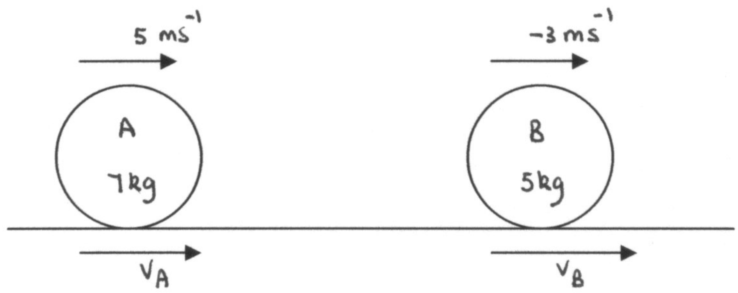
Resolve perpendicular to slope		M1
$R = 6.5\, g \cos \alpha$		A1
$= 6.5 \times 9.8 \times \frac{12}{13}$		
$= 58.8$		
$F = \frac{1}{5} \times 58.8$	ft	B1
$= 11.76$ N		
N2l down slope	3 terms, dimensionally correct	M1
$6.5g \sin \alpha - F = 6.5\, a$		A1 A1
$a = \dfrac{6.5 \times 9.8 \times \frac{5}{13} - 11.76}{6.5}$		
$= \underline{1.96 \text{ ms}^{-2}}$	cao	A1

6.(b)



N2L up slope	4 terms, dimensionally correct	M1
$T - F - 6.5\, g \sin \alpha = 6.5\, a$		A1
Constant speed, therefore $a = 0$	s.i.	B1
$T = 11.76 + 6.5 \times 9.8 \times \frac{5}{13}$		
$= \underline{36.26 \text{ N}}$	cao	A1

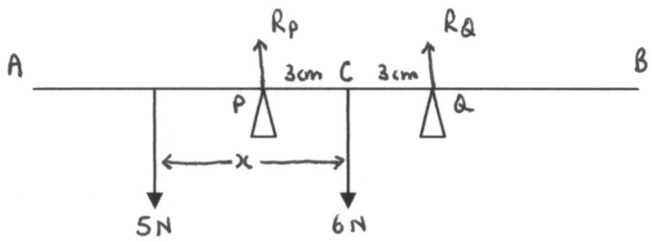
7.



Momentum			M1
	$7 \times 5 + 5 \times (-3) = 7v_A + 5v_B$		A1
Restitution			M1
	$v_B - v_A = -0.2(-3 - 5)$		A1
	$5v_B - 5v_A = 8$		
Subtract	$12 v_A = 12$		m1
	$v_A = \underline{1 \text{ ms}^{-1}}$	cao	A1
	$v_B = \underline{2.6 \text{ ms}^{-1}}$	cao	A1

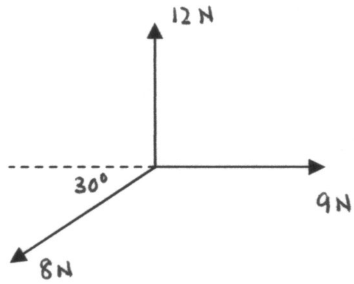
Sphere *A* is moving in the same direction as before impact. Sphere *B* has reversed its direction of motion. A1

8.



Moments about *P* to obtain equation			M1
	$5(x - 3) = 6 \times 3 - 6 \times R_Q$		A1 B1
	$5x - 15 = 18 - 6R_Q$		
	$6R_Q = 33 - 5x$		
For limiting equilibrium	$R_Q = 0$	si	M1
	$33 - 5x = 0$		
	$5x = 33$		
	$x = \underline{6.6}$	cao	A1

9.



Resolve perpendicular to 12 N			M1
	$X = 9 - 8\cos 30°$		A1
	$= 9 - 8 \times \frac{\sqrt{3}}{2}$		
	$= 2.0718$		
Resolve parallel to 12 N			M1
	$Y = 12 - 8\sin 30°$		A1
	$= 12 - 8 \times 0.5$		
	$= 8$		
Magnitude of resultant $= \sqrt{8^2 + \quad 2.0718^2}$			M1
	$= \underline{8.264 \text{ N}}$	ft X, Y	A1
	$\theta = \tan^{-1}\left(\frac{8}{2{,}0718}\right)$		M1
	$= \underline{75\ 48°}$	cao	A1

10.(a)

	Area	from *AB*	from *AC*	
triangle *ABC*	108	6	4	B1
rectangle *DEFG*	20	5	2.5	B1
lamina	88	*x*	*y*	B1

Moments about *AB* to obtain equation			M1
	$20 \times 5 + 88x = 108 \times 6$	ft	A1
	$x = \underline{6.23}$	cao	A1
Moments about *AC* to obtain equation			M1
	$20 \times 2.5 + 88y = 108 \times 4$	ft	A1
	$y = \underline{4.34}$	cao	A1

