GCE MARKING SCHEME

SUMMER 2016

MATHEMATICS – C1
0973/01

INTRODUCTION

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GCE MATHEMATICS – C1

SUMMER 2016 MARK SCHEME

1.	(*a*)	(i)	Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$		M1
			Gradient of $AB = {}^1/_2$	(or equivalent)	A1
		(ii)	A correct method for finding the equation of AB using the candidate's value for the gradient of AB.		M1
			Equation of AB : $y - 2 = {}^1/_2\,(x - 4)$ (or equivalent) (f.t. the candidate's value for the gradient of AB)		A1
			Equation of AB : $2y = x$ (or equivalent) (f.t. one error if both M1's are awarded)		A1
	(*b*)	A correct method for finding the length of $AB(AC)$			M1
		$AB = \sqrt{125}$			A1
		$AC = \sqrt{80}$			A1
		$k = {}^5/_4$		(c.a.o.)	A1
	(*c*)	(i)	Equation of BD : $x = 4$		B1
		(ii)	**Either:**		
			An attempt to find the gradient of a line perpendicular to AB using the fact that the product of the gradients of perpendicular lines = –1.		M1
			An attempt to find the gradient of the line passing through C and D using the coordinates of C and D.		M1
			$-2 = \dfrac{m - 5}{4 - (-2)}$ (o.e.) (Equating candidate's derived expressions for gradient, f.t. candidate's gradient of AB)		M1
			$m = -7$	(c.a.o.)	A1
			Or:		
			An attempt to find the gradient of a line perpendicular to AB using the fact that the product of the gradients of perpendicular lines = –1.		M1
			An attempt to find the equation of line perpendicular to AB passing through C(or D) (f.t. candidate's gradient of AB)		M1
			$m - 5 = -2[4 - (-2)]$ (substituting coordinates of unused point in the candidate's derived equation)		M1
			$m = -7$	(c.a.o.)	A1

2. $\dfrac{5\sqrt{7} + 4\sqrt{2}}{3\sqrt{7} + 5\sqrt{2}} = \dfrac{(5\sqrt{7} + 4\sqrt{2})(3\sqrt{7} - 5\sqrt{2})}{(3\sqrt{7} + 5\sqrt{2})(3\sqrt{7} - 5\sqrt{2})}$ M1

Numerator: $15 \times 7 - 25 \times \sqrt{7} \times \sqrt{2} + 12 \times \sqrt{2} \times \sqrt{7} - 20 \times 2$ A1

Denominator: $63 - 50$ A1

$\dfrac{5\sqrt{7} + 4\sqrt{2}}{3\sqrt{7} + 5\sqrt{2}} = 5 - \sqrt{14}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $3\sqrt{7} + 5\sqrt{2}$

3. y-coordinate at $P = 11$ B1

An attempt to differentiate, at least one non-zero term correct M1

$\dfrac{dy}{dx} = 12 \times (-2) \times x^{-3} + 7$ A1

An attempt to substitute $x = 2$ in candidate's derived expression for $\dfrac{dy}{dx}$ m1

Use of candidate's derived numerical value for $\dfrac{dy}{dx}$ as gradient in the equation of the tangent at P m1

Equation of tangent to C at P: $y - 11 = 4(x - 2)$ (or equivalent)

(f.t. only candidate's derived value for y-coordinate at P) A1

4. $(\sqrt{3} - 1)^5 = (\sqrt{3})^5 + 5(\sqrt{3})^4(-1) + 10(\sqrt{3})^3(-1)^2 + 10(\sqrt{3})^2(-1)^3$
$+ 5(\sqrt{3})(-1)^4 + (-1)^5$ (five or six terms correct) B2

(**If B2 not awarded**, award B1 for three or four correct terms)

$(\sqrt{3} - 1)^5 = 9\sqrt{3} - 45 + 30\sqrt{3} - 30 + 5\sqrt{3} - 1$ (six terms correct) B2

(**If B2 not awarded**, award B1 for three, four or five correct terms)

$(\sqrt{3} - 1)^5 = -76 + 44\sqrt{3}$ (f.t. one error) B1

5. (*a*) $a = 2, \quad b = -12$ B1 B1

(*b*) $x^2 + 4x - 8 = 2x + 7$ M1

An attempt to collect terms, form and solve the quadratic equation in x either by correct use of the quadratic formula or by writing the equation in the form $(x + n)(x + m) = 0$, where $n \times m$ = candidate's constant m1

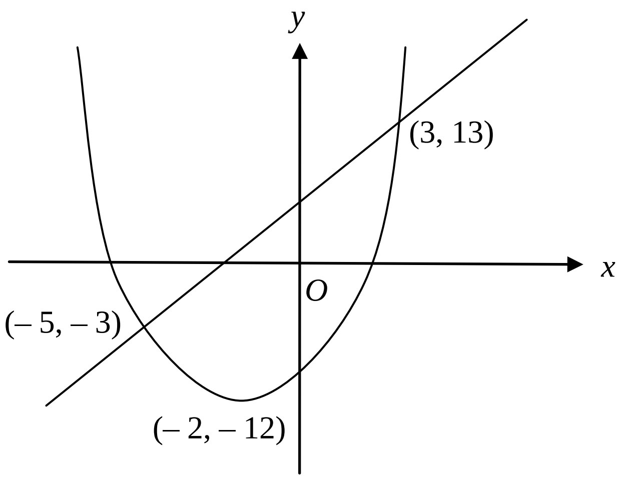
$x^2 + 2x - 15 = 0 \Rightarrow (x - 3)(x + 5) = 0 \Rightarrow x = 3, x = -5$

(both values, c.a.o.) A1

When $x = 3, y = 13$, when $x = -5, y = -3$

(both values, f.t. one slip) A1

(*c*)



A positive quadratic graph M1

Minimum point (– 2, – 12) marked

(f.t. candidate's values for *a*, *b*) A1

A straight line with positive gradient and positive *y*-intercept B1

Both points of intersection (– 5, – 3), (3, 13) marked

(f.t candidate's solutions to part(*b*)) B1

6. (*a*) An expression for $b^2 - 4ac$, with at least two of *a*, *b* or *c* correct M1

$b^2 - 4ac = 8^2 - 4 \times 9 \times (-2k)$ A1

$b^2 - 4ac > 0$ m1

$k > -{}^{8}/_{9}$ (o.e.)

[f.t. only for $k < {}^{8}/_{9}$ from $b^2 - 4ac = 8^2 - 4 \times 9 \times (2k)$] A1

(*b*) Attempting to rewrite the inequality in the form $5x^2 - 7x - 6 \geq 0$ and an attempt to find the critical values M1

Critical values $x = -0{\cdot}6, x = 2$ A1

A statement (mathematical or otherwise) to the effect that

$x \leq -0{\cdot}6$ or $2 \leq x$ (or equivalent)

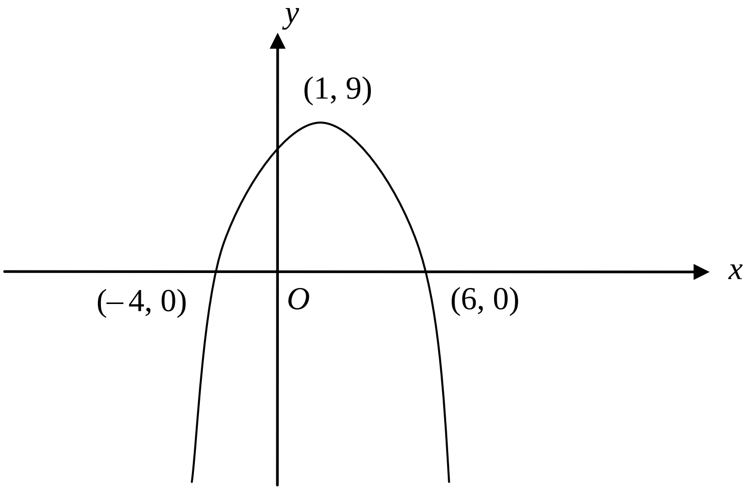
(f.t. candidate's derived critical values) A2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*)



Concave down curve with x-coordinate of maximum = 1	B1
y-coordinate of maximum = 9	B1
Both points of intersection with x-axis	B1

(*b*)

$g(x) = f(-x)$	B1
$g(x) = f(x + 2)$	B1

8. (*a*)

$y + \delta y = 10(x + \delta x)^2 - 7(x + \delta x) - 13$		B1
Subtracting y from above to find δy		M1
$\delta y = 20x\delta x + 10(\delta x)^2 - 7\delta x$		A1
Dividing by δx and letting $\delta x \to 0$		M1
$\dfrac{dy}{dx} = \lim_{\delta x \to 0} \dfrac{\delta y}{\delta x} = 20x - 7$	(c.a.o.)	A1

(*b*)

$\dfrac{dy}{dx} = 4 \times \dfrac{1}{2} \times x^{-1/2} + (-1) \times 45 \times x^{-2}$		B1, B1
Either $9^{-1/2} = \dfrac{1}{3}$ **or** $9^{-2} = \dfrac{1}{81}$ (or equivalent fraction)		B1
$\dfrac{dy}{dx} = \dfrac{1}{9}$ (or equivalent)	(c.a.o.)	B1

9. (a) **Either:** showing that $f(2) = 0$
Or: trying to find $f(r)$ for at least two values of r M1
$f(2) = 0 \Rightarrow x - 2$ is a factor A1
$f(x) = (x - 2)(8x^2 + ax + b)$ with one of a, b correct M1
$f(x) = (x - 2)(8x^2 + 18x - 5)$ A1
$f(x) = (x - 2)(4x - 1)(2x + 5)$
(f.t. only $8x^2 - 18x - 5$ in above line) A1

Special case

Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 3 marks

(b) **Either:** $f(2{\cdot}25) = 0{\cdot}25 \times 8 \times 9{\cdot}5$
(at least two terms correct, f.t. candidate's derived expression for f) M1
$f(2{\cdot}25) = 19$ [f.t. only for $f(2{\cdot}25) = -1{\cdot}25$ from $f(x) = (x - 2)(4x + 1)(2x - 5)$] A1
Or: $f(2{\cdot}25) = 91{\cdot}125 + 10{\cdot}125 - 92{\cdot}25 + 10$
(at least two of the first three terms correct) M1
$f(2{\cdot}25) = 19$ (c.a.o.) A1

10. (a) $V = x(24 - 2x)(9 - 2x)$ M1
$V = 4x^3 - 66x^2 + 216x$ (convincing) A1

(b) $\frac{dV}{dx} = 12x^2 - 132x + 216$ B1
Putting derived $\frac{dV}{dx} = 0$ M1
$x = 2, (9)$ (f.t. candidate's $\frac{dV}{dx}$) A1
Stationary value of V at $x = 2$ is 200 (c.a.o) A1
A correct method for finding nature of the stationary point yielding a maximum value (for $0 < x < 4{\cdot}5$) B1

GCE MARKING SCHEME

SUMMER 2016

Mathematics – C2
0974/01

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GCE MATHEMATICS – C2

SUMMER 2016 MARK SCHEME

1.

