GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3 AS/Advanced

SUMMER 2012

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2012 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

C1

1. (*a*) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^{4}/_{3}$ (or equivalent) A1

(*b*) A correct method for finding C M1

$C(-1, 3)$ A1

(*c*) Use of $m_{AB} \times m_L = -1$ to find gradient of L M1

A correct method for finding the equation of L using candidate's coordinates for C and candidate's gradient for L. M1

Equation of L: $y - 3 = {}^{3}/_{4}\,[x - (-1)]$ (or equivalent)

(f.t. candidate's coordinates for C and candidate's gradient for AB) A1

Equation of L: $3x - 4y + 15 = 0$ (convincing, c.a.o.) A1

(*d*) (i) Substituting $x = 7$, $y = k$ in equation of L M1

$k = 9$ A1

(ii) A correct method for finding the length of $CA(DA)$ M1

$CA = 5$ (f.t. candidate's coordinates for C) A1

$DA = \sqrt{125}$ A1

(iii) $\text{Sin } ADC = \dfrac{CA}{DA} = \dfrac{5}{\sqrt{125}}$

(f.t. candidate's derived values for CA and DA) M1

$\text{Sin } ADC = \dfrac{CA}{DA} = \dfrac{1}{\sqrt{5}}$ (c.a.o.) A1

2. (*a*) $\dfrac{10}{7 + 2\sqrt{11}} = \dfrac{10(7 - 2\sqrt{11})}{(7 + 2\sqrt{11})(7 - 2\sqrt{11})}$ M1

Denominator: $49 - 44$ A1

$\dfrac{10}{7 + 2\sqrt{11}} = \dfrac{10(7 - 2\sqrt{11})}{5} = 2(7 - 2\sqrt{11}) = 14 - 4\sqrt{11}$ (c.a.o.) A1

Special case
If M1 not gained, allow B1 for correctly simplified denominator following multiplication of top and bottom by $7 + 2\sqrt{11}$

(*b*) $(4\sqrt{3})^2 = 48$ B1

$\sqrt{8} \times \sqrt{50} = 20$ B1

$\dfrac{5\sqrt{63}}{\sqrt{7}} = 15$ B1

$(4\sqrt{3})^2 - (\sqrt{8} \times \sqrt{50}) - \dfrac{5\sqrt{63}}{\sqrt{7}} = 13$ (c.a.o.) B1

3. (*a*) $\dfrac{dy}{dx} = 4x - 11$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\dfrac{dy}{dx}$ m1

Use of candidate's numerical value for $\dfrac{dy}{dx}$ as gradient of tangent at P m1

Equation of tangent at P: $y - (-1) = -3(x - 2)$ (or equivalent) (c.a.o.) A1

(*b*) Gradient of tangent at $Q = 9$ B1

An attempt to equate candidate's expression for $\dfrac{dy}{dx}$ and candidate's derived value for gradient of tangent at Q M1

$4x - 11 = 9 \Rightarrow x = 5$
(f.t. one error in candidate's expression for $\dfrac{dy}{dx}$) A1

4. $(1 - 2x)^6 = 1 - 12x + 60x^2 - 160x^3 + \ldots$ B1 B1 B1 B1
(– 1 for further incorrect simplification)

5. (*a*) $a = 3$ B1

$b = -2$ B1

$c = 17$ B1

(*b*) Stationary value = 17 (f.t. candidate's value for c) B1

This is a minimum B1

6. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1
$b^2 - 4ac = (2k - 1)^2 - 4(k^2 - k + 2)$ A1
$b^2 - 4ac = -7$ (c.a.o.) A1
candidate's value for $b^2 - 4ac < 0$ ($\Rightarrow$ no real roots) A1

(*b*) Finding critical values $x = -6$, $x = {}^2/_3$ B1
A statement (mathematical or otherwise) to the effect that
$x < -6$ or ${}^2/_3 < x$ (or equivalent)
(f.t critical values ± 6, $\pm {}^2/_3$ only) B2
Deduct 1 mark for each of the following errors
the use of $\leq$ rather than $<$
the use of the word 'and' instead of the word 'or'

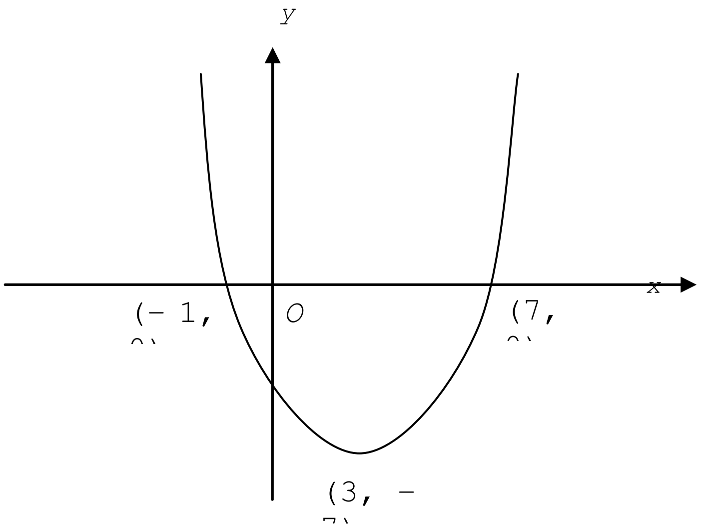
7. (*a*) $y + \delta y = 3(x + \delta x)^2 - 7(x + \delta x) + 5$ B1
Subtracting y from above to find δy M1
$\delta y = 6x\delta x + 3(\delta x)^2 - 7\delta x$ A1
Dividing by δx and letting $\delta x \to 0$ M1
$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 6x - 7$ (c.a.o.) A1

(*b*) Required derivative = ${}^2/_3 \times {}^1/_4 \times x^{-3/4} + 12 \times (-3) \times x^{-4}$ B1, B1

8. (*a*) Attempting to find $f(r) = 0$ for some value of r M1
$f(2) = 0 \Rightarrow x - 2$ is a factor A1
$f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct M1
$f(x) = (x - 2)(6x^2 - 7x - 3)$ A1
$f(x) = (x - 2)(3x + 1)(2x - 3)$ (f.t. only $6x^2 + 7x - 3$ in above line) A1
$x = 2, -{}^1/_3, {}^3/_2$ (f.t. for factors $3x \pm 1$, $2x \pm 3$) A1
Special case
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 4 marks

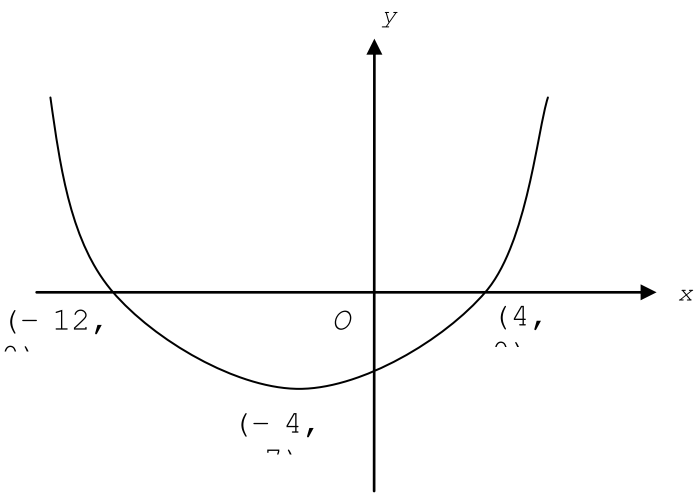
(*b*) Use of $g(a) = 11$ M1
$a^3 - 53 = 11 \Rightarrow a = 4$ A1

9. (*a*)



Concave up curve and y-coordinate of minimum = – 7 B1
x-coordinate of minimum = 3 B1
Both points of intersection with x-axis B1

(*b*)



Concave up curve and y-coordinate of minimum = – 7 B1
x-coordinate of minimum = – 4 B1
Both points of intersection with x-axis B1

10. (*a*) $\frac{dy}{dx} = 3x^2 + 6x$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = 0, -2$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(0, -1)$ and $(-2, 3)$ (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

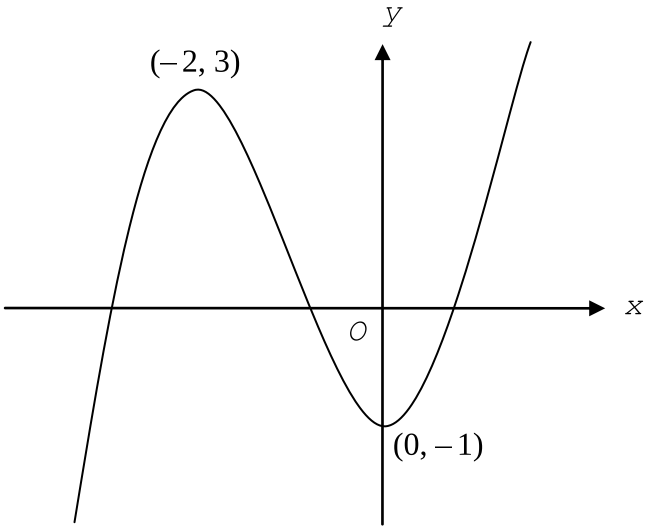
either $(0, -1)$ is a minimum point

or $(-2, 3)$ is a maximum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) One positive root (f.t. the number of times the candidate's curve crosses the positive *x*-axis) B1

C2

1.

1	0·5		
1·25	0·53935989		
1·5	0·603022689		
1·75	0·718421208	(5 values correct)	B2
2	1	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$I \approx \frac{0{\cdot}25}{2} \times \{0{\cdot}5 + 1 + 2(0{\cdot}53935989 + 0{\cdot}603022689 + 0{\cdot}718421208)\}$

$I \approx 5{\cdot}221607574 \div 8$

$I \approx 0{\cdot}652700946$

$I \approx 0{\cdot}6527$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}2$

1	0·5		
1·2	0·52999894		
1·4	0·573539334		
1·6	0·640184399		
1·8	0·753778361		
2	1	(all values correct)	B1

Correct formula with $h = 0{\cdot}2$ M1

$I \approx \frac{0{\cdot}2}{2} \times \{0{\cdot}5 + 1 + 2(0{\cdot}52999894 + 0{\cdot}573539334 + 0{\cdot}640184399 + 0{\cdot}753778361)\}$

$I \approx 6{\cdot}495002069 \div 10$

$I \approx 0{\cdot}6495002069$

$I \approx 0{\cdot}6495$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) $10\cos^2\theta + 3\cos\theta = 4(1 - \cos^2\theta) - 2$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$14\cos^2\theta + 3\cos\theta - 2 = 0 \Rightarrow (2\cos\theta + 1)(7\cos\theta - 2) = 0$

$\Rightarrow \cos\theta = \frac{2}{7}$, $\cos\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 73{\cdot}40°, 286{\cdot}60°$ B1

$\theta = 120°, 240°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) $3x - 21° = -54°, 234°, 306°, 594$ (one value) B1

$x = 85°, 109°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) Use of $\frac{\sin\phi}{\cos\phi} = \tan\phi$ M1

$\tan\phi = 0{\cdot}2$ A1

$\phi = 11{\cdot}31°, 191{\cdot}31°$ (f.t $\tan\phi = a$) B1

3. (*a*) $11^2 = 5^2 + x^2 - 2 \times 5 \times x \times {}^2/_5$ (correct use of cos rule) M1

An attempt to collect terms, form and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + b)(x + d)$, with $b \times d$ = candidate's constant m1

$x^2 - 4x - 96 = 0 \Rightarrow x = 12$ (c.a.o.) A1

(*b*) $\frac{\sin XZY}{32} = \frac{\sin 19°}{15}$

(substituting the correct values in the correct places in the sin rule) M1

$XZY = 44°, 136°$ (at least one value) A1

Use of angle sum of a triangle = 180° M1

$YXZ = 117°, 25°$ (both values)

(f.t. candidate's values for XZY provided both M's awarded) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or:

$2S_n = [a + a + (n - 1)d]$ n times M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $a + 2d + a + 3d + a + 9d = 79$ B1

$a + 5d + a + 6d = 61$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = 3$, $d = 5$ (both values) (c.a.o.) A1

(*c*) $a = 15$, $d = -2$ B1

$S_n = \frac{n}{2}[2 \times 15 + (n - 1)(-2)]$ (f.t. candidate's d) M1

$S_n = n(16 - n)$ (c.a.o.) A1

5. (*a*) $a + ar = 72$ B1

$a + ar^2 = 120$ B1

An attempt to solve candidate's equations simultaneously by correctly eliminating a M1

$3r^2 - 5r - 2 = 0$ (convincing) A1

(*b*) An attempt to solve quadratic equation in r, either by using the quadratic formula or by getting the expression into the form $(ar + b)(cr + d)$, with $a \times c = 3$ and $b \times d = -2$ M1

$(3r + 1)(r - 2) = 0 \Rightarrow r = -\frac{1}{3}$ A1

$a \times (1 - \frac{1}{3}) = 72 \Rightarrow a = 108$ (f.t. candidate's derived value for r) B1

$S_\infty = \frac{108}{1 - (-\frac{1}{3})}$ (correct use of formula for S_∞, f.t. candidate's derived values for r and a) M1