10.(b)	$\tan\theta = \frac{4.34}{6.23}$		M1
	$\theta = \underline{34.9°}$	ft x, y	A1

Mathematics M2 (June 2008)

Markscheme

1.(a) Using $T = \frac{\lambda x}{l}$ with $T = 12$, $x = 0.55 - 0.3$, $l = 0.3$ M1

$12 = \frac{\lambda(0.55 - 0.3)}{0.3}$ A1

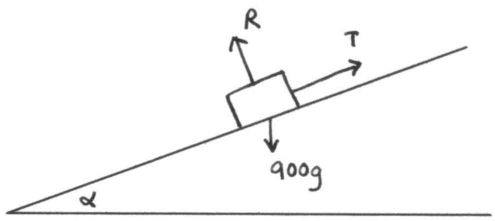
$\lambda = \frac{12 \times 0.3}{0.25}$

$= \underline{14.4 \text{ N}}$ cao A1

1.(b) Using EE $= \frac{1}{2} \times \frac{14.4 \times 0.25^2}{0.3}$ ft λ M1 A1

$= \underline{1.5 \text{ J}}$ ft λ A1

2.



$T = \frac{32 \times 1000}{16}$ si M1

$= 2000$ N

N2L up plane M1

$T - F - 900\, g \sin \alpha = 0$ A1

Resistive force $F = 2000 - 900 \times 9.8 \times \frac{8}{49}$

$= \underline{560 \text{ N}}$ cao A1

3.(a)	Using $F = ma$ $\quad 5a = 15t^2 - 60t$		M1
	$a = 3t^2 - 12t$		
	When $t = 2$ $\quad a = 12 - 24$		
	$= -12$		
	Therefore magnitude of acceleration = <u>-12 ms^{-1}</u>		A1
3.(b)	$v = \int 3t^2 - 12t \, dt$		M1
	$= t^3 - 6t^2 \; (+ \; C)$		A1
	When $t = 0$, $v = 35$		m1
	$C = 35$		A1
	$v = \underline{t^3 - 6t^2 + 35}$		
3.(c)	Least value of v when $a = 0$		M1
	$3t(t - 4) = 0$		
	$t = (0 \text{ or}) \; 4$	ft v	A1
	Therefore least value of $v = 4^3 - 6 \times 4^2 + 35$		
	$= \underline{3 \text{ ms}^{-1}}$	ft v	A1
3.(d)	Required distance $= \int_2^8 t^3 - 6t^2 + 35 \, dt$	attempt to integrate v	M1
	$= \left[\frac{t^4}{4} - 2t^3 + 35t \right]_2^8$	correct integration	A1
	$= (16 \times 64 - 16 \times 64 + 35 \times 8) - (4 - 16 + 70)$		m1
	$= 280 - 58$		
	$= \underline{222 \text{ m}}$	cao	A1
4.(a)	Difference in PE = $2232 \times 90\,g - 2128 \times 90\,g$ +	PE	M1 A1
	$= (1968624 - 1876896)$		
	$= 91728$ J		
	Difference in KE = $0.5 \times 90 \times 35^2 - 0.5 \times 90 \times 2^2$	KE	M1 A1
	$= 55125 - 180$		
	$= 54945$ J		
	Work done against resistance = 36783 J		M1 A1
4.(b)	Work done = $1335\,R = 36783$		M1
	$R = \underline{27.55 \text{ N}}$	ft WD if M's gained in (a)	A1

5.(a) Using $s = ut + 0.5at^2$ with $u = 14$, $a = (-)9.8$, $s = 8.4$ — M1

$8.4 = 14t - 4.9t^2$ — A1

$7t^2 - 20t + 12 = 0$ — attempt to solve — m1

$(7t - 6)(t - 2) = 0$

$t = 6/7, 2$ — A1

As particle is on the way down, $t = 2$

Therefore horizontal distance of wall $= 2 \times 12$ — M1

$= \underline{24 \text{ m}}$ — cao — A1

5.(b) Using $v = u + at$ with $u = 14$, $a = (-)9.8$, $t = 2$ — M1

$v = 14 - 9.8 \times 2$ — ft t — A1

$= \underline{-5.6 \text{ ms}^{-1}}$ — ft t — A1

Therefore speed of motion $= \sqrt{5.6^2 + 12^2}$ — M1

$= \underline{13.24 \text{ ms}^{-1}}$ — ft v — A1

$\theta = \tan^{-1}\left(\frac{5.6}{12}\right)$ — M1

$= \underline{25°}$ to the horizontal. — ft v — A1

6.(a) $\underline{AB} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) - (2\mathbf{i} + \mathbf{j} + \mathbf{k})$ — M1