3	0·6032888847		
3·75	0·5666103111		
4·5	0·5348655099		
5·25	0·5067878888		
6	0·4815614791	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}75$ M1

$$I \approx \frac{0{\cdot}75}{2} \times \{0{\cdot}6032888847 + 0{\cdot}4815614791 + 2(0{\cdot}5666103111 + 0{\cdot}5348655099 + 0{\cdot}5067878888)\}$$

$I \approx 4{\cdot}301377783 \times 0{\cdot}75 \div 2$

$I \approx 1{\cdot}613016669$

$I \approx 1{\cdot}613$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}6$

3	0·6032888847		
3·6	0·5734992875		
4·2	0·5470655771		
4·8	0·5232474385		
5·4	0·5015353186		
6	0·4815614791	(all values correct)	B1

Correct formula with $h = 0{\cdot}6$ M1

$$I \approx \frac{0{\cdot}6}{2} \times \{0{\cdot}6032888847 + 0{\cdot}4815614791 + 2(0{\cdot}5734992875 + 0{\cdot}5470655771 + 0{\cdot}5232474385 + 0{\cdot}5015353186)\}$$

$I \approx 5{\cdot}375545607 \times 0{\cdot}6 \div 2$

$I \approx 1{\cdot}612663682$

$I \approx 1{\cdot}613$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $6\sin^2\theta + 1 = 2(1 - \sin^2\theta) - 2\sin\theta$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$8\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow (4\sin\theta - 1)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{4}$, $\sin\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 14{\cdot}48°, 165{\cdot}52°$ B1

$\theta = 210°, 330°$ B1, B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) $3x - 57° = -39°, 141°, 321°, 501°$ (one correct value) B1

$x = 6°, 66°, 126°$ B1 B1 B1

Note: Subtract (from final three marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) $\sin\phi \geq -1, \cos\phi \geq -1$ and thus $2\sin\phi + 4\cos\phi > -7$ E1

3. (*a*) $(x + 5)^2 = 7^2 + x^2 - 2 \times 7 \times x \times -\frac{3}{5}$ (correct use of cos rule) M1

$x^2 + 10x + 25 = 49 + x^2 + 8{\cdot}4x$ A1

$1{\cdot}6x = 24 \Rightarrow x = 15$ (convincing) A1

(*b*) $\sin B\hat{A}C = \frac{4}{5}$ B1

Area of triangle $ABC = \frac{1}{2} \times 7 \times 15 \times \frac{4}{5}$

(substituting the correct values in the correct places in the area formula, f.t. candidate's derived value for $\sin B\hat{A}C$) M1

Area of triangle ABC = 42 (cm^2) A1

(*c*) $\frac{1}{2} \times 20 \times AD = 42$

(f.t. candidate's derived value for area of triangle ABC) M1

$AD = 4{\cdot}2$ (cm)

(f.t. candidate's derived value for area of triangle ABC) A1

4. (*a*) This is an A.P. with $a = 6$, $d = 2$ (s.i.) M1

(i) 20th term $= 6 + 2 \times 19$

(f.t. candidate's values for a and d) M1

20th term $= 44$ (c.a.o.) A1

(ii) $\frac{n}{2}[2 \times 6 + (n - 1) \times 2] = 750$

(f.t. candidate's values for a and d) M1

Rewriting above equation in a form ready to be solved

$2n^2 + 10n - 1500 = 0$ or $n^2 + 5n - 750 = 0$ or $n(n + 5) = 750$ or

$n^2 + 5n = 750$ (f.t. candidate's values for a and d) A1

$n = 25$ (c.a.o.) A1

(*b*) (i) $t_{11} + t_{14} = 50$ B1

(ii) $S_{24} = \frac{24}{2} \times 50$ M1

$S_{24} = 600$ A1

5. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) **Either:** $\frac{a(1 - r^5)}{1 - r} = 275$

Or: $a + ar + ar^2 + ar^3 + ar^4 = 275$ B1

$\frac{a}{1 - r} = 243$ B1

An attempt to solve these equations simultaneously by eliminating a

M1

$243r^5 = -32$ (or $-243r^5 = 32$) A1

$r = -\frac{2}{3}$ (c.a.o.) A1

$a = 405$ (f.t. candidate's derived value for r) A1

6. (*a*) $3 \times \frac{x^{3/4}}{3/4} - 9 \times \frac{x^{7/2}}{7/2} + c$ B1, B1

(–1 if no constant term present)

(*b*) Area = $\int_1^4 \left(2x^2 + \frac{6}{x^2}\right) dx$ (use of integration) M1

$\frac{2x^3}{3} + 6 \times (-1) \times x^{-1}$ (correct integration) A1, A1

Area = (128/3 – 6/4) – (2/3 – 6/1) (an attempt to substitute limits) m1

Area = 93/2 or 46·5 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indices) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) **Either:**

$(3x + 1) \log_{10} 4 = \log_{10} 22$

(taking logs on both sides and using the power law) M1

$x = \frac{\log_{10} 22 - \log_{10} 4}{3 \log_{10} 4}$ (o.e.) A1

$x = 0{\cdot}41$ (f.t. one slip, see below) A1

Or:

$3x + 1 = \log_4 22$ (rewriting as a log equation) M1

$x = \frac{\log_4 22 - 1}{3}$ A1

$x = 0{\cdot}41$ (f.t. one slip, see below) A1

Note: an answer of $x = -0{\cdot}41$ from $x = \frac{\log_{10} 4 - \log_{10} 22}{3 \log_{10} 4}$

earns M1 A0 A1

an answer of $x = 1{\cdot}08$ from $x = \frac{\log_{10} 22 + \log_{10} 4}{3 \log_{10} 4}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

(*c*) Correct use of power law B1

At least one correct use of addition or subtraction law B1

$\log_d (36/9z) = 1$ (o.e.) (f.t. one incorrect term) B1

$z = \frac{4}{d}$ (c.a.o.) B1

8.	(*a*)	(i)	$A(-3, 10)$	B1
			A correct method for finding the radius	M1
			Radius = $\sqrt{50}$	A1
		(ii)	Use of shortest distance = OA – radius	M1
			Shortest distance = $\sqrt{109} - \sqrt{50} = 3\cdot37$	
			(f.t. candidate's derived radius)	A1
	(*b*)	(i)	An attempt to substitute $(3x - 1)$ for y in the equation of C_1	M1
			$x^2 - 6x + 8 = 0$ (or $10x^2 - 60x + 80 = 0$)	A1
			$x = 2, x = 4$	
			(correctly solving candidate's quadratic, both values)	A1
			Points of intersection P and Q are (2, 5), (4, 11) (c.a.o.)	A1
		(ii)	$BP^2(BQ^2) = 20$ or $BP(BQ) = \sqrt{20}$	
			(f.t. candidate's derived coordinates for P or Q)	B1
			Use of $(x - 6)^2 + (y - 7)^2 = BP^2(BQ^2)$	
			(f.t. candidate's derived coordinates for P or Q)	M1
			$(x - 6)^2 + (y - 7)^2 = 20$ (c.a.o.)	A1

9.	Area of sector $AOB = {}^1/_2 \times r^2 \times 2\cdot15$	B1
	Area of sector $BOC = {}^1/_2 \times r^2 \times (\pi - 2\cdot15)$	B1
	${}^1/_2 \times r^2 \times 2\cdot15 - {}^1/_2 \times r^2 \times (\pi - 2\cdot15) = 26$	M1
	$r^2 = \dfrac{52}{4\cdot3 - \pi}$ (o.e.)	A1
	$r = 6\cdot7$	A1

GCE MARKING SCHEME

SUMMER 2016

Mathematics – C3
0975/01

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GCE MATHEMATICS – C3

SUMMER 2016 MARK SCHEME

1. (*a*)

0	1		
$\pi/20$	$1\cdot025402923$		
$\pi/10$	$1\cdot111347018$		
$3\pi/20$	$1\cdot296432399$		
$\pi/5$	$1\cdot695307338$	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = \pi/20$ M1

$$I \approx \frac{\pi/20}{3} \times \{1 + 1\cdot695307338 + 4(1\cdot025402923 + 1\cdot296432399) + 2(1\cdot111347018)\}$$

$I \approx 14\cdot20534263 \times (\pi/20) \div 3$

$I \approx 0\cdot7437900006$

$I \approx 0\cdot74379$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$$\int_0^{\pi/5} e^{\sec^2 x}\,dx = e^1 \times \int_0^{\pi/5} e^{\tan^2 x}\,dx$$ M1

$$\int_0^{\pi/5} e^{\sec^2 x}\,dx \approx 2\cdot02183$$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $3\operatorname{cosec}\theta(\operatorname{cosec}\theta - 1) = 5(\operatorname{cosec}^2\theta - 1) - 9$

(correct use of $\cot^2\theta = \operatorname{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\operatorname{cosec}\theta$, either by using the quadratic formula or by getting the expression into the form $(a\operatorname{cosec}\theta + b)(c\operatorname{cosec}\theta + d)$, with $a \times c$ = candidate's coefficient of $\operatorname{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$2\operatorname{cosec}^2\theta + 3\operatorname{cosec}\theta - 14 = 0 \Rightarrow (\operatorname{cosec}\theta - 2)(2\operatorname{cosec}\theta + 7) = 0$

$\Rightarrow \operatorname{cosec}\theta = 2$, $\operatorname{cosec}\theta = -\frac{7}{2}$

$\Rightarrow \sin\theta = \frac{1}{2}$, $\sin\theta = -\frac{2}{7}$ (c.a.o.) A1

$\theta = 30°, 150°$ B1

$\theta = 196{\cdot}6°, 343{\cdot}4°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) Correct use of $\operatorname{cosec}\phi = \frac{1}{\sin\phi}$ and $\sec\phi = \frac{1}{\cos\phi}$ (o.e.) M1

$\tan\phi = -\frac{2}{3}$ A1

$\phi = 146{\cdot}31°, 326{\cdot}31°$ (f.t. for negative $\tan\phi$) A1

3. $\frac{d}{dx}(x^2) = 2x$ $\quad\frac{d}{dx}(2x) = 2$ $\quad\frac{d}{dx}(21) = 0$ B1

$\frac{d}{dx}(3xy) = 3x\frac{dy}{dx} + 3y$ B1

$\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{6}{9} = \frac{2}{3}$ (c.a.o.) B1

4. (*a*) candidate's x-derivative $= 12\cos 3t$ B1

candidate's y-derivative $= -6\sin 3t$ B1

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = -\dfrac{1}{2}\tan 3t$ (c.a.o.) A1

(*b*) (i) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = -\dfrac{3}{2}\sec^2 3t$ (f.t. $\dfrac{dy}{dx} = k\tan 3t$ or $k\dfrac{\sin 3t}{\cos 3t}$ only) B1

Use of $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\dfrac{d^2y}{dx^2} = -\dfrac{1}{8}\sec^3 3t$ or $\dfrac{-1}{8\cos^3 3t}$ (c.a.o.) A1

(ii) $\dfrac{d^2y}{dx^2} = -\dfrac{1}{y^3}$ (f.t. $\dfrac{d^2y}{dx^2} = m\sec^3 3t$ or $\dfrac{m}{\cos^3 3t}$ only) B1

5. (*a*) Denoting the end points of the chord by A, B

Length of arc $AB = 3\theta$ B1

Length of chord $AB = 2 \times 3 \times \sin(\theta/2)$ (convincing) B1

$3\theta + 6\sin(\theta/2) = 13{\cdot}5 \Rightarrow \theta + 2\sin(\theta/2) = 4{\cdot}5$

(convincing) B1

(*b*) $\theta_0 = 2{\cdot}5$

$\theta_1 = 2{\cdot}602030761$ (θ_1 correct, at least 2 places after the point) B1