$S_\infty = 81$ (c.a.o.) A1

6. (*a*) $3 \times \frac{x^{3/2}}{3/2} - 2 \times \frac{x^{-2/3}}{-2/3} + c$ B1 B1

(–1 if no constant term present)

(*b*) (i) $36 - x^2 = 5x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = -36$ m1

$(x - 4)(x + 9) = 0 \Rightarrow A(4, 20)$ (c.a.o.) A1

$B(6, 0)$ B1

(ii) Area of triangle = 40 (f.t. candidate's coordinates for A) B1

Area under curve $= \int_4^6 (36 - x^2)\,dx$ (use of integration) M1

$\int 36\,dx = 36x$ **and** $\int x^2\,dx = \frac{x^3}{3}$ B1

Area under curve = [(216 – 216/3) – (144 – 64/3)]

(substitution of candidate's limits) m1

= 64/3

Use of candidate's, x_A, x_B as limits and trying to find total area by adding area of triangle and area under curve m1

Total area = 40 + 64/3 = 184/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indices) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) **Either:**

$(x/2 - 3) \log_{10} 9 = \log_{10} 6$

(taking logs on both sides and using the power law) M1

$x = \dfrac{2(\log_{10} 6 + 3 \log_{10} 9)}{\log_{10} 9}$ A1

$x = 7{\cdot}631$ (f.t. one slip, see below) A1

Or:

$x/2 - 3 = \log_9 6$ (rewriting as a log equation) M1

$x = 2(\log_9 6 + 3)$ A1

$x = 7{\cdot}631$ (f.t. one slip, see below) A1

Note: an answer of $x = -4{\cdot}369$ from $x = \dfrac{2(\log_{10} 6 - 3 \log_{10} 9)}{\log_{10} 9}$

earns M1 A0 A1

an answer of $x = 3{\cdot}815$ from $x = \dfrac{\log_{10} 6 + 3 \log_{10} 9}{\log_{10} 9}$

earns M1 A0 A1

an answer of $x = 1{\cdot}908$ from $x = \dfrac{(\log_{10} 6 + 3 \log_{10} 9)}{2\log_{10} 9}$

earns M1 A0 A1

an answer of $x = 4{\cdot}631$ from $x = \dfrac{2\log_{10} 6 + 3 \log_{10} 9}{\log_{10} 9}$

earns M1 A0 A1

Note: Answer only with no working earns 0 marks

(*c*) $\log_a (x - 2) + \log_a (4x + 1) = \log_a [(x - 2)(4x + 1)]$ (addition law) B1

$2 \log_a (2x - 3) = \log_a (2x - 3)^2$ (power law) B1

$(x - 2)(4x + 1) = (2x - 3)^2$ (removing logs) M1

$x = 2{\cdot}2$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (*a*) $A(2, -3)$ B1

A correct method for finding the radius M1

Radius = $\sqrt{12}$ A1

(*b*) $AT^2 = 61$ (f.t. candidate's coordinates for A) B1

Use of $RT^2 = AT^2 - AR^2$ M1

$RT = 7$ (f.t. candidate's radius and coordinates for A) A1

9. Area of sector $POQ = {}^1/_2 \times r^2 \times 1{\cdot}12$ B1

Area of triangle $POQ = {}^1/_2 \times r^2 \times \sin(1{\cdot}12)$ B1

$10{\cdot}35 = {}^1/_2 \times r^2 \times 1{\cdot}12 - {}^1/_2 \times r^2 \times \sin(1{\cdot}12)$

(f.t. candidate's expressions for area of sector and area of triangle) M1

$r^2 = \dfrac{2 \times 10{\cdot}35}{(1{\cdot}12 - 0{\cdot}9)}$ (o.e.) (c.a.o.) A1

$r = 9{\cdot}7$ (f.t. one numerical slip) A1

C3

1.	(*a*)	0	1		
		0·25	1·064494459		
		0·5	1·284025417		
		0·75	1·755054657	(5 values correct)	B2
		1	2·718281828	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$$I \approx \frac{0{\cdot}25}{3} \times \{1 + 2{\cdot}718281828 + 4(1{\cdot}064494459 + 1{\cdot}755054657) + 2(1{\cdot}284025417)\}$$

$I \approx 17{\cdot}56452913 \times 0{\cdot}25 \div 3$

$I \approx 1{\cdot}463710761$

$I \approx 1{\cdot}4637$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*) $\int_0^1 e^{x^2+3}\,dx = e^3 \times \int_0^1 e^{x^2}\,dx$ M1

$\int_0^1 e^{x^2+3}\,dx = 29{\cdot}399$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $\phi = 360° - \theta$ or $\phi = -\theta$ and noting that $\cos\theta = \cos\phi$ B1

$\sin\theta \neq \sin\phi$ (including correct evaluations) B1

(*b*) $13\tan^2\theta = 5(1 + \tan^2\theta) + 6\tan\theta$.

(correct use of $\sec^2\theta = 1 + \tan^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\tan\theta$, either by using the quadratic formula or by getting the expression into the form $(a\tan\theta + b)(c\tan\theta + d)$, with $a \times c$ = candidate's coefficient of $\tan^2\theta$ and $b \times d$ = candidate's constant m1

$8\tan^2\theta - 6\tan\theta - 5 = 0 \Rightarrow (4\tan\theta - 5)(2\tan\theta + 1) = 0$

$\Rightarrow \tan\theta = \frac{5}{4}$, $\tan\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 51{\cdot}34°, 231{\cdot}34°$ B1

$\theta = 153{\cdot}43°, 333{\cdot}43°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\tan\theta$ = +, –, f.t. for 3 marks, $\tan\theta$ = –, –, f.t. for 2 marks
$\tan\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(-3x - 2) = -3$ B1

$\frac{d}{dx}(-4x^2y) = -4x^2\frac{dy}{dx} - 8xy$ B1

$\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$x = 3, y = 1 \Rightarrow \frac{dy}{dx} = \frac{6}{42} = \frac{1}{7}$ (c.a.o.) B1

(*b*) (i) Differentiating $\sin at$ and $\cos at$ with respect to t, at least one correct M1

candidate's x-derivative = $a\cos at$,

candidate's y-derivative = $-a\sin at$ (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\tan at$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = -a\sec^2 at$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = -\sec^3 at$ (c.a.o.) A1

4. $f(x) = \cos x - 5x + 2$

An attempt to check values or signs of $f(x)$ at $x = 0$, $x = \pi/4$ M1

$f(0) = 3 > 0$, $f(\pi/4) = -1{\cdot}22 < 0$

Change of sign $\Rightarrow f(x) = 0$ has root in $(0, \pi/4)$ A1

$x_0 = 0{\cdot}6$

$x_1 = 0{\cdot}565067123$ B1

$x_2 = 0{\cdot}568910532$

$x_3 = 0{\cdot}568497677$

$x_4 = 0{\cdot}568542145 = 0{\cdot}56854$ (x_4 correct to 5 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 0{\cdot}568535$, $x = 0{\cdot}568545$ M1

$f(0{\cdot}568535) = 1{\cdot}563 \times 10^{-5} > 0$, $f(0{\cdot}568545) = -3{\cdot}975 \times 10^{-5} < 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}56854$ correct to five decimal places A1

Note: 'change of sign' must appear at least once

5. (*a*) $\frac{dy}{dx} = \frac{a + bx}{7 + 2x - 3x^2}$ (including $a = 1$, $b = 0$) M1

$\frac{dy}{dx} = \frac{2 - 6x}{7 + 2x - 3x^2}$ A1

(*b*) $\frac{dy}{dx} = e^{\tan x} \times f(x)$ ($f(x) \neq 1, 0$) M1

$\frac{dy}{dx} = e^{\tan x} \times \sec^2 x$ A1

(*c*) $\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$ ($f(x), g(x) \neq 1, 0$) M1

$\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$ (either $f(x) = \frac{1}{\sqrt{1 - x^2}}$ or $g(x) = 10x$) A1

$\frac{dy}{dx} = 5x^2 \times \frac{1}{\sqrt{1 - x^2}} + 10x \times \sin^{-1} x$ A1

6. (*a*) (i) $\int 3e^{2-x/4}\,dx = k \times 3e^{2-x/4} + c$ $(k = 1, -^1/_4, 4, -\mathbf{4})$ M1

$\int 3e^{2-x/4}\,dx = -4 \times 3e^{2-x/4} + c$ A1

(ii) $\int \frac{9}{(2x-3)^6}\,dx = \frac{k \times 9 \times (2x-3)^{-5}}{-5} + c$ $(k = 1, 2, \mathbf{^1/_2})$ M1

$\int \frac{9}{(2x-3)^6}\,dx = \frac{9 \times (2x-3)^{-5}}{-5 \times 2} + c$ A1

(iii) $\int \frac{7}{3x+1}\,dx = k \times 7 \times \ln|3x+1| + c$ $(k = 1, 3, \mathbf{^1/_3})$ M1

$\int \frac{7}{3x+1}\,dx = {^7/_3} \times \ln|3x+1| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \sin 2x\,dx = k \times \cos 2x$ $(k = -1, -2, {^1/_2}, -\mathbf{^1/_2})$ M1

$\int \sin 2x\,dx = -\frac{1}{2} \times \cos 2x$ A1

$k \times (\cos 2a - \cos 0) = {^1/_4}$

(f.t. candidate's value for k) M1

$\cos 2a = {^1/_2}$ (c.a.o.) A1

$a = \pi/6$ (f.t. $\cos 2a = b$ provided both M's are awarded) A1

7. (*a*) $9|x-3| = 6$ B1

$x - 3 = \pm {^2/_3}$ (f.t. candidate's $a|x-3| = b$, with at least one of a, b correct) B1

$x = {^{11}/_3}, {^7/_3}$ (f.t. candidate's $a|x-3| = b$, with at least one of a, b correct) B1

(*b*) Trying to solve either $5x - 2 \leq 3$ or $5x - 2 \geq -3$ M1

$5x - 2 \leq 3 \Rightarrow x \leq 1$

$5x - 2 \geq -3 \Rightarrow x \geq -{^1/_5}$ (both inequalities) A1

Required range: $-{^1/_5} \leq x \leq 1$ (f.t. one slip) A1

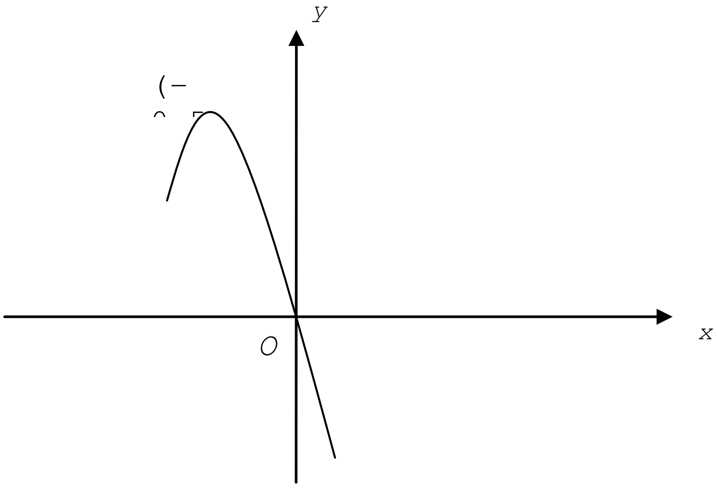
Alternative mark scheme

$(5x-2)^2 \leq 9$ (forming and trying to solve quadratic) M1

Critical points $x = -{^1/_5}$ and $x = 1$ A1

Required range: $-{^1/_5} \leq x \leq 1$ (f.t. one slip) A1

8.



Concave down curve passing through the origin with maximum point in the second quadrant B1

x-coordinate of stationary point = – 0·5 B1

y-coordinate of stationary point = 8 B1

9. (*a*) (i) $f'(x) = \dfrac{(x^2+5)\times f(x) - (x^2+3)\times g(x)}{(x^2+5)^2}$ ($f(x), g(x) \neq 1$) M1

$f'(x) = \dfrac{(x^2+5)\times 2x - (x^2+3)\times 2x}{(x^2+5)^2}$ A1

$f'(x) = \dfrac{4x}{(x^2+5)^2}$ (c.a.o.) A1

$f'(x) < 0$ since numerator is negative and denominator is positive B1

(ii) $R(f) = ({}^3/_5, 1)$ B1 B1

(*b*) (i) $x^2 = \dfrac{3-5y}{y-1}$ (o.e.) (condone any incorrect signs) M1

$x = (\pm)\left(\dfrac{3-5y}{y-1}\right)^{1/2}$ (f.t. at most one incorrect sign) A1

$x = -\left(\dfrac{3-5y}{y-1}\right)^{1/2}$ (f.t. at most one incorrect sign) A1

$f^{-1}(x) = -\left(\dfrac{3-5x}{x-1}\right)^{1/2}$ (c.a.o.) A1

(ii) $R(f^{-1}) = (-\infty, 0)$, $D(f^{-1}) = ({}^3/_5, 1)$,
(both intervals, f.t. candidate's $R(f)$) B1

10. $gg(x) = (3(g(x))^2 + 7)^{1/2}$ or $gg(x) = g((3x^2 + 7)^{1/2})$ M1

$gg(x) = (3(3x^2 + 7) + 7)^{1/2}$ A1

An attempt to solve the equation by squaring both sides M1

$gg(x) = 8 \Rightarrow 9x^2 = 36$ (o.e.) (c.a.o.) A1

$x = \pm 2$ (c.a.o.) A1

C4

1. (*a*) $f(x) \equiv \dfrac{A}{(x+1)} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-2)^2}$ (correct form) M1

$11 + x - x^2 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 1, C = 3, B = -2$ (2 correct coefficients) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = -\dfrac{1}{(x+1)^2} + \dfrac{2}{(x-2)^2} - \dfrac{6}{(x-2)^3}$ (o.e.)