$= \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ — cao — A1

6.(b) Work done by $\mathbf{F} = \mathbf{F}.\underline{AB}$

$= (\mathbf{i} - 4\mathbf{j} + \mathbf{k}).(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ — M1

$= 1 + 8 + 1$ — ft — m1

$= \underline{10 \text{ J}}$ — ft — A1

7.(a) $\boldsymbol{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$ — used — M1

$= \underline{3 \cos 3t\,\mathbf{i} - 10 \sin 5t\,\mathbf{j} + 9t^2\,\mathbf{k}}$ — A2

7.(b) $\boldsymbol{r}_A = (-8t - 2)\mathbf{i} + (3t + 3)\mathbf{j}$

$\boldsymbol{r}_B = (-16t + 11)\mathbf{i} + (9t - 8)\mathbf{j}$

$\boldsymbol{r}_A - \boldsymbol{r}_B = (8t - 13)\mathbf{i} + (-6t + 11)\mathbf{j}$ — M1

$|r_A - r_B|^2 = (-13 + 8t)^2 + (11 - 6t)^2$ — M1 A1

$= 169 - 208t + 64t^2 + 121 - 132t + 36t^2$

$= 290 - 340t + 100t^2$

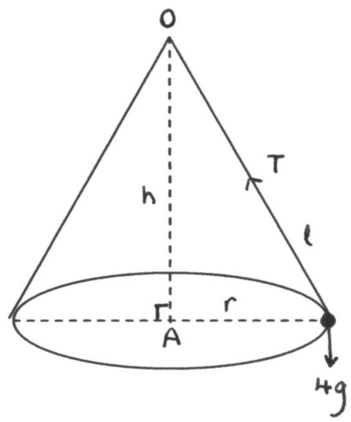
Minimum when $200t = 340$ — m1

$t = \underline{1.7}$ — cao — A1

Minimum distance $= \sqrt{290 - 340(1.7) + 100(1.7)^2}$ — M1

$= \underline{1}$ — cao — A1

8.



(a) N2L towards centre M1

$$T \sin \theta = \frac{mv^2}{r}$$ A1

$$= \frac{4 \times 2^2}{\frac{3}{7}} = \frac{112}{3}$$

Resolve vertically M1

$$T \cos \theta = 4 \times 9.8$$ A1

$$= 39.2$$

Dividing $\tan \theta = \frac{112}{3 \times 39.2}$ m1

$$\angle AOP = \theta = \tan^{-1}\left(\frac{112}{3 \times 39.2}\right)$$

$= \underline{43.6°}$ cao A1

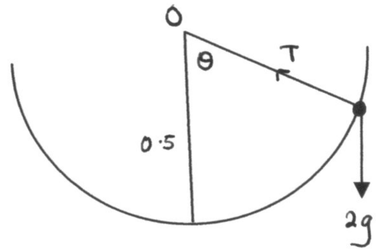
(b) $T = \frac{39.2}{\cos(43.6^\circ)}$

$= \underline{54.13 \text{ N}}$ ft angle A1

(c) $l = \frac{3}{7 \sin(43.6^\circ)}$

$= \underline{0.62 \text{ m}}$ ft angle B1

9.



(a) Energy considerations M1

$-2\,g \times 0.5 \cos\theta + 0.5\,mv^2 = -2\,g \times 0.5 \cos 60° + 0.5 \times 2 \times 4^2$ A1A1

$v^2 = g\cos\theta - 0.5\,g + 16$

$v^2 = 16 + g(\cos\theta - 0.5)$ A1

$= \underline{g\cos\theta + 11.1}$

(b) N2L towards centre M1

$T - mg\cos\theta = \dfrac{mv^2}{r}$ A1

$T = 2g\cos\theta + \dfrac{2}{0.5}\left(16 + g\cos\theta - \dfrac{1}{2}g\right)$ m1

$= 2g\cos\theta + 64 + 4g\cos\theta - 2g$

$= 6g\cos\theta - 2g + 64$ A1

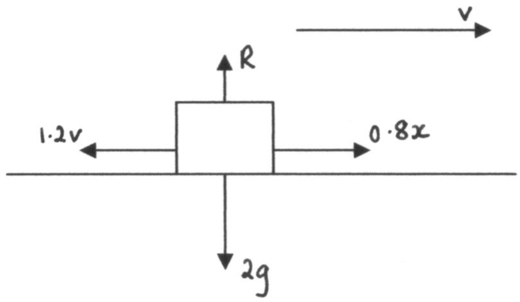
$T = \underline{58.8\cos\theta + 44.4}$

Mathematics M3 (June 2008)