$\theta_2 = 2{\cdot}572341396$

$\theta_3 = 2{\cdot}580466315 = 2{\cdot}58$ (θ_3 correct to 2 decimal places) B1

Let $f(\theta) = \theta + 2\sin(\theta/2) - 4{\cdot}5$

An attempt to check values or signs of $f(\theta)$ at $\theta = 2{\cdot}575$, $\theta = 2{\cdot}585$

M1

$f(2{\cdot}575) = -4{\cdot}72 \times 10^{-3} < 0$, $f(2{\cdot}585) = 8{\cdot}05 \times 10^{-3} > 0$ A1

Change of sign $\Rightarrow \theta = 2{\cdot}58$ correct to two decimal places A1

6. (*a*) $\frac{dy}{dx} = \frac{f(x)}{\cos x}$ (including $f(x) = 1$) M1

$\frac{dy}{dx} = -\frac{\sin x}{\cos x}$ A1

$\frac{dy}{dx} = -\tan x$ (f.t. only for $\tan x$ from $\frac{dy}{dx} = \frac{\sin x}{\cos x}$) A1

(*b*) $\frac{dy}{dx} = \frac{\mathbf{1/3}}{\mathbf{1 + (x/3)^2}}$ **or** $\frac{1}{1 + (x/3)^2}$ **or** $\frac{1/3}{1 + (1/3)x^2}$ M1

$\frac{dy}{dx} = \frac{1/3}{1 + (x/3)^2}$ A1

$\frac{dy}{dx} = \frac{3}{9 + x^2}$ $\left\{\text{f.t. only for } \frac{dy}{dx} = \frac{9}{9 + x^2} \text{ from } \frac{1}{1 + (x/3)^2}\right\}$ A1

(*c*) $\frac{dy}{dx} = e^{6x} \times f(x) + (3x - 2)^4 \times g(x)$ M1

$\frac{dy}{dx} = e^{6x} \times f(x) + (3x - 2)^4 \times g(x)$
(either $f(x) = 4 \times 3 \times (3x - 2)^3$ or $g(x) = 6e^{6x}$) A1

$\frac{dy}{dx} = e^{6x} \times 12 \times (3x - 2)^3 + (3x - 2)^4 \times 6e^{6x}$ (all correct) A1

$\frac{dy}{dx} = e^{6x} \times 18x \times (3x - 2)^3$ (c.a.o.) A1

7. (*a*) (i) $\int 7e^{5 - \frac{3}{4}x}\,dx = k \times 7e^{5 - \frac{3}{4}x} + c$ $\quad (k = 1, -{}^3/_4, {}^4/_3, -\boldsymbol{{}^4/_3})$ M1

$\int 7e^{5 - \frac{3}{4}x}\,dx = -\frac{28}{3}e^{5 - \frac{3}{4}x} + c$ A1

(ii) $\int \sin(2x/3 + 5)\,dx = k \times \cos(2x/3 + 5) + c$

$(k = -1, -{}^2/_3, {}^3/_2, -\boldsymbol{{}^3/_2})$ M1

$\int \sin(2x/3 + 5)\,dx = -\frac{3}{2} \times \cos(2x/3 + 5) + c$ A1

(iii) $\int \frac{8}{(9 - 10x)^3}\,dx = \frac{8}{-2k} \times (9 - 10x)^{-2} + c$

$(k = 1, 10, \mathbf{-10}, -{}^1/_{10})$ M1

$\int \frac{8}{(9 - 10x)^3}\,dx = \frac{2}{5} \times (9 - 10x)^{-2} + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \frac{1}{4x + 3}\,dx = k \times \ln(4x + 3)$ $\quad (k = 1, 4, \boldsymbol{{}^1/_4})$ M1

$\int \frac{1}{4x + 3}\,dx = 1/4 \times \ln(4x + 3)$ A1

$k \times [\ln(6 \times 4 + 3) - \ln(4a + 3)] = 0{\cdot}1986$ $\quad (k = 1, 4, \boldsymbol{{}^1/_4})$ m1

$\frac{27}{4a + 3} = e^{0{\cdot}7944}$ (o.e.) (c.a.o.) A1

$a = 2{\cdot}3$ (f.t. $a = 4{\cdot}8$ for $k = 1$ and $a = 5{\cdot}7$ for $k = 4$) A1

8. (*a*) Choice of a, b, c, d such that a is a factor of c and b is a factor of d M1

Correctly verifying that the candidate's a, b, c, d are such that $(a + b)$ is **not** a factor of $(c + d)$ and a statement to the effect that this is the case A1

(*b*) Trying to solve $5x + 4 = -7x$ M1

Trying to solve $5x + 4 = 7x$ M1

$x = -1/3, x = 2$ (c.a.o.) A1

$x = -1/3$ (c.a.o.) A1

Alternative mark scheme

$(5x + 4)^2 = (-7x)^2$ (squaring both sides) M1

$24x^2 - 40x - 16 = 0$ (at least two coefficients correct) A1

$x = -1/3, x = 2$ (c.a.o.) A1

$x = -1/3$ (c.a.o.) A1

(*c*) (i) $a = 5, -3$ B1

(ii) $b = -\frac{2}{3}$ B1

9. (*a*) $y - 8 = e^{4 - x/3}$. B1

An attempt to express equation as a logarithmic equation and to isolate x M1

$x = 3[4 - \ln(y - 8)]$ (c.a.o.) A1

$f^{-1}(x) = 3[4 - \ln(x - 8)]$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = [9, \infty)$ B1 B1

10. (*a*)

$$hh(x) = \frac{4 \times \frac{4x+3}{5x-4} + 3}{5 \times \frac{4x+3}{5x-4} - 4}$$ M1

$$hh(x) = \frac{16x + 12 + 15x - 12}{20x + 15 - 20x + 16}$$ A1

$hh(x) = x$ (convincing) A1

(*b*) $h^{-1}(x) = h(x)$ B1

$h^{-1}(-1) = h(-1) = \frac{1}{9}$ (awarded only if first B1 awarded) B1

GCE MARKING SCHEME

SUMMER 2016

Mathematics – C4
0976/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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GCE MATHEMATICS – C4

SUMMER 2016 MARK SCHEME

1. (*a*) $f(x) \equiv \dfrac{A}{(2x-1)} + \dfrac{B}{(x-3)^2} + \dfrac{C}{(x-3)}$ (correct form) M1

$17 + 4x - x^2 \equiv A(x-3)^2 + B(2x-1) + C(x-3)(2x-1)$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 3$, $B = 4$, $C = -2$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) $f'(x) = -\dfrac{6}{(2x-1)^2} - \dfrac{8}{(x-3)^3} + \dfrac{2}{(x-3)^2}$ (o.e.)

(f.t. candidate's derived values for *A*, *B*, *C*)

(second term) B1

(**both** the first and third terms) B1

2. (*a*) (i) $(1+2x)^{-1/2} = 1 - x + \dfrac{3x^2}{2}$ $(1-x)$ B1

$({}^3/_2 x^2)$ B1

(ii) $|x| < {}^1/_2$ or $-{}^1/_2 < x < {}^1/_2$ B1

(*b*) $6 - 6x + 9x^2 = 4 + 15x - x^2 \Rightarrow 10x^2 - 21x + 2 = 0$

(f.t. only candidate's quadratic expansion in (*a*)) M1

$x = 0{\cdot}1$ (f.t. only candidate's quadratic expansion in (*a*)) A1

3. (*a*) $4x^3 + 2x^3\dfrac{dy}{dx} + 6x^2y - 12y^3\dfrac{dy}{dx} = 0$ $\left(2x^3\dfrac{dy}{dx} + 6x^2y\right)$ B1

$\left(4x^3 - 12y^3\dfrac{dy}{dx}\right)$ B1

$\dfrac{dy}{dx} = \dfrac{2x^3 + 3x^2y}{6y^3 - x^3}$

(intermediary line required in order to be convincing) B1

(*b*) $2x^3 + 3x^2y = -2(6y^3 - x^3)$ M1

$y(3x^2 + 12y^2) = 0$ A1

$3x^2 + 12y^2 = 0 \Rightarrow x = 0, y = 0$ but not on curve A1

$y = 0 \Rightarrow x = \pm 2 \Rightarrow (2, 0), (-2, 0)$ (both points) A1

4. (*a*) (i) $\dfrac{6\tan x}{1 - \tan^2 x} + 16\cot^2 x = 0$ (o.e.)

(correct use of formula for $\tan 2x$) M1

$\dfrac{6\tan x}{1 - \tan^2 x} + \dfrac{16}{\tan^2 x} = 0$ (correct use of $\cot^2 x = \dfrac{1}{\tan^2 x}$) M1

$3\tan^3 x - 8\tan^2 x + 8 = 0$

(intermediary line required in order to be convincing) A1

(ii) $3\tan^3 x - 8\tan^2 x + 8 = (\tan x - 2)(3\tan^2 x + a\tan x + b)$

with one of a, b correct M1

$3\tan^3 x - 8\tan^2 x + 8 = (\tan x - 2)(3\tan^2 x - 2\tan x - 4)$ A1

$x = 63{\cdot}4°, 56{\cdot}9°, 139{\cdot}0°$

(rounding off errors are only penalised once) A1 A1 A1

(*b*) $R = 25$ B1

Correctly expanding $\cos(\theta + \alpha)$ and using either $25\cos\alpha = 24$ **or** $25\sin\alpha = 7$ **or** $\tan\alpha = \dfrac{7}{24}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 16{\cdot}26°$ (c.a.o) A1

Use of both critical values -25 and 25

(f.t candidate's derived value for R) M1

$25\cos(\theta + \alpha) = k$ has no solutions if $k < -25$ or $k > 25$

(f.t candidate's derived value for R) A1

5. (*a*) candidate's x-derivative $= -3t^{-2}$ (o.e.)

candidate's y-derivative $= 4$ (at least one term correct)

and use of

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = \dfrac{4}{-3t^{-2}}$ or $-\dfrac{4t^2}{3}$ (c.a.o.) A1

Equation of tangent at P: $y - 4p = -\dfrac{4p^2}{3}\left(x - \dfrac{3}{p}\right)$

(f.t. candidate's expression for $\dfrac{dy}{dx}$) m1

Equation of tangent at P: $3y = -4p^2x + 24p$

(intermediary line required in order to be convincing) A1

(*b*) Substituting $x = 1$, $y = 9$ in equation of tangent M1

$4p^2 - 24p + 27 = 0$ A1

$p = \dfrac{9}{2}, \dfrac{3}{2}$ (both values, c.a.o.) A1

Points are $(^2/_3, 18)$, $(2, 6)$ (f.t. candidate's values for p) A1

6. (*a*) $u = 2x + 1 \Rightarrow du = 2dx$ (o.e.) B1

$dv = e^{-3x}\,dx \Rightarrow v = -\frac{1}{3}e^{-3x}$ (o.e.) B1

$\int (2x+1)\,e^{-3x}\,dx = -\frac{1}{3}e^{-3x} \times (2x+1) - \int -\frac{1}{3}e^{-3x} \times 2dx$ (o.e.) M1

$\int (2x+1)\,e^{-3x}\,dx = -\frac{1}{3}e^{-3x} \times (2x+1) - \frac{2}{9}e^{-3x} + c$ (c.a.o.) A1