(f.t. candidate's values for *A*, *B*, *C*)

(at least one of the first two terms) B1

(third term) B1

$f'(0) = {}^{1}/_{4}$ (c.a.o.) B1

2. $3y^2\dfrac{dy}{dx} - 8x - 3x\dfrac{dy}{dx} - 3y = 0$

$\left(3y^2\dfrac{dy}{dx} - 8x\right)$ B1

$\left(-3x\dfrac{dy}{dx} - 3y\right)$ B1

Either $\dfrac{dy}{dx} = \dfrac{3y + 8x}{3y^2 - 3x}$ **or** $\dfrac{dy}{dx} = \dfrac{1}{3}$ (o.e.) (c.a.o.) B1

Equation of tangent: $y - (-3) = \dfrac{1}{3}(x - 2)$

$\left(\text{f.t. candidate's value for } \dfrac{dy}{dx}\right)$ B1

3. (*a*) $4(1 - 2\sin^2\theta) = 1 - 2\sin\theta$. (correct use of $\cos 2\theta = 1 - 2\sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$8\sin^2\theta - 2\sin\theta - 3 = 0 \Rightarrow (4\sin\theta - 3)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{3}{4}, \quad \sin\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 48{\cdot}59°, 131{\cdot}41°$ B1

$\theta = 210°, 330°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks
$\sin\theta$ = +, +, f.t. for 1 mark

(*b*) (i) $R = 17$ B1

Correctly expanding $\sin(x + \alpha)$ and using either $17\cos\alpha = 8$ **or** $17\sin\alpha = 15$ **or** $\tan\alpha = \frac{15}{8}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 61{\cdot}93°$ (c.a.o) A1

(ii) $\sin(x + \alpha) = \frac{11}{17}$ (f.t. candidate's value for R) B1

$x + 61{\cdot}93° = 40{\cdot}32°, 139{\cdot}68°, 400{\cdot}32°$,
(at least one value on R.H.S., f.t. candidate's values for α and R) B1

$x = 77{\cdot}75°, 338{\cdot}39°$ (c.a.o.) B1

(iii) Greatest possible value for k is 17 since greatest possible value for sin is 1 (f.t. candidate's value for R) E1

4.

$$\text{Volume} = \pi\int_3^4 \left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 dx$$ B1

$$\left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 = \left(x + 10 + \frac{25}{x}\right)$$ B1

$$\int\left(ax + b + \frac{c}{x}\right)dx = \frac{ax^2}{2} + bx + c\ln x,$$ where $c \neq 0$ and at least one of $a, b \neq 0$ B1

Correct substitution of correct limits in candidate's integrated expression of form $\frac{ax^2}{2} + bx + c\ln x$, where $c \neq 0$ and at least one of $a, b \neq 0$ M1

Volume = 65($\cdot$0059. . .) (c.a.o.) A1

5. $\left(1 + \frac{x}{3}\right)^{-1/2} = 1 - \frac{x}{6} + \frac{x^2}{24}$ — $\left(1 - \frac{x}{6}\right)$ B1

$\left(\frac{x^2}{24}\right)$ B1

$|x| < 3$ or $-3 < x < 3$ B1

$\left(\frac{16}{15}\right)^{-1/2} \approx 1 - \frac{1}{30} + \frac{1}{600}$ (f.t. candidate's coefficients) B1

$\sqrt{15} \approx \frac{581}{150}$ (c.a.o.) B1

6. (*a*) candidate's x-derivative = $2t$

candidate's y-derivative = 2 (at least one term correct)

and use of

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{1}{t}$ (o.e.) (c.a.o.) A1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ m1

Equation of normal at P: $y - 2p = -p(x - p^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$y + px = p^3 + 2p$ (convincing) (c.a.o.) A1

(*b*) (i) Substituting $x = 9$, $y = 6$ in equation of normal M1

$p^3 - 7p - 6 = 0$ (convincing) A1

(ii) A correct method for solving $p^3 - 7p - 6 = 0$ M1

$p = -1$ A1

$p = -2$ A1

P is either $(1, -2)$ or $(4, -4)$ (c.a.o.) A1

7. (*a*)

$u = x \Rightarrow du = dx$	(o.e.)	B1
$dv = e^{-2x}\,dx \Rightarrow v = -\frac{1}{2}e^{-2x}$	(o.e.)	B1
$\int x\,e^{-2x}\,dx = x \times -\frac{1}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x}\,dx$		M1
$\int x\,e^{-2x}\,dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + c$	(c.a.o.)	A1

(*b*)

$\int \frac{1}{x(1+3\ln x)}\,dx = \int \frac{k}{u}\,du$	($k = {}^{1}/_{3}$ or 3)	M1
$\int \frac{a}{u}\,du = a\ln u$		B1
$\int_1^e \frac{1}{x(1+3\ln x)}\,dx = k[\ln u]_1^4$ or $k[\ln(1+3\ln x)]_1^e$		B1
$\int_1^e \frac{1}{x(1+3\ln x)}\,dx = 0{\cdot}4621$	(c.a.o.)	A1

8. (*a*)

$\frac{dV}{dt} = -kV^3$	(where $k > 0$)	B1

(*b*)

$\int \frac{dV}{V^3} = -\int k\,dt$	(o.e.)	M1
$-\frac{V^{-2}}{2} = -kt + c$		A1
$c = -\frac{1}{7200}$	(c.a.o.)	A1
$V^2 = \frac{3600}{7200kt+1} = \frac{3600}{at+1}$	(convincing)	
where $a = 7200k$		A1

(*c*)

Substituting $t = 2$ and $V = 50$ in expression for V^2		M1
$a = 0{\cdot}22$		A1
Substituting $V = 27$ in expression for V^2 with candidate's value for a		M1
$t = 17{\cdot}9$	(c.a.o)	A1

9. (*a*) An attempt to evaluate **a.b** M1
Correct evaluation of **a.b** and $\mathbf{a.b} \neq 0 \Rightarrow$ **a** and **b** not perpendicular A1

(*b*) (i) $\mathbf{AB} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of *AB* M1

$\mathbf{r} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda\,(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*c*) $4 + 2\lambda = 2 - 2\mu$

$1 + \lambda = 6 + \mu$

$-6 + 2\lambda = p + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving the first two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = 2$, $\mu = -3$ (o.e.) (c.a.o.) A1

$p = 7$ from third equation

(f.t. candidates derived values for λ and μ) A1

10. $a^2 = 5b^2 \Rightarrow (5k)^2 = 5b^2 \Rightarrow b^2 = 5k^2$ B1

$\therefore$ 5 is a factor of b^2 and hence 5 is a factor of *b* B1

$\therefore$ *a* and *b* have a common factor, which is a contradiction to the original assumption B1

FP1

Ques	Solution	Mark	Notes
1	$S_n = \sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r$	**M1**	
	$= \dfrac{n^2(n+1)^2}{4} - \dfrac{n(n+1)}{2}$	**A1A1**	
	$= \dfrac{n(n+1)}{4}\left(n^2 + n - 2\right)$	**A1**	
	$= \dfrac{n(n-1)(n+1)(n+2)}{4}$	**A1**	
2(a)	$(1+2\mathrm{i})^2 = 1 + 4\mathrm{i} + 4\mathrm{i}^2$	**M1**	Award for 3 reasonable terms.
	$= -3 + 4\mathrm{i}$	**A1**	
	$z = \dfrac{(-3+4\mathrm{i})(2-\mathrm{i})}{(2+\mathrm{i})(2-\mathrm{i})}$	**M1**	
	$= \dfrac{-6+8\mathrm{i}+3\mathrm{i}-4\mathrm{i}^2}{5}$	**A1A1**	A1 numerator, A1 denominator FT 1 arithmetic slip from line 2
	$= \dfrac{-2+11\mathrm{i}}{5}$ $(-0.4 + 2.2\mathrm{i})$ cao	**A1**	
(b)			
	$r = \sqrt{5}$ (2.24)	**B1**	FT on line above for r.
	$\tan^{-1}(-5.5) = -1.39$ $(-79.6°)$ or $\tan^{-1}(5.5) = 1.39$ $(79.6°)$	**B1**	FT on line above for this B1
	$\theta = 1.75$ $(100.3°)$	**B1**	FT only if in 2nd or 3rd quad
3(a)	$\alpha + \beta = -\dfrac{1}{2}, \alpha\beta = 1$	**B1**	
	$\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = \dfrac{\alpha^3 + \beta^3}{\alpha\beta}$	**M1**	
	$= \dfrac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$	**M1A1**	
	$= \dfrac{(-1/2)^3 - 3\times 1\times(-1/2)}{1}$	**A1**	
	$= \dfrac{11}{8}$		
(b)	Consider $\dfrac{\alpha^2}{\beta}\times\dfrac{\beta^2}{\alpha} = \alpha\beta = 1$	**M1A1**	
	The required equation is $x^2 - \dfrac{11}{8}x + 1 = 0$ $(8x^2 - 11x + 8 = 0)$ cao	**B1**	

4(a)(i)	Cofactor matrix = $\begin{bmatrix} -13 & 9 & 1 \\ -18 & 13 & 1 \\ 14 & -10 & -1 \end{bmatrix}$	**M1** **A1**	Award M1 if at least 5 cofactors are correct.
	Adjugate matrix = $\begin{bmatrix} -13 & -18 & 14 \\ 9 & 13 & -10 \\ 1 & 1 & -1 \end{bmatrix}$	**A1**	No FT on cofactor matrix.
(ii)	Determinant = 3(7 – 20) +4(16 – 7) +2(5 – 4) = – 1	**M1** **A1**	
	Inverse matrix = $\begin{bmatrix} 13 & 18 & -14 \\ -9 & -13 & 10 \\ -1 & -1 & 1 \end{bmatrix}$	**A1**	FT the adjugate
(b)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 & 18 & -14 \\ -9 & -13 & 10 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix}$	**M1**	
	$= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$	**A1**	FT their inverse matrix.
5(a)	By reduction to echelon form, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -2 & -10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ k-6 \end{bmatrix}$ It follows now that k – 6 = – 2 k = 4	**M1** **A1** **A1** **M1** **A1**	
(b)	Put $z = \alpha$. Then $y = 1 - 5\alpha$ And $x = 7\alpha$	**M1** **A1** **A1**	
6	Putting n = 1, the expression gives 3 which is divisible by 3 so the result is true for n = 1 Assume that the formula is true for $n = k$. ($k^3 + 2k$ is divisible by 3 or $k^3 + 2k = 3N$)). Consider (for $n = k + 1$) $(k+1)^3 + 2(k+1)$ $= k^3 + 3k^2 + 3k + 1 + 2k + 2$ $= 3N - 2k + 3k^2 + 3k + 1 + 2k + 2$ $= 3(N + k^2 + k + 1)$ (This is divisible by 3), therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	**B1** **M1** **M1** **A1** **A1** **A1** **A1**	Award this M1 only if it is clearly stated that this is an assumption Do not award the second M1 if this is stated as an assumption but the three A1s may be awarded if either of the M1s is awarded

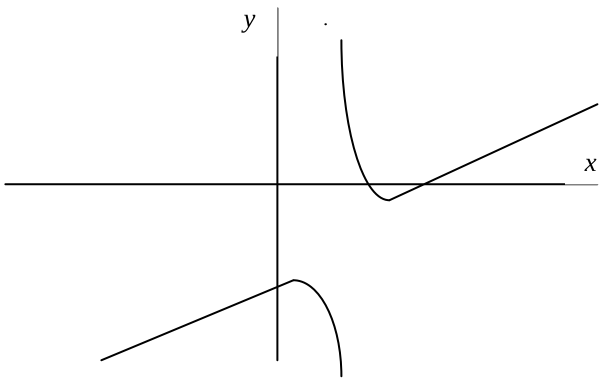
7(a)	Ref matrix in $y = x$ = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Translation matrix = $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Ref matrix in x-axis = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$	**M1**	
	$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	**A1**	
	$= \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$		
(b)	$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$	**M1**	
	$y - 2 = x, \; -x - 2 = y$	**A1**	
	$x = -2, y = 0$ cao	**A1A1**	