Markscheme

1,(a)	R = $35g$		B1
	Friction $F = \mu R$		
	$= \frac{1}{7} \times 35\,g$		B1
	$= 5g = 49$ N		
	N2L	dim. correct	M1
	$-F - 0.7\,v = 35\,a$		
	$35\frac{\mathrm{d}v}{\mathrm{d}t} + 49 + 0.7v = 0$	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	m1
	Divide by 0.7 $\quad 50\frac{\mathrm{d}v}{\mathrm{d}t} + 70 + v = 0$	convincing	A1
(b)	$-\int \frac{50}{70 + v}\,\mathrm{d}v = \int \mathrm{d}t$		M1
	$-50 \ln (70 + v) = t\,(+ \mathrm{C})$		A1 A1
	Either limits or initial conditions used		m1
	$t = 0, v = 28$ $\quad$ C = $-50 \ln 98$		A1
	$t = 50 \ln 98 - 50 \ln (70 + v)$		
	Body come to rest when $v = 0$	used	m1
	$t = 50 \ln (98/70)$		
	$= 50 \ln(7/5)$		
	$= 16.82$ s	cao	A1

2.(a)



(i)	N2L	$0.8x - 1.2v = 2a$	dim correct	M1
		$a = \dfrac{d^2x}{dt^2}, \quad v = \dfrac{dx}{dt}$		m1
	Multiply by 2.5	$5\dfrac{d^2x}{dt_2} + 3\dfrac{dx}{dt} - 2x = 0$		A1

(ii) Auxiliary equation

$5m^2 + 3m - 2 = 0$ B1

$(5m - 2)(m + 1) = 0$

$m = -1, 0.4$

General solution is $x = Ae^{0.4t} + Be^{-t}$ ft m values B1

When $t = 0, x = 0, v = 7$ used M1

$A + B = 0$

$v = \dfrac{dx}{dt} = 0.4Ae^{0.4t} - Be^{-t}$ ft x B1

$0.4A - B = 7$ ft x, v both A1

Solving simultaneously

$1.4A = 7$

$A = 5$

$B = -5$ both cao A1

$x = 5e^{0.4t} - 5e^{-t}$

(iii) $e^{0.4t}$ increases with t; e^{-t} decreases with t, therefore $e^{0.4t} - e^{-t}$ increases with t.
Hence $x = 5e^{0.4t} - 5e^{-t}$ increases with t. B1 B1

2.(b) For PI try $x = at + b$ M1

$3a - 2(at + b) = 20t - 70$ A1

Compare coefficients m1

$-2a = 20$

$a = -10$

$3a - 2b = -70$

$2b = 40$

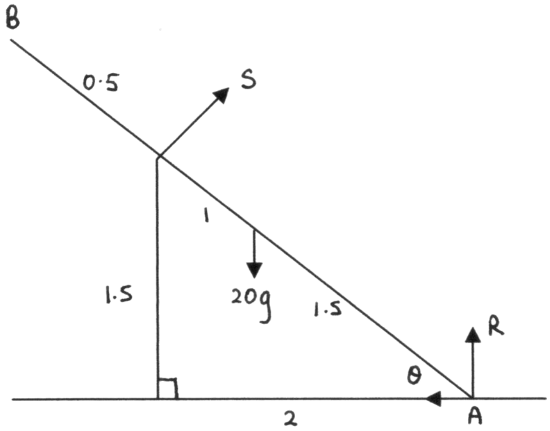
$b = 20$ both cao A1

Therefore general solution is $x = Ae^{0.4t} + Be^{-t} - 10t + 20$ ft a, b A1

3.(a)		$v^2 = \omega^2(a^2 - x^2)$	used	M1
	At A	$25 = \omega^2(a^2 - 9)$		A1
	At B	$14.0625 = \omega^2(a^2 - 16)$		A1
	Therefore	$25(a^2 - 16) = 14.0625(a^2 - 9)$		m1
		$10.9375\,a^2 = 273.4375$		
		$a^2 = 25$		
		$a = 5$	convincing	A1
		$\omega^2 = \dfrac{25}{25-9} = \dfrac{25}{16}$		
		$\omega = 1.25$		A1
		Period $= \dfrac{2\pi}{\omega} = \dfrac{8\pi}{5}$	convincing	M1 A1
(b)	Let	$x = 5\sin\left(\dfrac{5}{4}t\right)$		B1
	When $t = 2$,	$x = 5\sin(1.25 \times 2)$		M1
		$= 2.99$ m	cao	A1
(c)	Minimum speed	$= \omega a$	used	M1
		$= 5 \times 1.25$		
		$= 6.25\ \text{ms}^{-1}$	si ft ω	A1
	We require t such that	$6.25 \times 0.4 = 6.25\cos(1.25t)$		M1 B1
		$\cos(1.25t) = 0.4$		
		$t = 0.93$ s	cao	A1