(*b*) $\int \frac{\sqrt{(4 + 5\tan x)}}{\cos^2 x}\,dx = \int k \times u^{1/2}\,du$ ($k = {}^1/_5$ or 5) M1

$\int a \times u^{1/2}\,du = a \times \frac{u^{3/2}}{3/2}$ B1

Either: Correctly inserting limits of 4, 9 in candidate's $bu^{3/2}$

or: Correctly inserting limits of 0, $\pi/4$ in candidate's $b(4 + 5\tan x)^{3/2}$ M1

$\int_0^{\pi/4} \frac{\sqrt{(4 + 5\tan x)}}{\cos^2 x}\,dx = \frac{38}{15} = 2{\cdot}53$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

7. (*a*) $\frac{dV}{dt} = -kV^3$ B1

(*b*) $\int \frac{dV}{V^3} = -\int k\,dt$ (o.e.) M1

$-\frac{V^{-2}}{2} = -kt + c$ A1

$c = -\frac{A^{-2}}{2}$ (c.a.o.) A1

$2V^2 = \frac{2A^2}{(2A^2k)t + 1} \Rightarrow V^2 = \frac{A^2}{bt + 1}$ (convincing)

where $b = 2A^2k$ A1

(*c*) Substituting $t = 2$ and $V = \frac{A}{2}$ in an expression for V^2 M1

$b = \frac{3}{2}$ $\left(\text{or } k = \frac{3}{4A^2}\right)$ A1

Substituting $V = \frac{A}{4}$ in an expression for V^2 with candidate's value for b

or expression for k M1

$t = 10$ (c.a.o) A1

8. (*a*) (i) $\mathbf{AB} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1

$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*b*) (i) $1 + 2\lambda = -1 - 2\mu$

$3 + \lambda = 8 + \mu$

$-3 + 2\lambda = p + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving the first two equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = 2,\ \mu = -3$ (o.e.) (c.a.o.) A1

$p = 10$ from third equation (f.t. candidate's derived values for λ and μ provided the third equation is correct) A1

(ii) An attempt to evaluate $(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ M1

$(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}) = -1 \neq 0 \Rightarrow L$ and $(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ not perpendicular A1

9.

Volume $= \pi\displaystyle\int_{\pi/5}^{2\pi/5}(\cos x + \sin x)^2\,dx$ B1

$(\cos x + \sin x)^2 = \cos^2 x + \sin^2 x + 2\sin x\cos x$ B1

$\displaystyle\int(\cos^2 x + \sin^2 x)\,dx = x$ **or** $\left[\frac{x}{2} + \frac{1}{4}\sin 2x\right] + \left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]$ B1

$\displaystyle\int k\sin x\cos x\,dx = -\frac{k}{4}\cos 2x$ **or** $\frac{k}{2}\sin^2 x$ **or** $-\frac{k}{2}\cos^2 x$ B1

Substitution of limits in candidate's integrated expression
(awarded only if at least two of the previous three marks have been awarded)
M1

Volume = 3·73 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

10. Assume that there is a real value of x such that

$$\left| x + \frac{1}{x} \right| < 2$$

Then squaring both sides, we have:

$x^2 + \dfrac{1}{x^2} + 2 < 4$ B1

$x^2 + \dfrac{1}{x^2} - 2 < 0$ B1

$\left(x - \dfrac{1}{x} \right)^2 < 0$, which is impossible since the square of a real number cannot be negative B1

Alternative Mark Scheme

Assume that there is a real value of x such that

$$\left| x + \frac{1}{x} \right| < 2$$

Then squaring both sides, we have:

$x^2 + \dfrac{1}{x^2} + 2 < 4$ B1

$x^4 - 2x^2 + 1 < 0$ B1

$(x^2 - 1)^2 < 0$, which is impossible since the square of a real number cannot be negative B1

GCE MARKING SCHEME

SUMMER 2016

Mathematics – FP1
0977/01

INTRODUCTION

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Ques	Solution	Mark	Notes
1	$f(x+h)-f(x)=\dfrac{(x+h)^2}{(x+h+1)}-\dfrac{x^2}{x+1}$	M1A1	
	$=\dfrac{(x+h)^2(x+1)-x^2(x+h+1)}{(x+h+1)(x+1)}$	A1	
	$=\dfrac{x^3+x^2+2hx^2+2hx+h^2x+h^2-x^3-hx^2-x^2}{(x+h+1)(x+1)}$	A1	
	$=\dfrac{hx^2+2hx+h^2x+h^2}{(x+h+1)(x+1)}$	A1	
	$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$		
	$=\lim_{h\to 0}\dfrac{x^2+2x+hx+h}{(x+h+1)(x+1)}$	M1	
	$=\dfrac{x^2+2x}{(x+1)^2}$	A1	
2(a)	The rotation matrix = $\begin{bmatrix}0&-1&0\\1&0&0\\0&0&1\end{bmatrix}$	B1	
	The translation matrix = $\begin{bmatrix}1&0&1\\0&1&2\\0&0&1\end{bmatrix}$	B1	
	$\mathbf{T}=\begin{bmatrix}1&0&1\\0&1&2\\0&0&1\end{bmatrix}\begin{bmatrix}0&-1&0\\1&0&0\\0&0&1\end{bmatrix}$	M1	
	$=\begin{bmatrix}0&-1&1\\1&0&2\\0&0&1\end{bmatrix}$	A1	
(b)	The fixed point satisfies $\begin{bmatrix}0&-1&1\\1&0&2\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}=\begin{bmatrix}x\\y\\1\end{bmatrix}$	M1	FT their **T**
	$-y+1=x\,;\,x+2=y$	A1	
	(x,y) = $\left(-\dfrac{1}{2},\dfrac{3}{2}\right)$ cao	m1A1	

Ques	Solution	Mark	Notes
3	$S_n = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$	**M1**	
	$= \dfrac{n^2(n+1)^2}{4} + \dfrac{n(n+1)(2n+1)}{6}$	**A1A1**	
	$= \dfrac{n(n+1)(3n(n+1) \ \ +2(2n+1))}{12}$	**m1**	m1 for attempting to combine and take out two factors
	$= \dfrac{n(n+1)}{12}\left(3n^2+7n+2\right)$	**A1**	
	$= \dfrac{n(n+1)(n+2)(3n+1)}{12}$	**A1**	
4(a)	$\lvert z_1 \rvert = 2$; $\arg(z_1) = \dfrac{5\pi}{6}$	**B1B1**	
	$\lvert z_2 \rvert = \sqrt{2}$; $\arg(z_2) = \dfrac{\pi}{4}$	**B1B1**	
(b)	EITHER		
	$\lvert w \rvert = \dfrac{\lvert z_1 \rvert^2}{\lvert z_2 \rvert} = \dfrac{4}{\sqrt{2}}$	**M1A1**	FT from (a)
	$\arg(w) = 2\arg(z_1) - \arg(z_2) = \dfrac{17\pi}{12}$	**M1A1**	
	$w = \dfrac{4}{\sqrt{2}}\cos\left(\dfrac{17\pi}{12}\right) + \dfrac{4}{\sqrt{2}}\sin\left(\dfrac{17\pi}{12}\right)\mathrm{i}$	**M1**	
	$= -0.73 - 2.73\mathrm{i}$	**A1**	
	OR		
	${z_1}^2 = 2 - 2\sqrt{3}\mathrm{i}$	**(M1A1)**	
	$\dfrac{{z_1}^2}{z_2} = \dfrac{2-2\sqrt{3}\mathrm{i}}{(1+\mathrm{i})} \times \dfrac{1-\mathrm{i}}{1-\mathrm{i}}$	**(M1)**	
	$= \dfrac{2-2\sqrt{3}-(2\sqrt{3}+2)\mathrm{i}}{2}$	**(A1A1)**	A1 numerator, A1 denominator
	$= -0.73 - 2.73\mathrm{i}$	**(A1)**	
	OR		
	${z_1}^2 = 2 - 2\sqrt{3}\mathrm{i}$	**M1A1**	
	$a + \mathrm{i}b = \dfrac{2-2\sqrt{3}i}{1+\mathrm{i}}$		
	$(a+\mathrm{i}b)(1+\mathrm{i}) = 2 - 2\sqrt{3}\mathrm{i}$	**M1**	
	$a - b + i(a+b) = 2 - 2\sqrt{3}\mathrm{i}$	**A1**	
	$a - b = 2; a + b = -2\sqrt{3}$	**A1**	
	$\dfrac{{z_1}^2}{z_2} = -0.73 - 2.73\mathrm{i}$	**A1**	

Ques	Solution	Mark	Notes
5(a)(i)	det**M** = $2(\lambda+2)+5(-\lambda)+\lambda(-\lambda^2)$	M1A1	Or equivalent
	$= 4-3\lambda-\lambda^3$		
(ii)	Substituting $\lambda = 1$, det**M** =0 (therefore singular).	B1	
	$4-3\lambda-\lambda^3 = (1-\lambda)(\lambda^2+\lambda+4)$	M1A1	Do not accept unsupported answers
	The other two roots (of det**M** = 0) are complex since $b^2-4ac=-15$ so no other real values of λ result in a singular **M**. cao	A1	
(iii)	Using row operations,		
	$$\begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$	M1 A1	
	The first two (complete) rows are identical therefore consistent.	A1	
	Let $z = \alpha$.	M1	
	Then $y = \alpha + 1$.	A1	
	and $x = -3\alpha - 1$.	A1	
(b)	Now, **M** = $\begin{bmatrix} 2 & 5 & -1 \\ 0 & -1 & -1 \\ -1 & 2 & 1 \end{bmatrix}$		
	Cofactor matrix = $\begin{bmatrix} 1 & 1 & -1 \\ -7 & 1 & -9 \\ -6 & 2 & -2 \end{bmatrix}$	M1 A1	Award M1 if at least 5 elements correct
	Adjugate matrix = $\begin{bmatrix} 1 & -7 & -6 \\ 1 & 1 & 2 \\ -1 & -9 & -2 \end{bmatrix}$	A1	
	Det**M** = 8	B1	
	$\mathbf{M}^{-1} = \frac{1}{8}\begin{bmatrix} 1 & -7 & -6 \\ 1 & 1 & 2 \\ -1 & -9 & -2 \end{bmatrix}$	A1	FT from adjugate matrix and determinant

Ques	Solution	Mark	Notes
6	Let the roots be $\alpha, \frac{1}{\alpha}, \beta$. Then, $\alpha + \frac{1}{\alpha} + \beta = -\frac{b}{a}$ (i) $1 + \alpha\beta + \frac{\beta}{\alpha} = \frac{c}{a}$ (ii) $\beta = -\frac{d}{a}$ (iii) From (i), $\alpha + \frac{1}{\alpha} = -\frac{b}{a} + \frac{d}{a}$ From (ii), $\alpha + \frac{1}{\alpha} = \left(\frac{c}{a} - 1\right)\left(-\frac{a}{d}\right)$ Therefore $\frac{d-b}{a} = \left(\frac{c-a}{a}\right)\left(-\frac{a}{d}\right)$ $d^2 - bd = a^2 - ac$	**M1** **A1** **M1A1** **A1** **A1**	M1 attempting to eliminate one of the parameters
7	The result to be proved gives $x_1 = 2 + 1 = 3$ which is correct so true for $n = 1$. Let the result be true for $n = k$, ie $x_k = 2^k + k$ Consider (for $n = k + 1$) $x_{k+1} = 2(2^k + k) - k + 1$ $= 2^{k+1} + (k+1)$ Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$, the result is proved by induction.	**B1** **M1** **M1A1** **A1** **A1**	Award A1 for completely correct solution
8(a) **(b)**	Taking logs, $\ln f(x) = \sin x \ln x$ Differentiating, $\frac{f'(x)}{f(x)} = \cos x \ln x + \frac{\sin x}{x}$ $f'(x) = (x)^{\sin x}(\cos x \ln x + \frac{\sin x}{x})$ Consider $f'(0.35) = -0.00451...$ $f'(0.36) = 0.0156...$ The change of sign indicates a root between 0.35 and 0.36.	**M1** **A1A1** **A1** **B1** **B1** **B1**	Accept – 0.00646… Accept 0.0223…