8(a)	Taking logs, $\ln f(x) = x\ln x$ Differentiating, $\dfrac{f'(x)}{f(x)} = \ln x + 1$ $f'(x) = x^x(\ln x + 1)$	**B1** **B1B1** **B1**	B1 for LHS, B1 for RHS
(b)	At a stationary point, $f'(x) = 0$ $\ln x = -1$ $x = \dfrac{1}{\mathrm{e}}; y = \left(\dfrac{1}{\mathrm{e}}\right)^{1/\mathrm{e}}$ (0.368, 0.692)	**M1** **A1** **A1**	
(c)	Differentiating the expression in (a), $f''(x) = x^x(\ln x + 1)(\ln x + 1) + x^x \times \dfrac{1}{x}$ $= x^{x-1} + x^x(1 + \ln x)^2$. $f''(1/\mathrm{e}) = 1.88$ Since this is positive it is a minimum.	**B1B1** **B1** **B1**	B1 each term Accept 'Since the first term is positive and the second term zero, it is a minimum' FT the final B1 on the line above
9(a)	$x + \mathrm{i}y = \dfrac{1}{u + \mathrm{i}v}$ $= \dfrac{u - \mathrm{i}v}{u^2 + v^2}$ $x = \dfrac{u}{u^2 + v^2}$ $y = -\dfrac{v}{u^2 + v^2}$	**M1** **A1** **A1**	
(b)(i)	We are given that $-\dfrac{v}{u^2 + v^2} = \dfrac{mu}{u^2 + v^2} + 1$ $-v = mu + u^2 + v^2$ $u^2 + v^2 + mu + v = 0$ (This is the equation of a circle).	**M1** **A1** **A1**	
(ii)	Completing the square or quoting the standard results, Radius $= \dfrac{1}{2}\sqrt{m^2 + 1}$ Centre $\left(-\dfrac{1}{2}m, -\dfrac{1}{2}\right)$	**M1** **A1** **A1**	FT on their circle equation
(iii)	$v = -\dfrac{1}{2}$	**A1**	Accept $y = -\dfrac{1}{2}$

FP2

Ques	Solution	Mark	Notes
1	Putting $x = 2$, $4a - 8 = 8 - 2b$ The two derivatives are $2ax$ and $3x^2 - b$ Putting $x = 2$, $4a = 12 - b$ Solving, $a = 2$, $b = 4$ cao	M1A1 M1 A1 A1	
2	$u = e^x \Rightarrow du = e^x dx$, $[0,1] \rightarrow [1, e]$ $I = \int_1^e \frac{du/u}{u + 4/u}$ $= \int_1^e \frac{du}{u^2 + 4}$ $= \frac{1}{2}\left[\tan^{-1}(\frac{u}{2})\right]_1^e$ $= 0.236$	B1 B1 M1 A1 A1 A1	
3	Put $t = \tan(x/2)$ $\frac{3 \times 2t}{1+t^2} = t$ $t(t^2 - 5) = 0$ $t = 0$ giving $x/2 = n\pi \rightarrow x = 2n\pi$ $(360n^o)$ $t = \sqrt{5}$ giving $x/2 = 1.15026.. + n\pi$ $\rightarrow x = 2.30 + 2n\pi$ $(360n^o + 132^o)$ $t = -\sqrt{5}$ giving $x/2 = -1.15026.. + n\pi$ $\rightarrow x = -2.30 + 2n\pi$ $(360n^o - 132^o)$	M1 A1 M1A1 M1 A1 M1 A1	

4(a)	Let $\dfrac{3x^2-4x+1}{(x-2)(x^2+1)} \equiv \dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+1}$ $= \dfrac{A(x^2+1)+(Bx+C)(x-2)}{(x-2)(x^2+1)}$	**M1**	
	$x = 2$ gives $A = 1$	**A1**	
	Coeff of x^2 gives $A + B = 3$, $B = 2$	**A1**	
	Const term gives $A - 2C = 1$, $C = 0$	**A1**	
(b)	$\int_3^4 f(x)\mathrm{d}x = \int_3^4 \dfrac{1}{x-2}\mathrm{d}x + \int_3^4 \dfrac{2x}{x^2+1}\mathrm{d}x$	**M1**	
	$= [\ln(x-2)]_3^4 + [\ln(x^2+1)]_3^4$	**A1A1**	
	$= \ln 2 - \ln 1 + \ln 17 - \ln 10$	**A1**	
	$= \ln\left(\dfrac{34}{10}\right)$ or $\ln\left(\dfrac{17}{5}\right)$	**A1**	
5(a)	Consider $f(-x) = (-x)^2\sin(-x)$	**M1**	Accept a specific value for *x*.
	$= -x^2\sin x = -f(x)$	**A1**	
	f is therefore odd.	**A1**	
(b)	$\sin x$ is odd and x^n is even if *n* is even and odd if *n* is odd. si	**M1**	For a valid attempt. Accept a specific value for *x*.
	So *g* is even if *n* is odd	**A1**	
	and *g* is odd when *n* is even.	**A1**	

6(a)	Putting $x = 0$ gives (0, – 20/3) Putting $y = 0$, $\frac{2}{x-3} = 6 - x$ $(x-3)(6-x) = 2$ $x^2 - 9x + 20 = 0$ giving (4,0) ; (5,0) cao	**B1** **M1** **A1** **A1** **A1**	
(b)	Differentiating, $-\frac{2}{(x-3)^2} + 1 = 0$ $(x-3)^2 = 2$ $x = 3 + \sqrt{2}(4.41), y = 2\sqrt{2} - 3(-0.172)$ $x = 3 - \sqrt{2}(1.59), y = -3 - 2\sqrt{2})(-5.83)$	**M1** **A1** **A1** **A1** **A1**	Award A1A0 for the 2 x values only.
(c)	The asymptotes are $x = 3$ $y = x - 6$	**B1** **B1**	
(d)		**G1** **G1**	General shape of both branches. Correct shape including asymptotic behaviour.

7(a)(i)	Completing the square, $(y-1)^2 = 8x - 24$ The vertex is therefore (3,1)	**M1** **A1** **A1**	 FT on 1 arithmetic slip
(ii)	In the usual notation, a = 2 si The focus is (5,1)	**B1** **B1**	
(iii)	The equation of the directrix is x = 1	**B1**	
(b)(i)	Substituting $y = mx$, $m^2x^2 - 2mx - 8x + 25 = 0$	**M1** **A1**	
(ii)	For coincident roots, $(2m+8)^2 = 100m^2$ $3m^2 - m - 2 = 0$ Solving using a valid method, m = 1, – 2/3	**M1** **A1** **A1** **M1** **A1**	
8(a)	The result is true for n = 1 since it gives $(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos 1\theta + \mathrm{i}\sin 1\theta$ Let the result be true for $n = k$, ie $(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$ Consider $(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos\theta + \mathrm{i}\sin\theta)^k(\cos\theta + \mathrm{i}\sin\theta)$ $= (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$ $\cos k\theta\cos\theta - \sin k\theta\sin\theta$ $+\mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ $= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$ True for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1 the result is proved by induction.	**B1** **M1** **M1** **A1** **A1** **A1** **A1**	
(b)(i)	Consider $\left(w(\cos 2\pi/3 + \mathrm{i}\sin 2\pi/3\right)^3$ $= w^3(\cos 2\pi + \mathrm{i}\sin 2\pi)$ $= z \times 1 = z$ Showing that $\left(w(\cos 2\pi/3 + \mathrm{i}\sin 2\pi/3)\right)$ is a cube root of z.	**M1** **A1** **A1**	
(ii)	The real cube root of – 8 is – 2. The other cube roots are $-2(\cos 2\pi/3 + \mathrm{i}\sin 2\pi/3) = 1 - \sqrt{3}\mathrm{i}$ $-2(\cos 4\pi/3 + \mathrm{i}\sin 4\pi/3) = 1 + \sqrt{3}\mathrm{i}$	**B1** **M1A1** **A1**	

FP3

Ques	Solution	Mark	Notes
1	$\int_0^1 x\sinh x\,\mathrm{d}x = \left[x\cosh x\right]_0^1 - \int_0^1 \cosh x\,\mathrm{d}x$	M1A1	
	$= \cosh 1 - \left[\sinh x\right]_0^1$	A1A1	
	$= \cosh 1 - \sinh 1$	A1	Do not accept an argument which evaluates this as 0.367879… and shows that this is also the numerical value of 1/e.
	$= \dfrac{\mathrm{e}^1 + \mathrm{e}^{-1}}{2} - \dfrac{\mathrm{e}^1 - \mathrm{e}^{-1}}{2}$	A1	
	$= \dfrac{1}{\mathrm{e}}$		
2(a)	The equation can be rewritten as $\sinh^2 x - \sinh x + 1 - k = 0$	M1A1	
	The condition for no real roots is $1 - 4(1-k) = 4k - 3 < 0$	m1	
	$k < \dfrac{3}{4}$	A1	
(b)	$\sinh^2 x - \sinh x - 2 = 0$ $(\sinh x - 2)(\sinh x + 1) = 0$	M1	
	$\sinh x = 2$	A1	
	$x = \sinh^{-1} 2 = \ln(2 + \sqrt{5})$	A1	
3	Let $f(x) = \tan^{-1} x$		
	$p = f(1) = \dfrac{\pi}{4}$	B1	
	$f'(x) = \dfrac{1}{1+x^2}; q = f'(1) = \dfrac{1}{2}$	M1A1	
	$f''(x) = -\dfrac{2x}{(1+x^2)^2}; r = \dfrac{f''(1)}{2} = -\dfrac{1}{4}$	M1A1	
	$f'''(x) = \dfrac{-2(1+x^2)^2 + 2(1+x^2).4x^2}{(1+x^2)^4}; s = \dfrac{f'''(1)}{6} = \dfrac{1}{12}$	M1A1	

4(a)	Consider $y = r\sin\theta$ $= 2\sin\theta\cos\theta - \sin^2\theta$	M1	
	$\dfrac{dy}{d\theta} = 2\cos^2\theta - 2\sin^2\theta - 2\sin\theta\cos\theta$	A1	
	$= 2\cos 2\theta - \sin 2\theta$	A1	
	The tangent is parallel to the initial line where $2\cos 2\theta = \sin 2\theta$	M1	
	$\tan 2\theta = 2$	A1	
	$\theta = 0.554, r = 1.18$	A1	Accept 31.7°
(b)	The curves intersect where $2\cos\theta - \sin\theta = 1 + \sin\theta$	M1	
	$2\cos\theta - 2\sin\theta = 1$	A1	
	EITHER Putting $t = \tan(\theta/2)$	M1	
	$\dfrac{2(1-t^2)}{1+t^2} - \dfrac{4t}{1+t^2} = 1$	A1	
	$3t^2 + 4t - 1 = 0$	A1	
	$\tan(\theta/2) = \dfrac{-4+\sqrt{28}}{6}$ (0.21525..)	A1	
	$\theta = 0.424, r = 1.41$	A1	Accept 24.3°
	OR Putting $2\cos\theta - 2\sin\theta = r\cos(\theta + \alpha)$	M1	
	$\alpha = \pi/4$	A1	
	$r = 2\sqrt{2}$	A1	
	$\cos(\theta + \pi/4) = \dfrac{1}{2\sqrt{2}}$	A1	
	$\theta = 0.424, r = 1.41$	A1	Accept 24.3°

5	Putting $t=\tan(x/2)$ gives $\mathrm{d}x=\dfrac{2\mathrm{d}t}{1+t^2}$	B1	
	$(0,\pi/2)\to(0,1)$	B1	
	$I=\displaystyle\int_0^1 \frac{2\mathrm{d}t/(1+t^2)}{4(1-t^2)/(1+t^2)+3}$	M1	
	$=2\displaystyle\int_0^1 \frac{\mathrm{d}t}{7-t^2}$	A1	
	$=2\times\dfrac{1}{2\sqrt{7}}\left[\ln\left\lvert\dfrac{\sqrt{7}+t}{\sqrt{7}-t}\right\rvert\right]_0^1$ or $\dfrac{2}{\sqrt{7}}\left[\tanh^{-1}\left(\dfrac{t}{\sqrt{7}}\right)\right]_0^1$	A1	
	$=\dfrac{1}{\sqrt{7}}\left(\ln\left(\dfrac{\sqrt{7}+1}{\sqrt{7}-1}\right)-\ln(1)\right)$ or $\dfrac{2}{\sqrt{7}}\left(\tanh^{-1}\left(\dfrac{1}{\sqrt{7}}\right)-\tanh^{-1}(0)\right)$	A1	
	$=0.301$	A1	
6(a)	$I_n=\left[\theta^n\sin\theta\right]_0^{\pi/2}-n\displaystyle\int_0^{\pi/2}\theta^{n-1}\sin\theta\,\mathrm{d}\theta$	M1A1	.
	$=\left(\dfrac{\pi}{2}\right)^n-n\displaystyle\int_0^{\pi/2}\theta^{n-1}\sin\theta\,\mathrm{d}\theta$	A1	
	$=\left(\dfrac{\pi}{2}\right)^n+\left[n\theta^{n-1}\cos\theta\right]_0^{\pi/2}-n(n-1)I_{n-2}$	M1A1	
	$=\left(\dfrac{\pi}{2}\right)^n-n(n-1)I_{n-2}$		
(b)(i)	$I_0=\displaystyle\int_0^{\pi/2}\cos\theta\,\mathrm{d}\theta=\left[\sin\theta\right]_0^{\pi/2}=1$	B1	
	$I_4=\left(\dfrac{\pi}{2}\right)^4-12I_2$	M1	
	$=\left(\dfrac{\pi}{2}\right)^4-12\left(\left(\dfrac{\pi}{2}\right)^2-2I_0\right)$	A1	
	$=0.479$	A1	
(b)(ii)	$\displaystyle\int_0^{\pi/2}\theta^5\sin\theta\,\mathrm{d}\theta=-\left[\theta^5\cos\theta\right]_0^{\pi/2}+5\int_0^{\pi/2}\theta^4\cos\theta\,\mathrm{d}\theta$	M1A1	
	$=5I_4=2.4$	A1	FT their answer from (b)(i)