4.(a)		$v\dfrac{dv}{dx} + \dfrac{v^2}{90} + 10 = 0$	$a = v\dfrac{dv}{dx}$	M1
		$v\dfrac{dv}{dx} = -\dfrac{1}{90}(v^2 + 900)$		
		$90\int \dfrac{v}{v^2 + 900}\,dv = -\int dx$		M1
		$45\ln(v^2 + 900) = -x\ (+ C)$		A1 A1
	When $t = 0, x = 0, v = 15$			m1
		$C = 45\ln(275 + 900) = 45\ln(1125)$		
		$x = 45\ln\left(\dfrac{1125}{v^2 + 900}\right)$	oe	A1
(b)	Greatest height when $v = 0$			M1
		$x = 45\ln\left(\dfrac{1125}{900}\right)$		
		$= 45\ln(1.25)$		
		$= 10.04$ m		A1

5.



(a)	Moments about A	dim correct	M1
	$S \times 2.5 = 20g \times 1.5\cos\theta$		A1 A1
	$\cos\theta = \frac{2}{2.5} = \frac{4}{5}$		B1
	$S = \frac{20 \times 9.8 \times 1.5 \times 0.8}{2.5}$		
	$= 94.08$ N	cao	A1
(b)	Resolve horizontally		M1
	$S\sin\theta = F$		A1
	$F = 94.08 \times 0.6$		
	$= 56.448$ N	ft S	A1
	Resolve vertically		M1
	$R = 20g + S\cos\theta$		A1
	$= 20 \times 9.8 + 94.08 \times 0.8$		
	$= 120\ 736$ N	ft S s.i.	A1
	Limiting friction $F = \mu R$	used	M1
	$\mu = \frac{56.448}{120.736}$		
	$= 0.47$	cao	A1

6.	Using $v^2 = u^2 + 2as$ with $u = 0$, $a = (-)9.8$, $s = 0$		M1
	$v^2 = 2 \times 9.8 \times 1.6$		A1
	$v = 5.6$ ms^{-1}		A1
	Impulse = change in momentum	applied to both particles	M1
	For A $J = 7v'$		B1
	For B $J = 3 \times 5.6 - 3v'$	ft v	A1
	Solving $7v' = 16.8 - 3v'$		m1
	$10v' = 16.8$		
	$v' = 1.68$	ft v	A1
	$J = 11.76$ Ns	ft v	A1

A/AS Mathematics - S1 – June 2008

Mark Scheme

1	(a)	P(2 red) = $\frac{2}{9}\times\frac{1}{8}$ or $\frac{\binom{2}{2}}{\binom{9}{2}}$ (1/36)	B1
		P(2 grn) = $\frac{3}{9}\times\frac{2}{8}$ or $\frac{\binom{3}{2}}{\binom{9}{2}}$ (1/12)	B1
		P(2 yel) = $\frac{4}{9}\times\frac{3}{8}$ or $\frac{\binom{4}{2}}{\binom{9}{2}}$ (1/6)	B1
		P(same) = $\frac{1}{36}+\frac{1}{12}+\frac{1}{6}=\frac{5}{18}$	B1
	(b)	P(diff) = 1 – P(same) = $\frac{13}{18}$	M1A1
2	(a)	Using $P(A\cup B)=P(A)+P(B)-P(A\cap B)$	M1
		$0.4 = 0.2 + P(B) - 0.2P(B)$	A1
		$0.8P(B) = 0.2$	A1
		$P(B) = 0.25$	A1
	(b)	EITHER	
		$P(A$ only$) = 0.15$	B1
		$P(B$ only$) = 0.2$	B1
		Reqd prob = 0.35	B1
		OR	
		P(Exactly one) = P(A∪B) – P(A∩B)	M1
		= 0.4 – 0.2 × 0.25	A1
		= 0.35	A1
	(c)	Reqd prob = $\frac{0.2\times 0.75}{0.35}$	B1B1
		= 3/7	B1
		[FT for $P(B)$ in (b) and (c)]	