Ques	Solution	Mark	Notes
9(a)	$u+\mathrm{i}v=(x+\mathrm{i}[y+2])^2$ $= x^2+2\mathrm{i}x(y+2)-(y+2)^2$ Equating real and imaginary parts, $u=x^2-(y+2)^2$; $v=2x(y+2)$	**M1** **A1** **M1** **A1**	
(b)	Substituting $y = x - 1$, $u=x^2-(x+1)^2=-(2x+1)$ $v=2x(x+1)$ Eliminating x, $v=-(u+1)\left(-\frac{(u+1)}{2}+1\right)$ $=\left(\frac{u^2-1}{2}\right)$ or equivalent	**M1** **A1** **M1** **A1**	FT from (a) provided equally difficult

GCE MARKING SCHEME

SUMMER 2016

Mathematics – FP2
0978/01

INTRODUCTION

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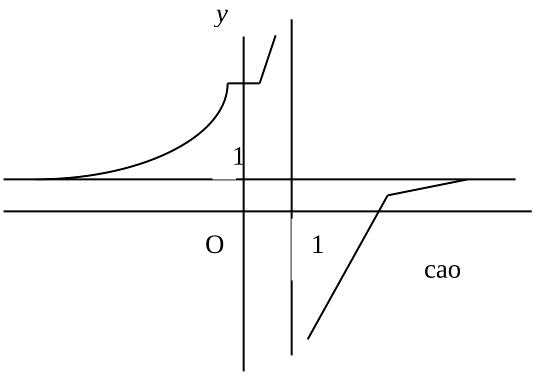
Ques	Solution	Mark	Notes
1	Putting $u = x^2$, $du = 2x dx$, $[0, \sqrt{2}]$ becomes $[0,2]$	B1B1	
	$I = \frac{1}{2}\int_0^2 \frac{du}{\sqrt{(16-u^2)}}$	M1	Valid attempt to substitute
	$= \frac{1}{2}\left[\sin^{-1}\left(\frac{u}{4}\right)\right]_0^2$	A1	
	$= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)$	A1	
	$= \frac{\pi}{12}$	A1	
2(a)(i)	$(3-i)^2 = 9-6i-1 = 8-6i$	M1A1	
(ii)	$(3-i)^4 = (8-6i)^2 = 64-96i-36 = 28-96i$	B1	Convincing
(b)	The 4th roots are $3 - i$ and $-3 + i$ and $1 + 3i$, $-1 - 3i$	B1 B1B1	Must start with 3 – i and rotate
3(a)	$\cos 4\theta + i\sin 4\theta = (\cos\theta + i\sin\theta)^4$	M1	
	$= 4i\cos^3\theta\sin\theta - 4i\cos\theta\sin^3\theta$ + real terms	m1	
	$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	A1	
	$\frac{\sin 4\theta}{\sin\theta} = 4\cos\theta(1-\sin^2\theta-\sin^2\theta)$ $= 4\cos\theta(1-2\sin^2\theta)$	A1	
(b)	EITHER $\int_{\pi/6}^{\pi/4}\frac{\sin 4\theta}{\sin\theta}d\theta = 4\int_{\pi/6}^{\pi/4}\cos\theta\cos 2\theta d\theta$	M1	
	$= 2\int_{\pi/6}^{\pi/4}[\cos\theta + \cos 3\theta]d\theta$	A1	
	$= 2\left[\sin\theta + \frac{\sin 3\theta}{3}\right]_{\pi/6}^{\pi/4}$	A1	This line must be seen
	$= 0.219$	A1	
	OR $\int_{\pi/6}^{\pi/4}\frac{\sin 4\theta}{\sin\theta}d\theta = 4\int_{\pi/6}^{\pi/4}(1-2\sin^2\theta)d\sin\theta$	(M1A1)	
	$= 4\left[\sin\theta - \frac{2}{3}\sin^3\theta\right]_{\pi/6}^{\pi/4}$	(A1)	This line must be seen
	$= 0.219$	(A1)	

Ques	Solution	Mark	Notes
4	Substituting $t = \tan\left(\frac{x}{2}\right)$,		
	$\frac{2t}{1+t^2} + \frac{2t}{1-t^2} + t = 0$	**M1A1**	
	$\frac{2t(1-t^2) + 2t(1+t^2) + t(1+t^2)(1-t^2)}{(1+t^2)(1-t^2)} = 0$	**A1**	
	$\frac{2t - 2t^3 + 2t + 2t^3 + t - t^5}{(1+t^2)(1-t^2)} = 0$	**A1**	
	$t(5 - t^4) = 0$	**A1**	
	$t = 0$	**B1**	FT for $t^4 = n$
	$\frac{x}{2} = 0 + n\pi$ giving $x = 2n\pi$	**B1**	Penalise – 1for use of degrees throughout
	$t = \sqrt[4]{5}$	**B1**	
	$\frac{x}{2} = 0.981 + n\pi$ giving $x = 1.96 + 2n\pi$	**B1**	
	$t = -\sqrt[4]{5}$	**B1**	
	$\frac{x}{2} = -0.981 + n\pi$ giving $x = -1.96 + 2n\pi$	**B1**	
5(a)	Because $f(-x)$is neither equal to $f(x)$ or $-f(x)$, f is neither even nor odd.	**B1**	
(b)	Let		
	$\frac{3x^2 + x + 6}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx + C}{x^2 + 4}$	**M1**	
	$= \frac{A(x^2+4) + (x+2)(Bx+C)}{(x+2)(x^2+4)}$	**A1**	
	$A = 2$; $B = 1$; $C = -1$	**A1A1A1**	
(c)			FT their values from (a)
	$\int_0^1 f(x)\,dx = \int_0^1 \frac{2}{x+2}dx + \int_0^1 \frac{x}{x^2+4}dx - \int_0^1 \frac{1}{x^2+4}dx$	**M1**	
	$= 2[\ln(x+2)]_0^1 + \frac{1}{2}[\ln(x^2+4)]_0^1 - \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^1$	**A1A1A1**	
	$= 2\ln 3 - 2\ln 2 + \frac{1}{2}\ln 5 - \frac{1}{2}\ln 4 - \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$	**A1**	
	$= 0.691$	**A1**	

Ques	Solution	Mark	Notes
6(a)	If $x = a\sec\theta$ and $y = b\tan\theta$, then $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$ showing that the point $(a\sec\theta, b\tan\theta)$ lies on the hyperbola.	M1A1	
(b)(i)	EITHER $\frac{dx}{d\theta} = \sec\theta\tan\theta, \frac{dy}{d\theta} = \sec^2\theta$	M1	
	$\frac{dy}{dx} = \frac{\sec^2\theta}{\sec\theta\tan\theta}$	A1	
	$= \operatorname{cosec}\theta$	A1	
	OR $2x - 2y\frac{dy}{dx} = 0$	(M1)	
	$\frac{dy}{dx} = \frac{x}{y}$	(A1)	
	$= \frac{\sec\theta}{\tan\theta} = \operatorname{cosec}\theta$	(A1)	
	The gradient of the normal is $-\sin\theta$.	M1	
	The equation of the normal is $y - \tan\theta = -\sin\theta(x - \sec\theta)$ $x\sin\theta + y = 2\tan\theta$	A1	
(ii)	The normal meets the x-axis where $y = 0$, ie $x = 2\sec\theta, y = 0$	B1	
	The coordinates of the midpoint of PQ are $\left(\frac{\sec\theta + 2\sec\theta}{2}, \frac{\tan\theta + 0}{2}\right)$, ie	M1	
	$\left(\frac{3}{2}\sec\theta, \frac{1}{2}\tan\theta\right)$ cao	A1	
	EITHER This is the parametric form of a hyperbola showing that the locus of the midpoint is a hyperbola	A1	FT from midpoint
	OR $x = \frac{3}{2}\sec\theta, y = \frac{1}{2}\tan\theta$ $\Rightarrow \sec\theta = \frac{2}{3}x, \tan\theta = 2y$ $\Rightarrow \frac{x^2}{9/4} - \frac{y^2}{1/4} = 1$		FT from midpoint
	This is the equation of a hyperbola showing that the locus of the midpoint is a hyperbola	(A1)	
	Since $a = 3/2$ and $b = ½$, Eccentricity = $\sqrt{\frac{1.5^2 + 0.5^2}{1.5^2}} = \frac{\sqrt{10}}{3}$	M1A1	
	The coordinates of the foci are $\left(\pm\frac{\sqrt{10}}{2}, 0\right)$	A1	

Ques	Solution	Mark	Notes
7(a)	$x = 1$ cao $y = 1$ cao	B1 B1	Penalise – 1for extra asymptotes
(b)	$f(0) = 8$ giving the point (0,8) cao $f(x) = 0 \Rightarrow x = 2$ giving the point (2,0) cao	B1 B1	
(c)	$f'(x) = \dfrac{3x^2(x^3-1) - 3x^2(x^3-8)}{(x^3-1)^2}\left(= \dfrac{21x^2}{(x^3-1)^2}\right)$ The stationary point is (0,8). $f'(x) > 0$ on either side of the stationary point. It is a point of inflection.	M1A1 A1 M1 A1	
(d)	 cao	G1 G1 G1	RH branch approach to asymptotes LH branch approach to asymptotes Stationary point of inflection
(e)(i)	$f(-2) = 16/9$, $f(2) = 0$ $f(S) = (-\infty, 0] \cup [16/9, \infty)$ cao	B1 B1	
(ii)	$f(x) = -2 \Rightarrow x = \sqrt[3]{10/3}$ $f(x) = 2 \Rightarrow x = -\sqrt[3]{6}$ $f^{-1}(S) = (-\infty, -\sqrt[3]{6}] \cup [\sqrt[3]{10/3}, \infty)$ cao	M1A1 A1 A1	Accept 1.82 for $\sqrt[3]{6}$ and 1.49 for $\sqrt[3]{10/3}$