7(a)	The Newton-Raphson iteration is $x_{n+1} = x_n - \dfrac{(x_n - 2\tanh x_n)}{(1-2\text{sech}^2 x_n)}$	M1A1	
	$= \dfrac{x_n - 2x_n\text{sech}^2 x_n - x_n + 2\tanh x_n}{1-2\text{sech}^2 x_n}$	A1	
	$= \dfrac{-2x_n + 2\sinh x_n \cosh x_n}{\cosh^2 x_n - 2}$	A1	
	$= \dfrac{\sinh 2x_n - 2x_n}{\cosh^2 x_n - 2}$	A1	
(b)	$x_0 = 2$ $x_1 = 1.916216399$ $x_2 = 1.915008327$	B1	
	Rounding to three decimal places gives 1.915	B1	
	Let $f(x) = x - 2\tanh x$ $f(1.9155) = 4.1\times10^{-4}$ $f(1.9145) = -4.2\times10^{-4}$	M1	
	The change of sign shows $\alpha = 1.915$ correct 3dp	A1	The values are required
8(a)	The curve cuts the x-axis where $x = \cosh^{-1} 2 = \alpha$	B1	Seen or implied
	$\dfrac{dy}{dx} = -\sinh x$	B1	
	$1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + \sinh^2 x = \cosh^2 x$	B1	
	Arc length = $\int \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx$	M1	
	$= \int_{-\alpha}^{\alpha} \cosh x\,dx$	A1	
	$= [\sinh x]_{-\alpha}^{\alpha}$	A1	
	$= 2\sqrt{3}$ (3.46) cao	A1	
(b)	Curved surface area = $2\pi \int y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx$	M1	
	$= 2\pi \int_{-\alpha}^{\alpha} (2-\cosh x)\cosh x\,dx$	A1	
	$= 4\pi \int_{-\alpha}^{\alpha} \cosh x\,dx - \pi \int_{-\alpha}^{\alpha} (\cosh 2x + 1)$	A1	
	$= \pi\left[4\sinh x - \dfrac{1}{2}\sinh 2x - x\right]_{-\alpha}^{\alpha}$	A2	Minus 1 each error
	$= 2\pi(4\sqrt{3} - 2\sqrt{3} - \cosh^{-1} 2)$	A1	
	$=13.5$	A1	

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GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3 AS/Advanced

SUMMER 2012

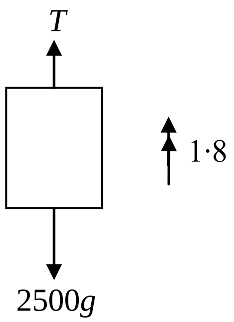
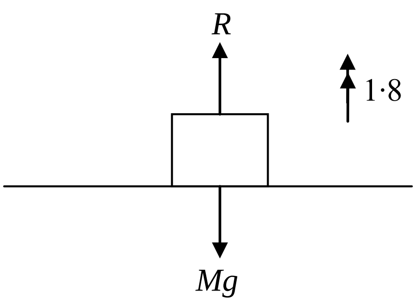
INTRODUCTION

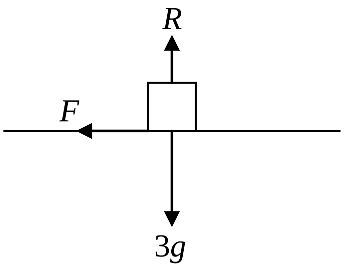
The marking schemes which follow were those used by WJEC for the Summer 2012 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

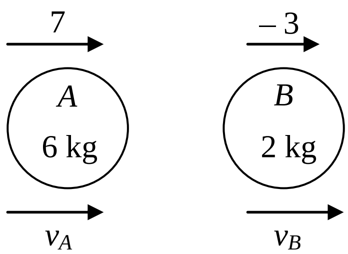
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

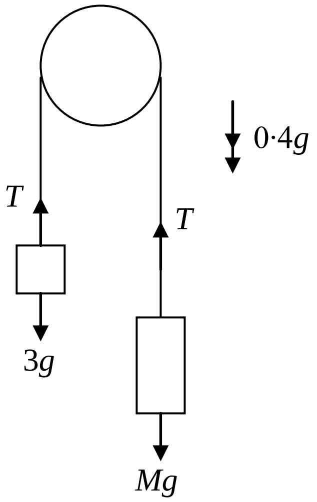
WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

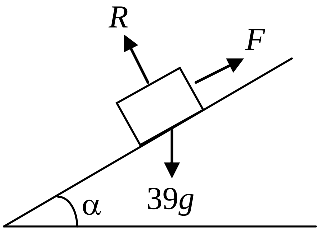
M1

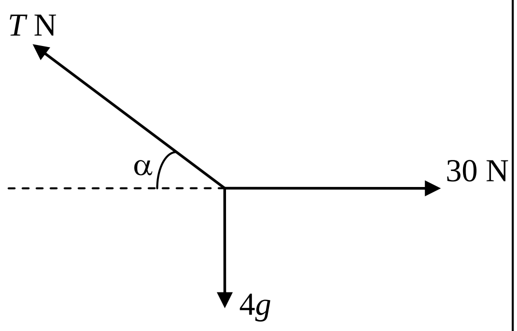
Q	Solution	Mark	Notes
1(a).			
	N2L dim correct equation attempted	M1	T, 2500g opposing
	$T - 2500g = 2500 \times a$	A1	Any form correct equ.
	$T = 2500(9.8 + 1.8)$		
	T = <u>29000 (N)</u>	A1	cao
1(b)			
	N2L attempted	M1	R, Mg opposing, no extra forces
	$R - Mg = Ma$	A1	Any form correct equ.
	$696 = M(9.8 + 1.8)$		
	M = <u>60 (kg)</u>	A1	cao

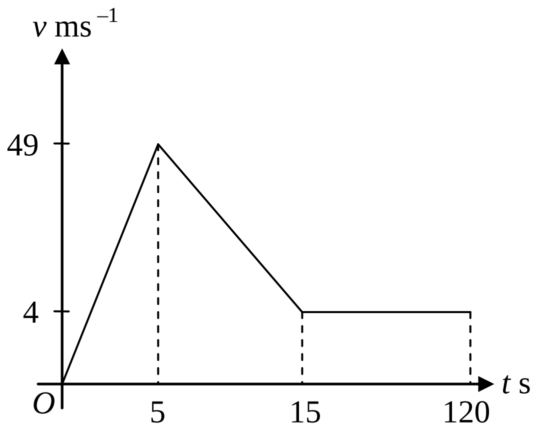
Q	Solution	Mark	Notes
2(a).			
	Resolve vertically R = 3g	B1	May be implied
	$F = \mu R = \frac{6}{49} \times 3 \times 9 \cdot 8$		
	F = <u>3.6 (N)</u>	B1	
	N2L F = ma	M1	used
	±3.6 = 3a		
	a = <u>-1.2 (ms^{-2})</u>	A1	needs to see -
2(b)	Using $v^2 = u^2 + 2as$ with u=9, v=0,a=(-)1.2	M1	allow sign errors, oe
	$0 = 9^2 + 2 \times (-1.2)$ s	A1	
	s = <u>33.75 (m)</u>	A1	allow -33.75

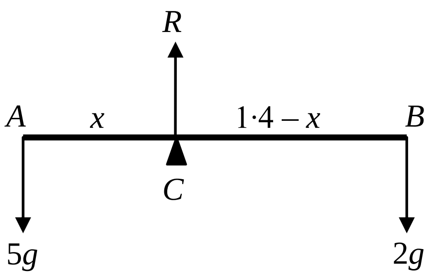
Q	Solution	Mark	Notes
3.			
3(a)	Conservation of momentum	M1	dim correct equation
	$6 \times 7 + 2 \times (-3) = 6v_A + 2v_B$	A1	
	$v_B = 2 v_A$	m1	used
	$42 - 6 = 6v_A + 2 \times 2v_A$		
	$36 = 10 v_A$		
	$v_A = 3.6$		
	$v_B = \underline{7.2\ (\text{ms}^{-1})}$	A1	
3(b)	Restitution equation	M1	attempted, ft c's vs, e on correct side. No more than one sign error.
	$7.2 - 3.6 = -e(-3 - 7)$	A1	
	$3.6 = 10e$		
	$e = \underline{0.36}$	A1	cao
3(c)	$I = 2 \times 7.2 - 2 \times (-3)$	M1	allow 6(7-3.6)
	$I = 14.4 + 6$		
	$I = \underline{20.4\ (\text{Ns})}$	A1	cao

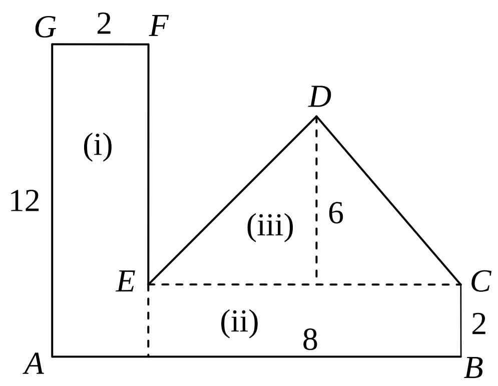
Q	Solution	Mark	Notes
4.			
	Apply N2L to B Mg – T = Ma	M1 A1	dim correct equation
	Apply N2L to A T – 3g = 3a	M1 A1	dim correct equation
	Adding	m1	correct method. dep on both M's
	Mg – 3g = 0.4g(M + 3) M – 3 = 0.4M + 1.2 0.6M = 4.2 M = <u>7</u>	A1	cao
	T = 3 × 9.8 + 3 × 0.4 × 9.8 T = <u>41.16 (N)</u>	A1	cao
	<u>Alternative solution</u> Apply N2L to A T – 3g = 3a T = 3(9.8 + 0.4 × 9.8) T = <u>41.16 (N)</u>	M1 A1 A1	dim. correct equation cao
	Apply N2L to B Mg – T = Ma 9.8M – 0.4 × 9.8M = 41.16 5.88M = 41.16 M = <u>7</u>	M1 A1 m1 A1	dim correct equation cao

Q	Solution	Mark	Notes
5.			
5(a)	Resolve perp to plane R = 39gcosα $R = 39 \times 9 \cdot 8 \times \frac{12}{13} = 352.8$ N F = μR F = 0.3 × 352.8 F = 105.84 N	M1 m1 A1	allow sin or cos si
	N2L down slope 39gsinα - F = 39a $39 \times 9 \cdot 8 \times \frac{5}{13} - 105 \cdot 84 = 39a$ a = 1.0554 a = <u>1.06 (ms^{-2})</u>	M1 A1 A1	dim correct equation, -F
5(b)	N2L up slope T – 39gsinα - F = 39a T = 147 + 105.84 + 39 × 0.4 T = <u>268.44 (N)</u>	M1 A1 A1	dim correct equation, all forces, sin/cos, -F cao

Q	Solution	Mark	Notes
6.			
	Resolve vertically	M1	
	$T\sin\alpha = 4g$	A1	
	Resolve horizontally	M1	
	$T\cos\alpha = 30$	A1	
	Dividing		
	$\tan\alpha = \dfrac{4 \times 9 \cdot 8}{30}$	m1	dep on both M's
	$\alpha = \underline{52.5(7)°}$	A1	cao
	$T^2 = (4 \times 9.8)^2 + (30)^2$	m1	
	$T = \underline{49.36\ (N)}$	A1	cao

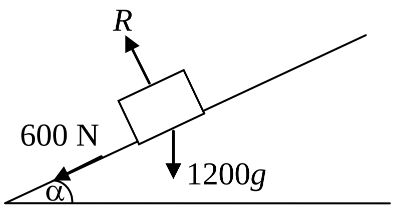
Q	Solution	Mark	Notes
7(a)	Using v = u + at with u=0, a=(±)9.8, t=5	M1	
	v = 0 + 9.8 × 5	A1	
	v = 49 (ms^{-1})	A1	accept -49
7(b)			
		B1	units, labels and correct shape starting (0,0)
		B1	(0, 0) to (5, v)
		B1	(5, v) to (15, 4)
		B1	(15, 4) to (120, 4)
7(c)	Distance = Area under graph	M1	oe
	Distance = 0.5 × 5 ×49 + 0.5(4 + 49) ×10 + 105 × 4	B1	any one correct area, ft graph
	Distance = 122.5 + 265 + 420		
	Distance = 807.5 (m)	A1	ft graph

Q	Solution	Mark	Notes
8.			
8(a)	Resolve vertically R = 5g + 2g R = 7g (N)	M1 A1	
8(b)	Moments about C 5gx = 2g(1.4 – x) 5x = 2.8 – 2x 7x = 2.8 x = 0.4 AC = 0.4 (m)	M1 A1 A1 A1	dim correct equation, no extra forces rhs correct lhs correct cao
	Alternative solution Moments about A 7gx = 2g × 1.4 x = 0.4 (m)	 M1 A1 A1 A1 SC1	 dim correct equation rhs correct lhs correct cao No marks at all, one correct moment, sc1.