3	(a)(i)	Total number of poss = 36	B1
		[Award if any evidence is seen that there are 36 possibilities]	
		Number giving equality = 6	B1
		Reqd prob = 6/36 = 1/6	B1
	(ii)	Number giving smaller score = 5 + 4 + 3 + 2 + 1 = 15	M1A1
		Reqd prob = 15/36 = 5/12	A1
	(b)	EITHER	
		Reduce the sample space to (5,1), (4,2), (3,3), (2,4), (1,5)	M1A1
		Reqd prob = 1/5	A1
		OR	
		$\text{Prob} = \dfrac{P(3,3)}{P(\text{Sum} = 6)}$	M1
		$= \dfrac{1/36}{5/36}$	A1
		= 1/5	A1
4	(a)	Prob = 0.9884 – 0.5697 or 0.4303 – 0.0116	B1B1
		= 0.4187 (cao)	B1
	(b)(i)	$\text{Prob} = e^{-3.25} \times \dfrac{3.25^5}{5!} = 0.117$	M1A1
	(ii)	$P(X \le 2) = e^{-3.25}\left(1 + 3.25 + \dfrac{3.25^2}{2}\right)$	M1A1
		= 0.370	A1
		[Special case : Award B1 for 0.370 with no working]	
5	(a)	P(B) = 0.25, P(C) = 0.25, P(D) = 0.5	M1A1
		[Award M1A0 for ¼, ¼, ¼ with no working]	
		$\text{P(Win)} = 0.25\times0.3 + 0.25\times0.4 + 0.5\times0.6$	M1A1
		= 0.475	A1
	(b)	$\text{P(D} \mid \text{win)} = \dfrac{0.5\times0.6}{0.475}$	B1B1
		= 0.632	B1
		[FT their initial probs in (b) but not in (a)]	
6	(a)	$\Sigma p = 1$	M1
		$9k = 1$	A1
		$k = 1/9$	AG
	(b)	$E(X) = \dfrac{2}{9}\times1 + \dfrac{3}{9}\times2 + \dfrac{4}{9}\times3$	M1A1
		= 2.22 (20/9)	A1
	(c)	$E\left(\dfrac{1}{X}\right) = \dfrac{2}{9}\times1 + \dfrac{3}{9}\times\dfrac{1}{2} + \dfrac{4}{9}\times\dfrac{1}{3}$	M1A1
		= 0.537 (29/54)	A1

7 (a) The number of sales, X, is B(50, 0.2) si B1

(i) $P(X = 12) = \binom{50}{12} \times 0.2^{12} \times 0.8^{38}$ or 0.8139 – 0.7107 or 0.2893 – 0.1861 M1

= 0.1032 or 0.1033 A1

(ii) $P(10 \le X \le 14) = 0.9393 - 0.4437$ or $0.5563 - 0.0607$ B1B1

= 0.4956 B1

(iii) Prob = $0.8 \times 0.8 \times 0.2$ M1A1

= 0.128 A1

(b) The week's pay Y is given by

$Y = 100 + 50X$ si B1

$E(Y) = 100 + 50 \times 50 \times 0.2 = 600$ M1A1

$SD = 50 \times \sqrt{50 \times 0.2 \times 0.8} = 141$ M1A1

[FT on their sensible Y]

8 (a) $P(0.25 \le X \le 0.75) = F(0.75) - F(0.25)$ M1

$= 4 \times 0.75^3 - 3 \times 0.75^4 - 4 \times 0.25^3 + 3 \times 0.25^4$ A1

= 0.6875 A1

(b) $F(0.6) = 0.4752$ B1

Since F(med) = 0.5, it follows that the median is greater than 0.6.

[FT on their $F(0.6)$] M1A1

(c) $f(x) = F'(x) = 12(x^2 - x^3)$ M1A1

(d) $E(X) = 12\int_0^1 x(x^2 - x^3)\,dx$ M1A1

$= 12\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$ A1

= 0.6 A1

A/AS level Mathematics - S2 June 2008

Mark Scheme -

1		$\bar{x} = \dfrac{32.6}{5}$, $\bar{y} = \dfrac{36.78}{6}$	B1
		SE of difference of means = $\sqrt{\dfrac{0.25^2}{5} + \dfrac{0.25^2}{6}}$	M1A1
		The 95% confidence limits are	
		$6.52 - 6.13 \pm 1.96 \times 0.1514$	m1A1
		giving [0.093, 0.687]	A1
2	(a)	E(X) = 4, Var(X) = 2.4	B1
		Using Var(X) = $E(X^2) - [E(X)]^2$	M1
		$E(X^2) = 2.4 + 16$	A1
		= 18.4	
		E(Y) = 9, Var(Y) = 6.3	B1
		Similarly,	
		$E(Y^2) = 6.3 + 81 = 87.3$	B1
	(b)	E(U)=E(X)E(Y) = 36	B1
		$E(X^2Y^2) = E(X^2)E(Y^2) = 1606.32$	B1
		Var(U) = $E(X^2Y^2) - [E(XY)]^2$	M1
		$= 1606.32 - 36^2 = 310.32$	A1
3	(a) (i)	$z = \dfrac{49 - 50}{2} = -0.5$	M1A1
		Prob = 0.3085	A1
	(ii)	Number weighing less than 49 g is B(6,0.3085) si	B1
		Required prob = $\dbinom{6}{3} \times 0.3085^3 \times 0.6915^3 = 0.194$	M1A1
		[Award M1A0 if combinatorial term omitted]	
	(b)	X-3Y is N(50-3 ×18, $2^2 + 9 \times 1.2^2$), ie N(- 4, 16.96)	M1B1B1
		$z = \dfrac{4}{\sqrt{16.96}} = (\pm)0.97$	A1
		Prob = 0.166	A1