GCE MARKING SCHEME

SUMMER 2016

Mathematics – FP3
0979/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Ques	Solution	Mark	Notes
1	Consider $x = r\cos\theta$ $= \cos\theta(1+2\tan\theta) = \cos\theta + 2\sin\theta$ $\frac{dx}{d\theta} = -\sin\theta + 2\cos\theta$ (The tangent is perpendicular to the initial line where) $\frac{dx}{d\theta} = 0$. $\sin\theta = 2\cos\theta$ $\tan\theta = 2$ $\theta = 1.11\ (63^\circ)$ This lies outside the domain for the curve, hence no point at which the tangent is perpendicular to the initial line.	**M1** **A1** **B1** **M1** **A1** **A1** **A1**	or $0 \le \theta \le \frac{\pi}{4} \Rightarrow 0 \le \tan\theta \le 1$
2(a)	$f(x) = \cos x + \cosh x$ $f'(x) = -\sin x + \sinh x$ $f''(x) = -\cos x + \cosh x$ $f'''(x) = \sin x + \sinh x$ $f^{(4)}(x) = \cos x + \cosh x (= f(x))$	B1 B1	Convincing
(b)(i)	$f(0) = 2$ $f'(0) = 0$ $f''(0) = 0$ $f'''(0) = 0$ $f^{(4)}(0) = 2$ This pattern repeats itself every four differentiations so $f^{(n)}(0) = 2$ if n is a multiple of 4 and zero otherwise. (Therefore the only terms in the Maclaurin series are those for which the power is a multiple of 4.)	B1 B1	Accept unsimplified expressions
(ii)	The first three terms are $2, \frac{x^4}{12}, \frac{x^8}{20160}$	B1	
(c)(i)	Substituting the series, $24 + x^4 + \frac{x^8}{1680} - x^4 = 36$ $x^8 = 20160$ $x = 3.45$	M1 A1 A1	
(ii)	Let $g(x) = 12(\cos x + \cosh x) - x^4 - 36$ Consider $g(3.445) = -0.0507\ldots$ $g(3.455) = 0.2312\ldots$ The change of sign confirms that the value of the root is 3.45 correct to 3 significant figures.	B1 B1	

Ques	Solution	Mark	Notes
3	Putting $t = \tan\left(\frac{x}{2}\right)$		
	$[0,\pi/2]$ becomes $[0,1]$	B1	
	$\mathrm{d}x = \frac{2\mathrm{d}t}{1+t^2}$	B1	
	$I = \int_0^1 \frac{2\mathrm{d}t/(1+t^2)}{3+5(1-t^2)/(1+t^2)}$	M1A1	
	$= \int_0^1 \frac{2\mathrm{d}t}{8-2t^2}$	A1	
	$= \int_0^1 \frac{\mathrm{d}t}{4-t^2}$	A1	
	$= \frac{1}{4}\left[\ln\left(\frac{2+t}{2-t}\right)\right]_0^1$	A1	
	$= \frac{1}{4}\ln 3 = \ln 3^{1/4}$	A1	
4(a)	The equation is $\cosh 2\theta - 8\cosh\theta - k = 0$ Substituting for $\cosh 2\theta$,	M1	
	$2\cosh^2\theta - 8\cosh\theta - (k+1) = 0$	A1	
	$\cosh\theta = \frac{8 \pm \sqrt{72+8k}}{4}$	m1	
	If $k < -9$, $72 + 8k < 0$ so no real solutions.	A1	
(b)	If $k = -8$,		
	$\cosh\theta = \frac{8 \pm \sqrt{8}}{4} = 1.292\ldots, 2.707\ldots$	M1A1	
	$\theta = 0.75, 1.65$	A1	Allow ±
(c)(i)	There is a repeated root when $k = -9$	B1	
(ii)	There will be only one real root if the smaller root of the quadratic equation in (a) < 1, ie	M1	
	$\frac{8-\sqrt{72+8k}}{4} < 1$	A1	
	$\sqrt{72+8k} > 4$	M1	Allow $k = -9$ to be included here
	$k > -7$	A1	

Ques	Solution	Mark	Notes
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\sin x}{1+\cos x}$	**B1**	
	$1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1+\frac{\sin^2 x}{(1+\cos x)^2}$	**M1**	
	$= \frac{1+2\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$	**A1**	
	$= \frac{2+2\cos x}{(1+\cos x)^2}$ $= \frac{2}{(1+\cos x)}$	**A1**	
(b)	METHOD 1		
	Arc length $= \sqrt{2}\int_0^{\pi/2}\sqrt{\frac{1}{(1+\cos x)}}\mathrm{d}x$	**M1**	
	$= \sqrt{2}\int_0^{\pi/2}\sqrt{\frac{1}{2\cos^2(x/2)}}\mathrm{d}x$	**m1**	
	$= \int_0^{\pi/2}\sec(x/2)\mathrm{d}x$	**A1**	
	$= 2\left[\ln\left(\sec(x/2)+\tan(x/2)\right)\right]_0^{\pi/2}$	**A1**	
	$= 2\ln(1+\sqrt{2})$	**A1**	Award this A1 if the 2 is missing
	$= \ln(3+2\sqrt{2})$	**A1**	
	METHOD 2		
	Arc length $= \sqrt{2}\int_0^{\pi/2}\sqrt{\frac{1}{(1+\cos x)}}\mathrm{d}x$	M1	
	Put $t=\tan\left(\frac{x}{2}\right)$; $\mathrm{d}x = \frac{2\mathrm{d}t}{1+t^2}$	m1	
	Arc length $= \sqrt{2}\int_0^1\sqrt{\frac{1}{(1+(1-t^2)/(1+t^2))}}\times\frac{2\mathrm{d}t}{1+t^2}$	A1	
	$= 2\int_0^1\sqrt{\frac{1}{(1+t^2)}}\mathrm{d}t$	A1	
	$= 2\ln\left[t+\sqrt{1+t^2}\right]_0^1$	A1	Allow $\sinh^{-1}(t)$
	$= 2\ln\left[1+\sqrt{2}\right] = \ln(3+2\sqrt{2})$	A1	

Ques	Solution	Mark	Notes
6(a)(i)	Let $f(x) = (3 - \sinh x)^{\frac{1}{5}}$		
	$f'(x) = \frac{1}{5}(3 - \sinh x)^{-\frac{4}{5}} \times (-\cosh x)$	M1A1	
	$f'(1) = -0.1907...$	A1	
	Since this is less than 1 in modulus, the sequence is convergent.	A1	
	Let $g(x) = \sinh^{-1}(3 - x^5)$		
	$g'(x) = \frac{1}{\sqrt{1 + (3 - x^5)^2}} \times (-5x^4)$	M1A1	
	$g'(1) = -2.236...$	A1	
	Since this is greater than 1 in modulus, the sequence is divergent.	A1	
(ii)	Successive approximations are 1 1.127828325 1.100939212 1.107049937 1.105684578 1.105990816	M1A1	
	(since the sequence oscillates) the value of the root is 1.106 correct to three decimal places.	A1 A1	
(b)	The Newton-Raphson iteration is $x \to x - \frac{x^5 + \sinh x - 3}{5x^4 + \cosh x}$	M1A1	
	Successive approximations are 1 1.126056647 1.106546041 1.105935334 1.105934755 1.105934754	M1A1	Allow any starting value
	The value of the root is 1.105935 correct to six decimal places.	A1 A1	This last value must be seen for A1

Ques	Solution	Mark	Notes
7(a)	$I_n = -\frac{1}{2}\int_0^{\pi} x^n \mathrm{d}(\cos 2x)$	M1	
	$= -\frac{1}{2}\left[x^n \cos 2x\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} nx^{n-1}\cos 2x\mathrm{d}x$	A1A1	
	$= -\frac{\pi^n}{2} + \frac{n}{4}\int_0^{\pi} x^{n-1}\mathrm{d}(\sin 2x)$	M1	
	$= -\frac{\pi^n}{2} + \frac{n}{4}\left[x^{n-1}\sin 2x\right]_0^{\pi} - \frac{n(n-1)}{4}I_{n-2}$	A1A1	
	$= -\frac{\pi^n}{2} - \frac{n(n-1)}{4}I_{n-2}$		
(b)	$I_0 = \int_0^{\pi}\sin 2x\mathrm{d}x = -\frac{1}{2}\left[\cos 2x\right]_0^{\pi} = 0$	B1	
	$I_4 = -\frac{\pi^4}{2} - 3I_2$	M1	
	$= -\frac{\pi^4}{2} - 3\left(-\frac{\pi^2}{2} - \frac{1}{2}I_0\right)$	A1	FT their I_0 for this A1
	$= -34$ cao	A1	

GCE MARKING SCHEME

SUMMER 2016

Mathematics – M1
0980/01

INTRODUCTION

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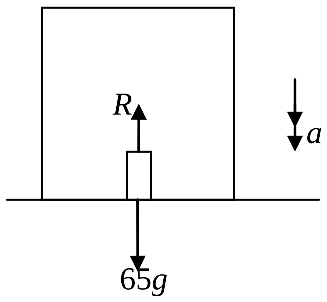
GCE Mathematics - M1

Summer 2016 Mark Scheme

Q	Solution	Mark	Notes
1.			



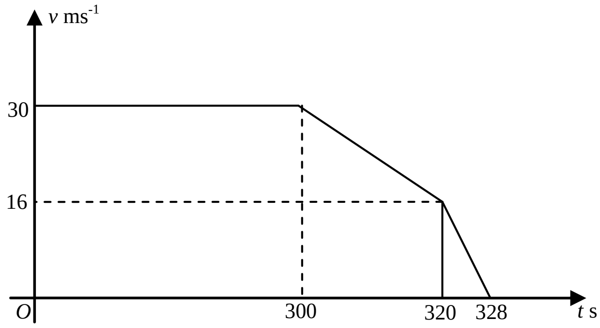
	N2L applied man	M1	*R* and 65g opposing. dim correct
	$65g - R = 65a$	A1	
	1^{st} stage, $a = 3.2$		
	$R = 65(9.8 - 3.2)$		
	$R =$ <u>429 (N)</u>	A1	cao
	2^{nd} stage, $a = 0$		
	$R = 65 \times 9.8$		
	$R =$ <u>637 (N)</u>	B1	cao
	3^{rd} stage, $a = -2.4$		
	$R = 65(9.8 + 2.4)$		
	$R =$ <u>793 (N)</u>	A1	cao

Q	Solution	Mark	Notes
2(a)	Apply N2L to B	M1	dim correct, all forces $5g$ and T opposing
	$5g - T = 5a$	A1	
	Apply N2L to A	M1	dim correct, all forces T and $2g$ opposing
	$T - 2g = 2a$	A1	
	Adding		
	$5g - 2g = 7a$	m1	one variable eliminated, Dep on both M's
	$a = \underline{4.2 \text{ ms}^{-2}}$	A1	cao
	$T = \underline{28 \text{ N}}$	A1	cao
2(b)	Upwards positive		
(i)	Using $v = u + at$, u=0. a=(±)4.2,t=2	M1	cand's a
	$v = 0 + 4.2\times2$		
	$v = \underline{8.4 \text{ (ms}^{-1})}$	A1	ft a
(ii)	$s=ut+0.5at^2$, s=(±)18.9,u=(±)8.4,a=(±)9.8	M1	cand's v, one sign error
	$-18.9 = 8.4t + 0.5 \times -9.8 \times t^2$	A1	ft v
	$7t^2 - 12t - 27 = 0$	m1	recognition of quadratic and attempt to solve
	$(7t + 9)(t - 3) = 0$		
	$t = \underline{3 \text{ (s)}}$	A1	cao

Q	Solution	Mark	Notes
3(a)	$I = 3 \times 4$		
	$= \underline{12\ (\text{Ns})}$	B1	
3(b)	Conservation of momentum	M1	attempted, equation, dim correct.
	$3\times4 + 11\times0 = 3v_A + 11v_B$	A1	correct equation
	$3v_A + 11v_B = 12$		
	Restitution	M1	one sign error only
	$v_B - v_A = -\frac{1}{4}(0 - 4)$	A1	correct equation, any form
	$v_B - v_A = 1$		
	$3v_A + 11v_B = 12$		
	$-3v_A + 3v_B = 3$		
	Adding	m1	
	$14v_B = 15$		
	$v_B = \underline{\frac{15}{14}\ (\text{ms}^{-1})}$	A1	cao
	$v_A = \underline{\frac{1}{14}\ (\text{ms}^{-1})}$	A1	cao
3(c)	$\frac{6}{7} = e \times \frac{15}{14}$	M1	correct equation, any form
	$e = \frac{6}{7} \times \frac{14}{15}$		
	$e = \frac{4}{5} = \underline{0.8}$	A1	ft v_B if $> \frac{6}{7}$

Note: Accept g throughout conservation of momentum equation, whether crossed off or not.