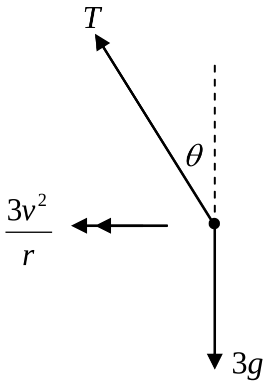
Q	Solution	Mark	Notes
9.			
9(a)	Area / from AG / from AB		
	(i) 24 1 6	B1	correct distances
	(ii) 12 5 1	B1	correct distances
	(iii) 18 5 4	B1	correct distances
	Lamina 54 x y	B1	areas all correct
	Moments about AG	M1	
	$54x = 24 \times 1 + 12 \times 5 + 18 \times 5$	A1	ft table if 2 or more B marks for distances gained.
	$x = \frac{29}{9} = 3.22$	A1	cao
	Moments about AB	M1	
	$54y = 24 \times 6 + 12 \times 1 + 18 \times 4$	A1	ft table
	$y = \frac{38}{9} = 4.22$	A1	cao
9(b)	$\theta = \tan^{-1}\left(\frac{x}{12 - y}\right)$	M1	correct triangle
	$= \tan^{-1}\left(\frac{29}{12 \times 9 - 38}\right)$	A1	correct equation, ft x, y
	$\theta = \underline{22.5°}$	A1	ft x and y

M2

Q	Solution	Mark	Notes
1.	s = $\int_0^{\frac{\pi}{6}} 4\cos 2t\, dt$	M1	limits not required
	s = [2sin2t]	A1	correct integration
	s = $2\sin\frac{\pi}{3}$ - 0		
	s = $\sqrt{3}$ = <u>1.732</u>	A1	cao
2(a)	N2L T = 7.5g	B1	
	Hooke's Law T = $\frac{245x}{5/3}$ (= 147x)	M1	
	7.5 × 9.8 = 147x		
	x = <u>0.5</u>	A1	cao
2(b)	Elastic Energy = $\frac{1}{2} \times \frac{x^2}{l}$	M1	used
	EE = $\frac{1}{2} \times \frac{245 \times 0\cdot5^2}{5/3}$		
	EE = <u>18.375 (J)</u>	A1	ft c's x value
3(a).	$\underline{v} = \frac{dr}{dt}$	M1	used
	$\underline{v}$ = (1 + 4t)$\underline{i}$ + (3t – 2)$\underline{j}$	A1	
	we required $\underline{v}$.(-$\underline{i}$ + 2$\underline{j}$) = 0	M1	
	-(1 + 4t) + 2(3t – 2) = 0	m1	
	-1 -4t + 6t -4 = 0		
	2t = 5		
	t = <u>2.5</u>	A1	cao
3(b)	$\underline{a} = \frac{dv}{dt}$	M1	used
	$\underline{a}$ = 4$\underline{i}$ + 3$\underline{j}$	A1	ft c's **v** provided constant
	$\underline{a}$ is independent of t and constant.		
	$\|a\| = \sqrt{4^2 + 3^2}$ = <u>5</u>	A1	ft constant **a**=x**i**+y**j**

Q	Solution	Mark	Notes
4.			
4(a)	$T = \frac{P}{v} = \frac{75\times1000}{25}$	M1	
	$T = 3000$ N		
	N2L up plane	M1	dim correct, all forces
	$T - 1200g\sin\alpha - 600 = 1200a$		A2 -1 each error
	$1200a = 3000 - 1200 \times 9.8 \times 0.1 - 600$		
	$a = \underline{1.02\ (\text{ms}^{-2})}$	A1	cao
4(b)	$T = \frac{90\times1000}{v}$	M1	
	N2L up plane	M1	dim correct, all forces
	$T - 1200g\sin\alpha - 600 = 1200a$		
	$a = 0$	m1	si
	$\frac{90000}{v} = 1776$		
	$v = \underline{50.7\ (\text{ms}^{-1})}$	A1	cao
5.	KE at A $= 0.5 \times 0.1 \times v^2$	B1	
	PE at A $= 0.1 \times 9.8 \times 0.5$	M1	
	PE at B $= 0.1 \times 9.8 \times 1.4$	A1	both or difference
	WD against resistance $= 6 \times 1.2$	B1	
	Work-energy principle	M1	all terms included
	$0.05 v^2 = 7.2 + 0.1 \times 9.8 \times 0.9$	A1	correct equation
	$v^2 = 161.64$		
	$v = \underline{12.7\ (\text{ms}^{-1})}$	A1	cao

Q	Solution	Mark	Notes
6(a).	$u_H = V\cos\alpha$ (= 0.8V)	M1	attempt to resolve
	$u_V = V\sin\alpha$ (= 0.6V)	A1	both answers correct
6(b)	Consider horizontal motion		
	$0.8V \times T = 12$	M1	
	$VT = 15$	A1	correctly obtained
6(c)	Consider vertical motion		
	$s = ut + 0.5at^2$ with s=(±)5.4, u=0.6V, t=T a=(±)9.8	M1	
	$-5.4 = 0.6VT - 4.9T^2$	A1	
	$-5.4 = 0.6 \times 15 - 4.9T^2$		
	$4.9T^2 = 14.4$		
	$T = \frac{12}{7}$	A1	cao
	$\frac{12}{7}V = 15$		
	$V = \underline{8.75}$	A1	cao
6(d)	Using v = u + at with u=5.25, a=(±)9.8, $t=\frac{12}{7}$	M1	
	$v = 5.25 - 9.8 \times \frac{12}{7}$		
	$v = -11.55$	A1	si, cao
	$u_H = 0.8 \times 8.75 = 7$	B1	
	Speed $= \sqrt{11.55^2 + 7^2}$	M1	
	Speed = $\underline{13.5\ (ms^{-1})}$	A1	

Q	Solution	Mark	Notes
7.			
7(a)	Resolve vertically	M1	
	$T\cos\theta = mg$	A1	
	$\theta = \cos^{-1}\left(\frac{3\times 9\cdot 8}{88\cdot 2}\right)$		
	$\theta = \underline{70.5°}$	A1	cao
7(b)	N2L towards centre	M1	attempted
	$T\sin\theta = ma$	A1	
	$a = r\omega^2$	m1	used
	$r = \frac{T\sin\theta}{m\omega^2}$		
	length of string $= l$		
	$l\sin\theta = r$	m1	
	$l = \frac{r}{\sin\theta}$		
	$l = \frac{T}{m\omega^2} = \frac{88\cdot 2}{3\times 2\cdot 8^2}$		
	$l = \underline{3.75\text{ (m)}}$	A1	cao
	Alternative Solution		
	N2l towards centre	M1	attempted
	$T\sin\theta = ma$	A1	
	$a = r\omega^2$	m1	used
	$88.2\sin\theta = 3\times r\times 2.8^2$		
	$r = 3.53553$ m		
	$AP = \frac{r}{\sin\theta}$	m1	
	$AP = \underline{3.75\text{ (m)}}$	A1	cao

Q	Solution	Mark	Notes
8(a)	$\underline{v} = \frac{1}{3}[(14\underline{i} - 5\underline{j}) - (8\underline{i} + 7\underline{j})]$	M1	
	$\underline{v} = \frac{1}{3}(6\underline{i} - 12\underline{j})$		
	$\underline{v} = (2\underline{i} - 4\underline{j})$	A1	
8(b)	$\underline{r_S} = (8\underline{i} + 7\underline{j}) + (2\underline{i} - 4\underline{j})t$	M1 A1	
	$\underline{r_S} = (8 + 2t)\underline{i} + (7 - 4t)\underline{j}$		
8(c)	$\underline{r_B} = (x\underline{i} + y\underline{j})(t - 10)$	M1 A1	
	$\underline{r_B} = x(t - 10)\underline{i} + y(t-10)\underline{j}$		
	At t = 50 $\underline{r_S} = \underline{r_B}$	M1	
	$8 + 2t = x(t - 10)$	m1	
	$40x = 108$		
	$x = \underline{2.7}$	A1	cao
	$7 - 4 \times 50 = 40y$		
	$y = \underline{-4.825}$	A1	cao
	<u>Alternative solution</u>		
	At t = 50		
	$\underline{r_S} = 108\underline{i} - 193\underline{j}$		
	$\underline{r_B} = 40x\underline{i} + 40y\underline{j}$	M1 A1	
	$\underline{r_S} = \underline{r_B}$	M1	si
	$40x = 108$	m1	
	$x = \underline{2.7}$	A1	cao
	$40y = -193$		
	$y = \underline{-4.825}$	A1	cao

Q	Solution	Mark	Notes
9(a).	Conservation of energy	M1	
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgl(1 - \cos\theta)$	A1 A1	
	At max height, $v=0$, $\cos\theta=\frac{2}{3}$, $l=1.2$	m1	
	$\frac{1}{2}u^2 = 9.8 \times 1.2(1 - \frac{2}{3})$		
	$u^2 = 2 \times 9.8 \times 1.2 \times \frac{1}{3}$		
	$u = \underline{2.8\ (ms^{-1})}$	A1	cao
	$v^2 = u^2 - 2gl(1 - \cos\theta)$		
	$v^2 = 2.8^2 - 2 \times 9.8 \times 1.2(1- \cos\theta)$		
	$\underline{v^2 = 23.52\cos\theta - 15.68}$	A1	cao
9(b)	N2L towards centre	M1	
	$T - mg\cos\theta = mv^2/l$	A1	
	$T = 3 \times 9.8\cos\theta + \frac{3}{1\cdot 2}(23.52\cos\theta - 15.68)$	m1	
	$T = 29.4\cos\theta + 58.8\cos\theta - 39.2$		
	$T = \underline{88.2\cos\theta - 39.2}$	A1	cao
9(c)	Greatest value of T when $\cos\theta = 1$		
	$T = 88.2 - 39.2$		
	$T = \underline{49\ (N)}$	B1	
	Least value of T when $\cos\theta = \frac{2}{3}$		
	$T = 88.2 \times \frac{2}{3} - 39.2$		
	$T = \underline{19.6\ (N)}$	B1	

M3

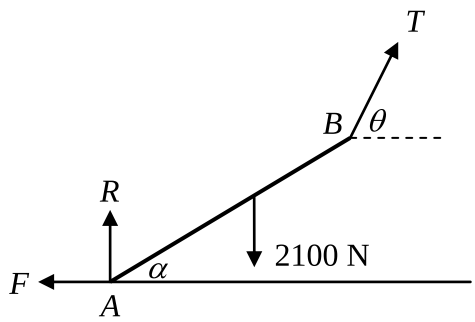
Q	Solution	Mark	Notes
1(a)	N2L $\frac{27000}{(t+3)^2} = 600a$	M1	+/-, no additional terms
	$\frac{45}{(t+3)^2} = \frac{dv}{dt}$	m1	use of dv/dt
	v = - $\frac{45}{(t+3)}$ (+ C)	A1	k/(t+3)
		A1	completely correct
	When t = 0, v = 0	m1	use of initial conditions
	C = 15		
	v = 15 - $\frac{45}{(t+3)}$	A1	
	As t $\to \infty$, v $\to$ 15	A1	ft similar expression
1(b)	v = $\frac{dx}{dt}$ = 15 - $\frac{45}{(t+3)}$	M1	
	x = 15t – 45 ln(t + 3) (+ C)	A1	ft similar expressions
	t = 0, x = 0 C = 45 ln3	A1	ft
	x = 15t + 45 ln$\left(\frac{3}{t+3}\right)$		
	When t = 6 x = 90 + 45 ln$\left(\frac{3}{9}\right)$	m1	
	x = 90 – 45 ln(3)		
	x = <u>40.56 (m)</u>	A1	cao

Q	Solution	Mark	Notes
2(a).	Using $v^2 = \omega^2(a^2 - x^2)$	M1	used
	$0.09 \times 3 = \omega^2(a^2 - 0.6^2)$	A1	
	$0.04 \times 5 = \omega^2(a^2 - 0.8^2)$	A1	
	$0.07 = 0.28\omega^2$	m1	
	$\omega = 0.5$		
	$0.2 = 0.25(a^2 - 0.64)$		
	$a = 1.2$	A1	
	Period $= \dfrac{2\pi}{\omega}$	M1	used
	Period $= \underline{4\pi}$	A1	
2(b)	$\ddot{x} = -\omega^2 x$	M1	used
	$\lvert\ddot{x}\rvert = 0.5^2 \times 0.6$		
	$\lvert\ddot{x}\rvert = \underline{0.15\ (\text{ms}^{-2})}$	A1	
2(c)	$x = 1.2\sin(0.5t)$	M1	used, accept cos
	At A, $0.6 = 1.2\sin(0.5t)$		
	$t = 2\sin^{-1}(0.5) = 1.0472$	A1	or 2.0944
	At B, $0.8 = 1.2\sin(0.5t)$		
	$t = 2\sin^{-1}(0.667) = 1.4595$	A1	or 1.6821
	Required $t = 1.4595 - 1.0472$		
	Required $t = \underline{0.412\ (\text{s})}$	A1	cao
2(d)	$x = a\sin(\omega t)$		
	$x = 1.2\sin(0.5t)$	M1	
	$x = 1.2\sin(0.5 \times 2\pi/3)$		
	$x = \underline{1.0392\ (\text{m})}$	A1	
2(e)	$v = a\omega\cos(\omega t)$	M1	oe
	$v = 1.2 \times 0.5\cos(0.5t)$		
	$v = 0.6\cos(0.5t)$	A1	
	When $t = 2\pi/3$, $v = 0.6\cos(0.5\times 2\pi/3)$		
	$v = 0.6\cos(\pi/3)$		
	$v = \underline{0.3\ (\text{ms}^{-1})}$	A1	cao