4 (a) $\bar{x} = \dfrac{48.32}{8}$ (= 6.04) B1

SE of $\bar{X} = \dfrac{0.05}{\sqrt{8}}$ (= 0.01767…) M1

[Award M1 only if the 8 appears in the denominator]

99% conf limits are

$6.04 \pm 2.576 \times 0.01767..$ m1A1

[M1 correct form, A1 2.576]

giving [5.99,6.09] A1

(b) We solve

$\dfrac{2.576 \times 2 \times 0.05}{\sqrt{n}} = 0.04$ M1

$n = 41.467$ A1

giving $n = 42$ B1

[FT on their n]

5 (a) $H_0 : p = 0.6$ versus $H_1 : p < 0.6$ B1

(b) Under H_0, X is B(20,0.6) si B1

and Y (No not germ) is B(20,0.4) (si) B1

p-value = P($X \le 9 \mid H_0$) = P(Y $\ge 11 \mid H_0$) M1

= 0.1275 A1

(c) X is now B(200,0.6) which is approx N(120,48) B1

$z = \dfrac{101.5 - 120}{\sqrt{48}}$ M1A1A1

[M1A1A0 if no continuity correction]

= - 2.67 A1

p-value = 0.00379 A1

Strong evidence to support Malcolm (or germination prob less than 0.6). B1

[No c/c gives z = - 2.74, p = 0.00307, wrong c/c gives z = - 2.81, p = 0.00248]

6 (a)(i) AC = $\sqrt{X^2 + X^2} = \sqrt{2}X$ (si) B1

We require P(√2X > 8) = P($X > 8/\sqrt{2}$) M1A1

$= \dfrac{6 - 8/\sqrt{2}}{6-4} = 0.172$ (3 - 2v2) A1

(ii) Area = $\dfrac{X^2}{2}$ B1

We require P($\dfrac{X^2}{2} < 10$) = P(X < √20) M1A1

$= \dfrac{\sqrt{20} - 4}{6-4} = 0.236$ (v5 – 2) A1

(b) EITHER

$f(x) = 0.5$ for $4 \le x \le 6$ B1

Expected area = $\int_4^6 0.5x^2 \times 0.5\,dx$ M1

$= 0.25\left[\dfrac{x^3}{3}\right]_4^6$ A1

$= \dfrac{38}{3}$ A1

OR

$E(X^2) = \text{Var}(X) + (E(X))^2$ M1

$= \dfrac{2^2}{12} + 5^2 (= \dfrac{76}{3})$ A1A1

Expected area = $E\left(\dfrac{X^2}{2}\right) = \dfrac{38}{3}$ A1

7 (a) $H_0 : \mu = 2.5$ versus $H_1 : \mu \ne 2.5$ B1

(b)(i) Under H_0 S is Po(15) B1

P(S ≤ 8) = 0.0374 B1

P(S ≥ 23) = 0.0327 B1

Significance level = 0.0701 B1

(ii) If μ = 2, S is Po(12) B1

We require P(9 ≤ X ≤ 22) = 0.9970 – 0.1550 M1A1

= 0.842 A1

(c) Under H_0, S is now Po(250) ≈ N(250,250) B1

$z = \dfrac{269.5 - 250}{\sqrt{250}}$ M1A1A1

= 1.23 A1

[Award M1A1A0 if the c/c is incorrect or omitted]

Prob from tables = 0.1093 A1

p-value = 0.2186 B1

Accept H_0 (context not required but accept if correct)). B1

[No c/c gives z = 1.26, prob = 0.1038, p-value = 0.2076;
incorrect c/c gives z = 1.30, prob = 0.0968, p-value = 01936]

AS/A Mathematics - S3 – June 2008

Markscheme

1. (a) $P(3£2)) = \frac{6}{10}\times\frac{5}{9}\times\frac{4}{8} = \frac{1}{6}$ B1

$P(3£1) = \frac{4}{10}\times\frac{3}{9}\times\frac{2}{8} = \frac{1}{30}$ B1

$P(2£2,1£1) = \frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times 3 = \frac{1}{2}$ B1

$P(1£2,2£1) = \frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}\times 3 = \frac{3}{10}$ B1

[Do not accept sampling with replacement]

The sampling distribution is

Total	£3	£4	£5	£6
Prob	1/30	3/10	1/2	1/6

[-1 each error] M1A2

[FT on their probabilities]

(b) $E(\text{Total}) = \frac{1}{30}\times 3 + \frac{3}{10}\times 4 + \frac{1}{2}\times 5 + \frac{1}{6}\times 6 = 4.8$ M1A1