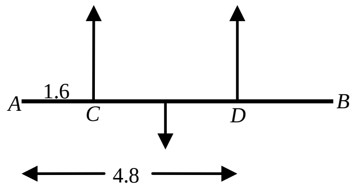
Q	Solution	Mark	Notes
4(a)			



		B1	(0, 30) to (300, 30)
		B1	(300, 30) to (320, 16)
		B1	(320, 16) to (328,0)
		B1	shape, units, labels
4(b)	Total distance = area under graph	M1	attempted
	$D = 300\times30 + 0.5\times(30+16)\times20 + 0.5\times16\times8$	B1	one correct area, ft graph
		A1	all correct, ft graph if shape correct.
	$D = 9000 + 460 + 64$		
	$D = \underline{9524\text{ (m)}}$	A1	cao

Q	Solution	Mark	Notes
5	Resolve in one direction	M1	obtain comp of resultant
	$X = 8\cos30° + 7\cos45°$		
	$- 15\cos60° - 12\cos50°$	A1	
	$X = -3.3355$		
	Resolve in perpendicular direction	M1	obtain comp of resultant
	$Y = 8\cos60° - 7\cos45°$		
	$- 15\cos30° + 12\cos40°$	A1	
	$Y = -4.7476$		
	Resultant^2 = $3.3355^2 + 4.7476^2$	m1	dep on both M's
	Resultant = <u>5.8N</u>	A1	cao
	Acceleration = $\dfrac{5\cdot8021777}{4}$		
	Acceleration = <u>1.45 (ms^{-2})</u>	A1	ft Resultant. Accept 1.5.

Q	Solution	Mark	Notes
6.			



Take moments about C	M1	dim correct moment equ.
$8g \times 1.4 = T_D \times 3.2$	B1	Any correct moment
	A1	correct equation
$T_D = \underline{3.5g\text{ (N)}} = \underline{34.3\text{ (N)}}$	A1	cao
Resolve vertically	M1	oe
$T_C + T_D = 8g = 78.4$	A1	
$T_C = \underline{4.5g\text{ (N)}} = \underline{44.1\text{ (N)}}$	A1	cao

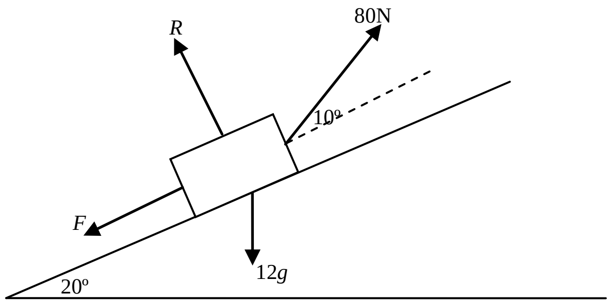
Note:
Simultaneous equations

First moment equation	M1 B1 A1
Second moment equation or resolution equation	M1 A1 (B1 if not previously awarded)
Answers	A1 A1

Equal tension

Moments about C/D	4 marks available
Moments about anywhere else	2 marks available.

Q	Solution	Mark	Notes
7			



7(a)	Resolve perpendicular to plane	M1	dim correct equation All forces No more than 1 sign error
	$R + 80 \sin 10° = 12g \cos 20°$ $R = 96.616$	A1	
	$F = \mu R = 0.2 \times 96.616$	M1	ft R (any correct form)
	$F = \underline{19.323\ (\text{N})}$	A1	cao
7(b)	Resolve parallel to plane	M1	dim correct equation All forces Allow sin/cos errors Friction subtracted from tension
	$80 \cos 10° - F - 12g \sin 20° = 12a$	A2	-1 each error, (ft F)
	$a = \underline{1.6\ (\text{ms}^{-2})}$	A1	cao

Note (for both parts)

If no g with 12,	M0 (possibly M1 for μR)
If 80 not resolved	M0
If g with 80	M0

Q	Solution	Mark	Notes
8	Use of $s = ut + 0.5at^2$ with s=460, t=20	M1	
	$460 = 20u + 0.5 \times a \times 400$	A1	
	$u + 10a = 23$		
	Use of $v = u + at$ with t=6, v=17	M1	
	$17 = u + 6a$	A1	
	$u + 6a = 17$		
	attempt to solve simultaneously	m1	one variable remains
	$4a = 6$		
	$a = \underline{1.5}$	A1	cao
	$u = \underline{8}$	A1	cao

Note:

3 or more equations	
First correct equation	M1 A1
All subsequent equations, eg 2 if 3 unknowns, 3 if 4 unknowns	M1 A1
All variables except one eliminated	m1
Correct answers	A1 A1

Q	Solution				Mark	Notes
9.		Area	AC	AB		
	ABC	54	4	3	B1	
	Circle	4π	4	3	B1	
	D	12π	6	4.5	B1	
	Lamina	$(54+8\pi)$	x	y	B1	expressions for areas, oe
	Moments about AC				M1	consistent areas and moments
	$54\times4 + 12\pi\times6 = (54+8\pi)x + 4\pi\times4$				A1	signs correct. Ft table if at least one B1 for c of m gained.
	x = <u>4.95 (cm)</u>				A1	cao
	Moments about AB				M1	consistent areas and moments
	$54\times3 + 12\pi\times4.5 = (54+8\pi)y + 4\pi\times3$				A1	signs correct. Ft table if at least one B1 for c of m gained.
	y = <u>3.71 (cm)</u>				A1	cao

<u>Alternative solution</u>

	Area	AC	AB		
ABC-Circle	$54-4\pi$	4	3	B1 B1	
D	12π	6	4.5	B1	
Lamina	$(54+8\pi)$	x	y	B1	expressions for areas, oe
Moments about AC				M1	consistent areas and moments
$(54-4\pi)\times4 + 12\pi\times6 = (54+8\pi)x$				A1	signs correct. Ft table if at least one B1 for c of m gained.
x = <u>4.95 (cm)</u>				A1	cao
Moments about AB				M1	consistent areas and moments
$(54-4\pi)\times3 + 12\pi\times4.5 = (54+8\pi)y$				A1	signs correct. Ft table if at least one B1 for c of m gained.
y = <u>3.71 (cm)</u>				A1	cao

GCE MARKING SCHEME

SUMMER 2016

Mathematics – M2
0981/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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GCE Mathematics - M2

Summer 2016 Mark Scheme

Q	Solution	Mark	Notes
1(a).	$x = \int 12t^2 - 7kt + 1 \mathrm{d}t$	M1	At least one power increased
	$x = 4t^3 - \frac{7k}{2}t^2 + t + (C)$	A1	correct integration
	$t = 0$, $x = 3$	m1	use of initial conditions
	$C = 3$		
	$x = 4t^3 - \frac{7k}{2}t^2 + t + 3$		
	$t = 2$, $x = 16$	m1	values substituted
	$16 = 32 - 14k + 2 + 3$		
	$k = \frac{3}{2}$	A1	cao
1(b).	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	M1	At least one power decreased
	$a = 24t - 10.5$	A1	correct differentiation ft k. accept k
	$F = 4(24t - 10.5)$	m1	4xa
	When $t = 5$		
	$F = 4(24\times5 - 10.5)$		
	F = <u>438 (N)</u>	A1	ft k. –ve values A0

Q	Solution	Mark	Notes
2(a)	$u_H = 24.5\cos30° = (12.25\sqrt{3})$	B1	
	$u_V = 24.5\sin30° = (12.25)$	B1	
	$s = ut + 0.5at^2$, $s=0$, $u=12.25$, $a=(\pm)9.8$	M1	oe complete method
	$0 = 12.25t - 0.5\times9.8\times t^2$	A1	
	$t = \dfrac{12\cdot25}{4\cdot9}$		
	$t = 2.5$	A1	
	Range $= 2.5 \times 12.25\sqrt{3}$		
	Range $=$ <u>53.04 (m)</u>	A1	cao
2(b)	$v^2 = u^2 + 2as$, $v=0$, $u=12.25$, $a=(\pm)9.8$	M1	oe complete method
	$0 = 12.25^2 - 2\times9.8\times s$	A1	ft u_V
	$s = \underline{7.65625} = \underline{7.66\text{ (m)}}$	A1	answers rounding to 7.7 ISW
2(c)	Required speed is 24.5 ms^{-1} downwards at an angle of 30° to the horizontal.	B1	

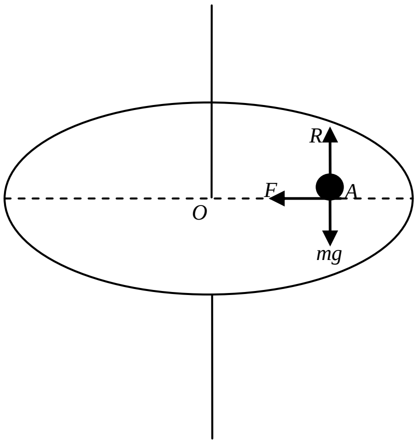
Q	Solution	Mark	Notes
3	$\mathbf{r} = \mathbf{p} + t\mathbf{v}$	M1	used
	$\mathbf{r}_A = (1 + 2t)\mathbf{i} + 5t\mathbf{j} - 4t\mathbf{k}$		
	$\mathbf{r}_B = (3 + t)\mathbf{i} + 3t\mathbf{j} - 5t\mathbf{k}$	A1	either correct, any form
	$\mathbf{r}_B - \mathbf{r}_A = (2 - t)\mathbf{i} - 2t\mathbf{j} - t\mathbf{k}$	M1	
	$AB^2 = x^2 + y^2 + z^2$	M1	
	$AB^2 = (2 - t)^2 + 4t^2 + t^2$	A1	cao
	$(AB^2 = 6t^2 - 4t + 4)$		
	Differentiate	M1	at least 1 power reduced
	$\frac{\mathrm{d}AB^2}{\mathrm{d}t} = 2(2 - t)(-1) + 10t \quad (= 12t - 4)$		
	$-4 + 2t + 10t = 0$	m1	equating to 0.
	$t = \frac{1}{3}$	A1	cao
	$(\text{least distance})^2 = (2 - \frac{1}{3})^2 + 5(\frac{1}{3})^2$		
	least distance $= \sqrt{\frac{10}{3}} = \underline{1.83\ (\text{m})}$	A1	cao

Q	Solution	Mark	Notes
4(a)	Conservation of momentum	M1	dimensionally correct
	$12 \times 600 = 1600 \times v$	A1	
	$v = \frac{9}{2}$ (ms^{-1})	A1	allow -ve
4(b)	Energy considerations	M1	
	$E = 0.5{\times}12{\times}600^2 + 0.5{\times}1600{\times}4.5^2$	A1	both expressions correct, Ft v in (a)
	$E = 2160000 + 16200$		
	$E = \underline{2176200\ (J)}$	A1	cao
	Energy dissipated by eg sound of cannon firing ignored.	E1	oe
4(c)	Work-energy principle	M1	used
	$F \times d = E$		
	$F \times 1.2 = 16200$		
	$F = \underline{13500\ (N)}$	A1	cao