Q	Solution	Mark	Notes
3.	Auxiliary equation		
	$2m^2 + 5m + 2 = 0$	B1	cao
	$(2m + 1)(m + 2) = 0$		
	$m = -0.5, -2$	B1	cao
	CF is $x = Ae^{-0.5t} + Be^{-2t}$	B1	ft solutions for m
	For PI, try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$5a + 2(at + b) = 6t + 5$	A1	
	Comparing coefficients	m1	
	$2a = 6$		
	$a = 3$		
	$15 + 2b = 5$		
	$b = -5$	A1	both answers cao
	General solution is		
	$x = Ae^{-0.5t} + Be^{-2t} + 3t - 5$	B1	ft CF and PI
	When $t = 0$, $x = 3$	M1	use of conditions in GS
	$3 = A + B - 5$		
	$A + B = 8$		
	$\frac{dx}{dt} = -0.5Ae^{-0.5t} - 2Be^{-2t} + 3$	B1	ft similar expressions
	When $t = 0$, $\frac{dx}{dt} = 2$		
	$2 = -0.5A - 2B + 3$		
	$0.5A + 2B = 1$		
	$A + 4B = 2$		
	$A + B = 8$		
	$3B = -6$		
	$B = \underline{-2}$	A1	cao
	$A = \underline{10}$	A1	cao

Q	Solution	Mark	Notes
4(a)	N2L F = ma	M1	used, no extra term
	$\dfrac{4}{2x+1} = 0 \cdot 5v\dfrac{dv}{dx}$	m1	use of vdv/dx
	$\int \dfrac{8}{2x+1}dx = \int v\,dv$	M1	separating variables
	$4\ln\|2x+1\| = \dfrac{1}{2}v^2 + C$	A1	kln(2x+1)
		A1	all correct
	$v^2 = 8\ln\|2x+1\| + C$		
	When $x = 3$, $v = 4$	m1	
	$16 = 8\ln 7 + C$		
	$C = 16 - 8\ln 7$	A1	ft kln(2x+1) + C
	$v^2 = 8\ln\left\|\dfrac{2x+1}{7}\right\| + 16$		
	When $x = 10$ $v^2 = 8\ln\left\|\dfrac{2\times10+1}{7}\right\| + 16$		
	$v^2 = 8\ln 3 + 16$		
	v = <u>4.98 (ms^{-1})</u>	A1	cao
4(b)	$v = 6$, $6^2 = 8\ln\left\|\dfrac{2x+1}{7}\right\| + 16$	M1	allow similar expressions
	$\ln\left\|\dfrac{2x+1}{7}\right\| = \dfrac{20}{8}$		
	$2x + 1 = 7e^{5/2}$	m1	correct inversion
	$x = 0.5[7e^{5/2} - 1]$		
	x = <u>42.1 (m)</u>	A1	cao

Q	Solution	Mark	Notes
5.	Using v = u + at with u=0, a=(±)9.8, t=2.5	M1	
	v = 9.8 × 0.5		
	v = 4.9 ms^{-1}	A1	
	Impulse = Change in momentum	M1	used
	For A J = 5v	B1	
	For B J = 2 × 4.9 – 2v	A1	
	Solving 5v = 9.8 – 2v	m1	
	7v = 9.8		
	v = 1.4 (ms^{-1})	A1	cao
	J = 5 × 1.4		
	J = 7 (Ns)	A1	cao

Q	Solution	Mark	Notes
6.(a)			
	$F = \mu R = \frac{3}{4}R$	M1	
	Moments about B	M1	dim correct equation, 3 terms, perp distance
	$R \times 2\cos\alpha + F \times 2\sin\alpha = 2100 \times 1\cos\alpha$	A3	-1 each error
	$R \times 2 \times \frac{12}{13} + \frac{3}{4}R \times 2 \times \frac{5}{13} = 2100 \times \frac{12}{13}$		
	$24R + \frac{15}{2}R = 25200$		
	$R = \underline{800\ (N)}$	A1	cao
6(b)	Resolve vertically	M1	
	$T\sin\theta = 2100 - R$	A1	
	$T\sin\theta = 1300$		
	Resolve horizontally	M1	
	$T\cos\theta = F$	A1	
	$T\cos\theta = \frac{3}{4} \times 800$		
	$T\cos\theta = 600$		
	$T = \sqrt{1300^2 + 600^2}$	m1	oe
	$T = \underline{1432\ (N)}$	A1	cao
	$\theta = \tan^{-1}\left(\frac{1300}{600}\right)$	m1	oe
	$\theta = \underline{65.2°}$	A1	cao

Ques	Solution	Mark	Notes
1(a)(i)	$P(A\cup B) = P(A) + P(B)$	M1	Award M1 for using formula
	$= 0.8$	A1	
(ii)	$P(A\cap B) = P(A)P(B) = 0.5 \times 0.3$	B1	
	$P(A\cup B) = P(A) + P(B) - P(A\cap B)$	M1	Award M1 for using formula
	$= 0.5 + 0.3 - 0.5 \times 0.3 = 0.65$	A1	
(b)	$P(A\cap B) = P(A) + P(B) - P(A\cup B) = 0.1$	B1	
	$P(B\|A) = \dfrac{P(A\cap B)}{P(A)}$	M1	Award M1 for using formula
	$= 0.2$	A1	
2(a)	$E(X^2) = \text{Var}(X) + [E(X)]^2$	M1	Award M1 for using formula
	$= 66$	A1	
(b)	$E(Y) = 3E(X) + 4$	M1	Award M1 for using formula
	$= 28$	A1	
	$\text{Var}(Y) = 3^2\,\text{Var}(X)$	M1	Award M1 for using formula
	$= 18$	A1	
3(a)	P(no white) $= \dfrac{4}{9}\times\dfrac{3}{8}\times\dfrac{2}{7}$ or $\dbinom{4}{3}\div\dbinom{9}{3}$	M1	
	$= \dfrac{1}{21}$	A1	
(b)	P(2 white) $= \dfrac{5}{9}\times\dfrac{4}{8}\times\dfrac{4}{7}\times 3$ or $\dbinom{5}{2}\times\dbinom{4}{1}\div\dbinom{9}{3}$	M1	M0 if 3 omitted.
	$= \dfrac{10}{21}$	A1	
(c)	EITHER		
	P(2 blue) $= \dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{6}{7}\times 3$ or $\dbinom{3}{2}\times\dbinom{6}{1}\div\dbinom{9}{3}$	M1A1	M0 if 3 omitted
	$= \left(\dfrac{3}{14}\right)$		
	P(2 the same) $= \dfrac{10}{21} + \dfrac{3}{14}$		
	$= \dfrac{29}{42}$ cao	A1	
	OR		
	P(2 the same) $= \dfrac{5}{9}\times\dfrac{4}{8}\times\dfrac{1}{7}\times 3 + \dfrac{5}{9}\times\dfrac{4}{8}\times\dfrac{3}{7}\times 3 + \dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\times 3 + \dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{5}{7}\times 3$	M1A1	M0 if 3 omitted Accept $\dbinom{5}{2}\times\dbinom{1}{1}\div\dbinom{9}{3}+\dbinom{5}{2}\times\dbinom{3}{1}\div\dbinom{9}{3}$ $+\dbinom{3}{2}\times\dbinom{1}{1}\div\dbinom{9}{3}+\dbinom{3}{2}\times\dbinom{5}{1}\div\dbinom{9}{3}$
	$= \dfrac{29}{42}$ cao	A1	

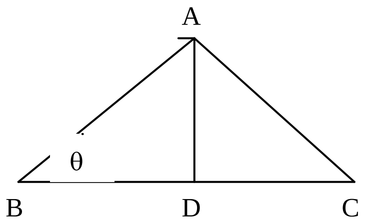
4(a)(i)	$P(X = 4) = \binom{10}{4} \times 0.75^4 \times 0.25^6$ $= 0.0162$	**M1** **A1**	Accept 0.9965 – 0.9803 or 0.0197 – 0.0035
(ii)	Let Y denote the number of games won by Dave so that Y is B(10,0.25). si We require $P(Y \le 4)$ $= 0.9219$	**M1** **m1** **A1**	
(b)	The number of games lasting less than 1 hr, G, is B(45,0.08) ≈ Poi(3.6). si $P(G > 6) = 0.0733$	**B1** **M1A1**	Award M1A0 for use of adjacent row or column. FT their mean
5(a)	$P(\text{CB}) = \frac{6}{10} \times \frac{8}{100} + \frac{4}{10} \times \frac{3}{100}$ $= 0.06$	**M1A1** **A1**	M1 Use of Law of Total Prob (Accept tree diagram)
(b)	$P(\text{F}\|\text{CB}) = \frac{12/1000}{0.06}$ $= 0.2$ cao	**B1B1** **B1**	FT denominator from (a) B1 num, B1 denom
6(a)	$\frac{1}{6}$	**B1**	
(b)	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$	**M1A1**	
(c)	$\frac{1}{6}, \frac{25}{216}$ and $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \left(\frac{625}{7776}\right)$	**M1A1**	Award M1A1 if only 3rd term given.
(d)	Prob $= \frac{1/6}{1 - 25/36}$ $= \frac{6}{11}$	**M1** **A1**	FT their answer to (a)
7(a)(i)	$P(X = 10) = \frac{e^{-12} \times 12^{10}}{10!}$ $= 0.105$	**M1** **A1**	Working must be shown. Accept 0.3472 – 0.2424 or 0.7576 – 0.6528
(ii)	$P(X > 10) = 1 - 0.3472 = 0.6528$	**M1A1**	Award M1 for adjacent row/col
(b)	Using tables, we see that $P(X \le 18) = 0.9626$ He needs to take 18 jars.	**M1** **A1**	Award M1A0 for 17 or 19

8(a)	$0 \le \theta \le 0.3$	**B1B1**	Accept use of <.
(b)	$E(X) = 2(0.3-\theta) + 3\times 2\theta + 4(0.7-\theta)$	**M1**	Use of $\Sigma x p_x$ with θ.
	$= 3.4$	**A1**	
	E(X) is therefore independent of θ		Need not be seen
(c)(i)	$E(X^2) = 4(0.3-\theta) + 9\times 2\theta + 16(0.7-\theta)$.	**M1**	Must include θ
	$= 12.4 - 2\theta$	**A1**	
	Var(X) $= 12.4 - 2\theta - 3.4^2$	**M1**	FT their $E(X)$ if possible
	$= 0.84 - 2\theta$	**A1**	
	$0.84 - 2\theta = 0.8^2$		
	$\theta = 0.1$ cao	**A1**	
(ii)	Possibilities are 3,3; 4,2 si	**B1**	
	P(Sum = 6) = 0.2×0.6×2 + 0.2×0.2	**M1**	Award M1A0 if 2 is missing in 1st term or present in 2nd term
	= 0.28	**A1**	FT their value of θ if sensible
9(a)(i)	$E(X) = \frac{1}{10}\int_1^2 x(2x+3x^2)\,dx$	**M1A1**	M1 for the integral of $xf(x)$, A1 for completely correct although limits may be left until 2nd line.
	$= \frac{1}{10}\left[\frac{2x^3}{3} + \frac{3x^4}{4}\right]_1^2$	**A1**	For evaluating the integral
	$= 1.59$	**A1**	
(ii)	$E(X^2) = \frac{1}{10}\int_1^2 x^2(2x+3x^2)\,dx$	**B1**	Integral and limits
	$= \frac{1}{10}\left[\frac{2x^4}{4} + \frac{3x^5}{5}\right]_1^2$	**B1**	Correct evaluation of integral
	$= 2.61$		
	Var(X) = 2.61 – 1.59² = 0.08	**M1A1**	FT their $E(X)$
(b)(i)	$F(x) = \int_1^x \frac{1}{10}(2t+3t^2)\,dt$	**M1**	Limits may be left until 2nd line
	$= \frac{1}{10}\left[t^2 + t^3\right]_1^x$	**A1**	
	$= \frac{1}{10}(x^2 + x^3 - 2)$ cao	**A1**	
(ii)	$P(X \le 1.4) = F(1.4)$	**M1**	FT their $F(x)$ if possible
	$= 0.27$	**A1**	
(iii)	The lower quartile is less than 1.4	**B1**	FT their answer to (a)(ii)
	since $F(1.4)$ is more than 0.25.	**B1**	