Mean value = $\frac{6}{10}\times 2 + \frac{4}{10}\times 1$ = 1.6 as A1

2 (a)(i) $\hat{p} = \frac{630}{1200} = 0.525$ B1

(ii) ESE = $\sqrt{\frac{0.525\times 0.475}{1200}}$ = 0.0144 M1A1

(b) 95% confidence limits are

$0.525 \pm 1.96\times 0.0144$ M1A1

giving [0.497,0.553] A1

(c) No, because 0.6 is not in the confidence interval. B1B1

3 (a) $\Sigma x = 3528$; $\Sigma x^2 = 1038840$ B1

UE of μ = 294 B1

UE of $\sigma^2 = \frac{1038840}{11} - \frac{3528^2}{11\times 12}$ M1

= 146.18 A1

(b) $H_0 : \mu = 300$ versus $H_1 : \mu < 300$ B1

Test stat = $\frac{294 - 300}{\sqrt{146.18/12}}$ M1A1

= -1.72 A1

DF = 11 B1

(i) At the 5% significance level, critical value = 1.796 B1

Accept farmer's claim with correct reasoning. B1

(ii) At the 10% significance level, critical value = 1.363 B1

Reject farmer's claim with correct reasoning. B1

[FT in (b) from (a) ; FT conclusions from their critical values but not from p-values]

4	(a)	UE of μ = 1815/75 (= 24.2)	B1
		UE of Var(X) = $\dfrac{44213}{74} - \dfrac{1815^2}{74 \times 75}$ (= 3.92)	M1A1
		[Accept division by 75 giving 3.87]	
		ESE = $\sqrt{\dfrac{3.92}{75}} = 0.229$ (3.87 gives 0.227)	M1A1
		Approx 90% confidence limits for μ are	
		$24.2 \pm 1.645 \times 0.229$	M1A1
		giving [23.8, 24.6] (cao)	A1
	(b)	No since the Central Limit Theorem ensures the approximate normality of $\overline{X}$.	B1
5	(a)	$H_0 : \mu_x = \mu_y$ versus $H_1 : \mu_x \neq \mu_y$	B1
	(b)	$\bar{x} = 1506/60 (= 25.1); \bar{y} = 1530/60 (= 25.5)$	B1B1
		$s_x^2 = \dfrac{38124}{59} - \dfrac{1506^2}{59 \times 60} = 5.4813...$ (/60 gives 5.39)	M1A1
		$s_y^2 = \dfrac{39327}{59} - \dfrac{1530^2}{59 \times 60} = 5.2881...$ (/60 gives 5.20)	A1
		[Accept division by 60]	
		Test stat = $\dfrac{25.5 - 25.1}{\sqrt{\dfrac{5.4813..}{60} + \dfrac{5.2881..}{60}}}$	M1A1
		= (±)0.94 (0.95)	A1
		Prob from tables = 0.1736 (0.1711)	A1
		p-value = 0.347 (0.342)	B1
		Insufficient evidence to doubt that the mean weights are equal.	B1
6	(a)	$P(X > 0) = \displaystyle\int_0^{1/2} (1 + \lambda x)\,dx$	M1
		$= \left[x + \dfrac{\lambda x^2}{2} \right]_0^{1/2} = \dfrac{1}{2} + \dfrac{\lambda}{8}$	A1AG
	(b)(i)	Y is B(n,1/2+λ/8)	M1
		$E(Y) = n\left(\dfrac{1}{2} + \dfrac{\lambda}{8}\right)$	A1
		$E(U) = 8 \times n\left(\dfrac{1}{2} + \dfrac{\lambda}{8}\right) \div n - 4 = \lambda$	M1A1
	(ii)	$\text{Var}(Y) = n\left(\dfrac{1}{2} + \dfrac{\lambda}{8}\right)\left(\dfrac{1}{2} - \dfrac{\lambda}{8}\right) = n\left(\dfrac{1}{4} - \dfrac{\lambda^2}{64}\right)$	M1A1
		$\text{Var}(U) = \dfrac{64}{n^2} \times n\left(\dfrac{1}{4} - \dfrac{\lambda^2}{64}\right)$	M1A1
		SE = $\sqrt{\dfrac{16 - \lambda^2}{n}}$	A1

7 (a) $S_{xy} = 23538 - 270 \times 517 / 6 = 273$ B1

$S_{xx} = 13900 - 270^2 / 6 = 1750$ B1

$b = \dfrac{273}{1750} = 0.156$ M1A1

$a = \dfrac{517 - 270 \times 0.156}{6}$ M1

$= 79.15$ A1

(b) Estimated length = 79.15 + 0.156 × 60 (= 88.51) B1

$\text{SE} = 0.15\sqrt{\dfrac{1}{6} + \dfrac{15^2}{1750}} (= 0.0815)$ M1A1

The 99% confidence interval is

$88.51 \pm 2.576 \times 0.0815$ M1A1

giving [88.3, 88.7] A1

[FT from (a)]

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