S2

Ques	Solution	Mark	Notes
1(a)	$E(X^2) = \text{Var}(X) + [E(X)]^2$	M1	Award M1 for **using** formula
	$= 27$	A1	
	Similarly, $E(Y^2) = 39$	A1	
(b)	$E(U) = E(X)E(Y)$	M1	
	$= 30$	A1	
	$E(X^2Y^2) = E(X^2)E(Y^2) = 27 \times 39$	B1	FT their $E(X^2), E(Y^2)$ but not their $E(X), E(Y)$
	$\text{Var}(U) = E(X^2Y^2) - [E(XY)]^2$	M1	Award M1 for **using** formula
	$= 27 \times 39 - 30^2 = 153$	A1	
2(a)(i)	$z = \frac{4.5 - 4.4}{0.2} = 0.5$	M1A1	
	$P(X > 4.5) = 0.3085$	A1	
(ii)	95th percentile = $\mu + 1.645\sigma$	M1	Award only for $\mu + z\sigma$
	$= 4.73$	A1	
(b)(i)	$E(2Y - X) = 0.8$	B1	
	$\text{Var}(2Y - X) = 4\text{Var}(Y) + \text{Var}(X)$	M1	
	$= 0.13$	A1	
(ii)	$z = \frac{0 - 0.8}{\sqrt{0.13}} = -2.22$ (Accept ±)	M1A1	FT their values from (b)(i)
	We require $P(2Y - X < 0)$	M1	
	Prob = 0.0132	A1	
(iii)	Let total weight = S		
	$E(S) = 2 \times 4.4 + 3 \times 2.6 = 16.6$	B1	
	$\text{Var}(S) = 2 \times 0.04 + 3 \times 0.0225 = 0.1475$	M1A1	
	$z = \frac{16 - 16.6}{\sqrt{0.1475}} = -1.56$	m1A1	
	Prob = 0.9406	A1	
3(a)	$\bar{x} = \frac{69.9}{75}$ (= 0.932)	B1	
	SE of $\bar{X} = \frac{0.1}{\sqrt{75}}$ (= 0.011547…)	B1	
	90% conf limits are $0.932 \pm 1.645 \times 0.011547\ldots$	M1A1	M1 correct form, A1 correct z. SE must have $\sqrt{75}$ in denom for M1.
	giving [0.913, 0.951]	A1	
(b)	If the method for finding the confidence interval is repeated a large number of times, then 90% of the intervals obtained will contain μ (or equivalent)	B1	Award B0 for any solution which suggests that the calculated interval contains μ with a probability of 0.9

4(a)	The total number of errors, X, is Poi(8) $P(X < 5) = 1 - 0.9004 = 0.0996$	**B1** **M1A1**	Award M1A0 for use of adjacent row/column
(b)(i) **(ii)**	$H_0 : \mu = 0.8; H_1 : \mu < 0.8$ Under H_0, number of errors is Poi(64) ≈ N(64,64). $z = \dfrac{60.5 - 64}{8}$ $= -0.4375$ p-value = 0.33 Insufficient evidence to reject H_0 /Accept H_0	**B1** **B1** **M1A1** **A1** **A1** **A1**	Award M1A0A1A1 for incorrect or no continuity correction No c/c gives $z = -\ 0.5$, $p = 0.31$ Incorrect c/c gives $z = -0.5625$, $p = 0.29$ FT their p-value
5(a) **(b)**	$H_0 : \mu_D = \mu_F ; H_1 : \mu_D \neq \mu_F$ $\bar{x}_D = \dfrac{890.4}{6} (= 148.4); \bar{x}_F = \dfrac{879}{6} (= 146.5)$ si SE of difference of means= $\sqrt{\dfrac{1.5^2}{6} + \dfrac{1.5^2}{6}}$ (0.866..) Test statistic = $\dfrac{148.4 - 146.5}{0.866..}$ = 2.19 Prob from tables = 0.01426 p-value = 0.02852 **Strong** evidence that there is a difference in mean distances for the two players. OR **Strong** evidence that David's mean is larger than Frank's mean.	**B1** **B1B1** **M1A1** **M1A1** **A1** **A1** **A1** **A1**	 FT arithmetic slip in evaluating means FT from previous line FT on their p-value

6(a)			
(b)(i)	Drop a perpendicular from A to BC. X = 2BD = 2ABcosθ = 4cosθ The probability density function of θ is $f(\theta) = \frac{2}{\pi}$ (for $0 < \theta < \pi/2$) si E(X) = $\int_0^{\pi/2} \frac{2}{\pi} \times 4\cos\theta \mathrm{d}\theta$ = $\frac{8}{\pi}[\sin\theta]_0^{\pi/2}$ = 8/π cao	**M1** **A1** **B1** **M1** **A1** **A1**	Accept any valid method Must be convincing Limits not required, award M1 for $\int K \times 4\cos\theta \mathrm{d}\theta, K \neq 1$ Limits required here
(ii)	P($X \le 3$) = P(cos$\theta \le 0.75$) = P($\theta \ge 0.723$) = $\frac{\pi/2 - 0.723}{\pi/2}$ = 0.54	**M1** **A1** **M1** **A1**	An answer of 0.46 is given M1A0M1A0
7(a)	Let X denote the number of white flowers produced. If bag is Type B, X is B(120,0.7) ≈ N(84,25.2) P(label A) = P(X < 70) $z = \frac{69.5 - 84}{\sqrt{25.2}}$ = – 2.89 (Accept ±) Prob = 0.00193	**M1A1** **M1A1** **A1** **A1**	Award M1A0A1A1 for incorrect or no c/c. 70.5 → z = – 2.69, p = 0.00357 70 → z = – 2.79, p = 0.00264 69→ z = – 2.99, p = 0.00139 68.5 → z = – 3.09, p = 0.001
(b)	If bag is of Type A, X is B(120,0.5) ≈ N(60,30) P(label B) = P(X ≥ 70) $z = \frac{69.5 - 60}{\sqrt{30}}$ = 1.73 (Accept ±) Prob = 0.0418	**M1A1** **M1A1** **A1** **A1**	Award M1A0A1A1 for incorrect or no c/c. 70.5 → z = 1.92, p = 0.02743 70 → z = 1.83, p = 0.03362 69 → z = 1.64, p = 0.0505 68.5 → z = 1.55, p = 0.06057

Ques	Solution	Mark	Notes
1(a)	The possibilities are		
		Numbers drawn	Mean	Median
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1 2 3	2	2
1 2 4	7/3	2
1 2 5	8/3	2
1 3 4	8/3	3
1 3 5	3	3
1 4 5	10/3	4
2 3 4	3	3
2 3 5	10/3	3
2 4 5	11/3	4
3 4 5	4	4		**B1** **B1** **B1**	B1 each column Special case – B2 if one combination is missing.
	The sampling distribution of the mean is		
	$\bar{x}$: 2, 7/3, 8/3, 3, 10/3, 11/3, 4 Prob: 1/10, 1/10, 2/10, 2/10, 2/10, 1/10, 1/10	**M1** **A1**	No FT from earlier work.
(b)	The sampling distribution of the median is		
	Median: 2, 3, 4 Prob: 3/10, 4/10, 3/10	**M1** **A1**	
2(a)	UE of μ = 99.03	**B1**	No working need be seen
	$\Sigma x^2 = 98088.11$	**B1**	
	UE of $\sigma^2 = \dfrac{98088.11}{9} - \dfrac{990.3^2}{9\times 10}$	**M1**	Answer only no marks
	= 2.08 (2.0778…)	**A1**	
(b)(i)	$H_0: \mu = 100; H_1: \mu \ne 100$	**B1**	
(ii)	$t = \dfrac{99.03-100}{\sqrt{2.0778../10}}$	**M1**	M0 if treated as z
	= – 2.13	**A1**	
	DF = 9 si	**B1**	
	Critical value = 2.262	**B1**	
	Insufficient evidence to reject the manager's claim or Accept the manager's claim	**B1**	FT their critical value but not their p-value obtained from using the normal distribution
	Because 2.13 < 2.262 or equivalent using the term 'acceptance region' or by means of a diagram	**B1**	

3(a)(i)	$\hat{p} = 0.45$	**B1**	
(ii)	ESE = $\sqrt{\dfrac{0.45 \times 0.55}{120}}$ = 0.0454.. si	**M1A1**	
	95% confidence limits are $0.45 \pm 1.96 \times 0.0454..$	**M1A1**	
	giving [0.361,0.539]	**A1**	
(b)(i)	This time, $\hat{p} = \dfrac{0.455 + 0.581}{2}$ = 0.518	**M1A1**	
(ii)	Width of CI = $2 \times 1.645\sqrt{\dfrac{0.518 \times 0.482}{n}}$	**M1**	
	= 0.581 – 0.455 = 0.126	**A1**	
	Solving, $n = \left(\dfrac{3.29}{0.126}\right)^2 \times 0.518 \times 0.482$	**M1**	Attempting to solve for *n*
	= 170	**A1**	
(iii)	*x* = 170 × 0.518 = 88	**B1**	FT their *n*
4(a)	$H_0 : \mu_A = \mu_B : H_1 : \mu_A \neq \mu_B$	**B1**	
(b)	$\bar{x} = 51.3; \bar{y} = 51.8$	**B1**	
	$s_x^2 = \dfrac{131659}{49} - \dfrac{2565^2}{49 \times 50} = 1.5204...$	**M1A1**	
	$s_y^2 = \dfrac{134232}{49} - \dfrac{2590^2}{49 \times 50} = 1.4285...$	**A1**	
	[Accept division by 50 giving 1.49… and 1.4]		
	SE = $\sqrt{\dfrac{1.5204..}{50} + \dfrac{1.4285}{50}}$	**M1**	
	= 0.2428.. (0.2404..)	**A1**	
	Test stat = $\dfrac{51.3 - 51.8}{0.2428..}$	**M1**	
	= 2.06 (2.08)	**A1**	
	p-value = 0.039 (0.038)	**A1**	
	Strong evidence for believing there is a difference in mean distances travelled (or that the Model A mean is less than the Model B mean).	**A1**	FT their p-value

5(a)	$\sum x = 15, \sum x^2 = 55, \sum y = 345.5, \sum xy = 1131.1$	**B2**	Minus 1 each error.
	$S_{xy} = 1131.1 - 15 \times 345.5/6 = 267.35$	**B1**	FT I error in sums.
	$S_{xx} = 55 - 15^2/6 = 17.5$	**B1**	
	$b = \dfrac{267.35}{17.5} = 15.3$	**M1** **A1**	
	$a = \dfrac{345.5 - 15 \times 15.277..}{6}$ = 19.4 (accept 19.3)	**M1** **A1**	
(b)	SE of $b = \dfrac{0.75}{\sqrt{17.5}}$ $(0.179\ldots)$	**M1A1**	FT their values from (a)
	99% confidence limits for β are $15.277 \pm 2.576 \times 0.179..$ giving $[14.8, 15.7]$	**M1A1** **A1**	
6(a)	$\text{E}(X) = \int_0^a x \times \dfrac{2x}{a^2}\,\text{d}x$	**M1**	Limits not required in this line
	$= \left[\dfrac{2x^3}{3a^2}\right]_0^a$	**A1**	
	$= \dfrac{2a}{3}$	**A1**	
	$\text{E}(X^2) = \int_{-1}^{1} x^2 \times \dfrac{2x}{a^2}\,\text{d}x$	**M1**	Limits not required in this line
	$= \left[\dfrac{2x^4}{4a^2}\right]_0^a$	**A1**	
	$= \dfrac{a^2}{2}$	**A1**	
	$\text{Var}(X) = \dfrac{a^2}{2} - \dfrac{4a^2}{9}$	**A1**	
	$= \dfrac{a^2}{18}$		

(b)(i)	$\mathrm{E}(U) = c\mathrm{E}(\bar{X})$ (or $c\mathrm{E}(X)$) $= c \times \frac{2a}{3}$	**M1**	Penalise the omission of E once in the question
	$E(U) = a \Rightarrow c = \frac{3}{2}$	**A1**	
	$\mathrm{Var}(U) = \frac{9}{4}\mathrm{Var}(\bar{X})$	**M1**	
	$= \frac{9}{4} \times \frac{a^2}{18n}$	**A1**	
	$= \frac{a^2}{8n}$	**A1**	
(ii)	$\mathrm{E}(V) = d\mathrm{E}(Y) = d \times \frac{2na}{2n+1}$	**M1**	
	$E(V) = a \Rightarrow d = \frac{2n+1}{2n}$	**A1**	
	$\mathrm{Var}(V) = \left(\frac{2n+1}{2n}\right)^2 \mathrm{Var}(Y)$	**M1**	
	$= \left(\frac{2n+1}{2n}\right)^2 \times \left(\frac{na^2}{(n+1)(2n+1)^2}\right)$	**A1**	
	$= \frac{a^2}{4n(n+1)}$	**A1**	
(iii)	$\frac{\mathrm{Var}(U)}{\mathrm{Var}(V)} = \frac{a^2}{8n} \div \frac{a^2}{4n(n+1)}$	**B1**	
	$= \frac{n+1}{2}$		
	V is the better estimator	**B1**	
	Because (for $n > 1$) it has the smaller variance	**B1**	

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