Q	Solution	Mark	Notes
5.	Hooke's Law	M1	used
	$30 = \dfrac{\lambda(0\cdot 95 - l)}{l}$	A1	
	$70 = \dfrac{\lambda(1\cdot 15 - l)}{l}$	A1	
	$\dfrac{70}{30} = \dfrac{(1.15 - l)}{(0\cdot 95 - l)}$	m1	getting to equation with 1 variable
	$7(0.95 - l) = 3(1.15 - l)$		
	$l = \underline{0.8}$	A1	cao
	$\lambda = \underline{160}$	A1	cao

Q	Solution	Mark	Notes
6(a)	$\mathbf{a} = \dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	sin to cos and coefficient multiplied
	$\mathbf{a} = 14\cos 2t\,\mathbf{i} - 18\sin 3t\,\mathbf{j}$	A1	
6(b)	$\mathbf{r} = \int 7\sin 2t\ \mathbf{i} + 6\cos 3t\,\mathbf{j}\,\mathrm{d}t$	M1	sin to cos and coefficient divided.
	$\mathbf{r} = -3.5\cos 2t\,\mathbf{i} + 2\sin 3t\,\mathbf{j} + (\mathbf{c})$	A1	
	$t = 0$, $\mathbf{r} = 0.5\,\mathbf{i} + 3\,\mathbf{j}$	m1	used
	$0.5\,\mathbf{i} + 3\,\mathbf{j} = -3.5\,\mathbf{i} + \mathbf{c}$		
	$\mathbf{c} = 4\,\mathbf{i} + 3\,\mathbf{j}$		
	When $t = \dfrac{\pi}{2}$	m1	substituted si
	$\mathbf{r} = -3.5\cos\pi\,\mathbf{i} + 2\sin\dfrac{3}{2}\pi\,\mathbf{j} + 4\,\mathbf{i} + 3\,\mathbf{j}$		
	$\mathbf{r} = (4 + 3.5)\,\mathbf{i} + (3 - 2)\,\mathbf{j}$		
	$\mathbf{r} = \underline{7.5\,\mathbf{i} + \mathbf{j}\text{ (m)}}$	A1	cao
OR			
	$\int_0^{\pi/2} 7\sin 2t\ \mathbf{i} + 6\cos 3t\,\mathbf{j}\,\mathrm{d}t$	(M1)	attempt to integrate
	$= [-3.5\cos 2t\,\mathbf{i} + 2\sin 3t\,\mathbf{j}]^{\pi/2}$	(A1)	correct integration
	$= 3.5\,\mathbf{i} - 2\,\mathbf{j} + 3.5\,\mathbf{i}$	(m1)	correct use of limits $0, \pi/2$
	$\mathbf{r} = 0.5\,\mathbf{i} + 3\,\mathbf{j} + 3.5\,\mathbf{i} - 2\,\mathbf{j} + 3.5\,\mathbf{i}$	(m1)	adding $0.5\,\mathbf{i} + 3\,\mathbf{j}$
	$\mathbf{r} = \underline{7.5\,\mathbf{i} + \mathbf{j}\text{ (m)}}$	(A1)	cao

Q	Solution	Mark	Notes
7.	K. Energy. at $A = 0.5 \times 70 \times v^2$	B1	
	K. Energy. at $A = 35v^2$		
	Let potential energy be 0 at A		
	P. Energy at $B = 70\times9.8\times(22\text{-}20)$	M1	mgh attempted
	P. Energy at $B = 70\times9.8\times2$	A1	correct for h=2, 20, 22
	P. Energy at $B = 1372$		
	Minimum K. Energy at $B = 0$		
	WD against resistance = 50×16	B1	
	WD against resistance = 800		
	Work-Energy Principle	M1	at least 3 energies
	$35v^2 = 1372 + 800$	A1	ft one arithmetic slip
	$v = \underline{7.88}$	A1	cao

Q	Solution	Mark	Notes
8			
	Resolve vertically $R = mg$	B1	
	$F = \mu R = 0.72mg$	B1	ft R, si
	If particle remains at A		
	$F \geq ma$	M1	accept =, used, No extra force
	$0.72mg \geq \dfrac{mv^2}{1\cdot6}$	A1	accept =
	$v^2 \leq 0.72 \times 9.8 \times 1.6$		
	$v \leq \underline{3.36}$	A1	cao, accept =
	Greatest value of v is $\underline{3.36}$		
	$\omega \leq \dfrac{3\cdot36}{1\cdot6}$		
	$\omega \leq \underline{2.1 \text{ rads}^{-1}}$	A1B1	accept =, ft v
	Greatest value of ω is $\underline{2.1 \text{ rads}^{-1}}$		

Q	Solution	Mark	Notes
9(a)	Conservation of energy	M1	KE and PE
	$0.5 \times m \times g + mg \times 4(1 - \cos\theta)$		
	$= 0.5 \times m \times v^2$	A1	KE both sides, oe
		A1	correct equation, any form
	$g + 8g(1 - \cos\theta) = v^2$		
	$v^2 = \underline{g(9 - 8\cos\theta)}$	A1	cao, simplified, ISW
9(b)	N2L towards centre of motion	M1	dim correct, 3 terms, $mg\cos\theta$ and R opposing
	$mg\cos\theta - R = \dfrac{mv^2}{4}$	A1	
	$R = mg\cos\theta - \dfrac{mg}{4}(9 - 8\cos\theta)$	m1	
	$R = \underline{3mg(\cos\theta - 0.75)}$	A1	cao, any form ISW
	P leaves the surface when R=0	M1	
	$\cos\theta = \underline{0.75}$	A1	cao
	$v^2 = \underline{g(9 - 8 \times 0.75)}$		
	$v^2 = \underline{3g} = \underline{29.4}$	A1	cao

GCE MARKING SCHEME

SUMMER 2016

Mathematics – M3
0982/01

INTRODUCTION

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GCE Mathematics - M3

Summer 2016 Mark Scheme

Q	Solution	Mark	Notes
1(a)	N2L applied to particle		
	$1800 - 120v = 60a$	M1	dim correct equation
	Divide by 60 and $a = \frac{\mathrm{d}v}{\mathrm{d}t}$		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 30 - 2v$	A1	convincing
1(b)	$\int \frac{dv}{30-2v} = \int dt$	M1	correct sep. of variables
	$-\frac{1}{2}\ln\lvert 30-2v \rvert = t\ (+\mathrm{C})$	A1A1	A1 for $\ln\lvert 30-2v \rvert$
			A2 all correct, any form.
	When $t = 0$, $v = 8$	m1	initial conditions used
	$\mathrm{C} = -\frac{1}{2}\ln 14$		
	$t = \frac{1}{2}\ln\left\lvert \frac{14}{30-2v} \right\rvert$		
	$e^{2t} = \frac{14}{30-2v}$	m1	correct inversion at any stage ft similar expression
	$30 - 2v = 14e^{-2t}$		
	$v = 15 - 7e^{-2t}$	A1	any correct simplified expression
	Limiting value of v = <u>15</u>	B1	cao. Allow if e^{-kt}, $k>0$.

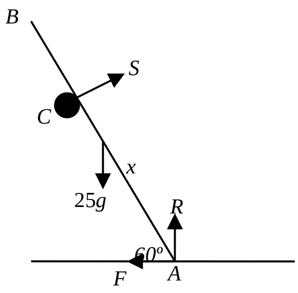
Q	Solution	Mark	Notes
2(a).	$x = A\sin\omega t + B\cos\omega t.$		
	$\frac{\mathrm{d}x}{\mathrm{d}t} = v = A\omega\cos\omega t - B\omega\sin\omega t.$	B1	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$	M1	
	Hence,		
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$	A1	convincing
	Therefore motion is SHM		
	Value of x at centre of motion = 0	B1	
	Amplitude a = value of x when v = 0		
	$A\omega\cos\omega t - B\omega\sin\omega t = 0$	M1	
	$\tan\omega t = \frac{A}{B}$		
	$\sin\omega t = \frac{A}{\sqrt{A^2+B^2}}$ $\cos\omega t = \frac{B}{\sqrt{A^2+B^2}}$	m1	either expression
	$a = A\frac{A}{\sqrt{A^2+B^2}} + B\frac{B}{\sqrt{A^2+B^2}}$		
	$a = \sqrt{A^2+B^2}$	A1	cao
2(b)(i)	using $v^2 = \omega^2(a^2 - x^2)$	M1	
	$25 = \omega^2(a^2 - 25)$		
	$169 = \omega^2(a^2 - 9)$	A1	either equation correct
	Subtract		
	$144 = 16\omega^2$	m1	oe
	$\omega = 3$		
	Amplitude = a		
	$25 = 3^2(a^2 - 25)$	m1	substitution
	Period $= \frac{2\pi}{\omega} = \frac{2\pi}{3}$	A1	cao
	$a^2 = \frac{250}{9}$, $a = \frac{5\sqrt{10}}{3}$ = <u>5.27 (m)</u>	A1	cao
2(b)(ii)	$x = \frac{5\sqrt{10}}{3}\sin(3t)$	M1	accept sin/cos, a, ω
	$x = \frac{5\sqrt{10}}{3}\sin(3\times 0.3)$	A1	ft derived a, ω
	x = <u>4.128 (m)</u>	A1	cao

Q	Solution	Mark	Notes
	<u>Alternative solution</u>		
2(a).	$x = A\sin\omega t + B\cos\omega t$.		
	$x = R\sin(\omega t + \varepsilon)$	M1	
	$A\sin\omega t + B\cos\omega t$		
	$= R\sin\omega t\cos\varepsilon + R\cos\omega t\sin\varepsilon$	m1	si
	$R\cos\varepsilon = A$		
	$R\sin\varepsilon = B$		
	$R = \sqrt{A^2 + B^2}$	A1	
	$\varepsilon = \tan^{-1}\left(\frac{B}{A}\right)$	A1	
	$x = \sqrt{A^2 + B^2}\sin(\omega t + \tan^{-1}\left(\frac{B}{A}\right))$		
	Therefore motion is SHM	A1	
	Value of x at centre of motion = 0	B1	
	Amplitude = $\sqrt{A^2 + B^2}$	A1	

Q	Solution	Mark	Notes
3	Auxiliary equation		
	$m^2 + 6m + 9 = 0$	M1	
	$(m + 3)^2 = 0$		
	$m = -3$ (twice)	A1	
	CF is $x = (A + Bt)e^{-3t}$	B1	ft values of m
	For PI, try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$\frac{d^2x}{dt^2} = 0$		
	$6a + 9(at + b) = 27t$	A1	
	Comparing coefficients	m1	
	$9a = 27$		
	$a = 3$		
	$18 + 9b = 0$		
	$b = -2$	A1	both values
	General solution is		
	$x = (A + Bt)e^{-3t} + 3t - 2$		
	When $t = 0$, $x = 0$	m1	used
	$0 = A - 2$		
	$A = 2$	A1	cao
	$\frac{dx}{dt} = -3(A + Bt)e^{-3t} + Be^{-3t} + 3$	B1	ft similar expressions
	When $t = 0$, $\frac{dx}{dt} = 0$,		
	$0 = -3A + B + 3$		
	$B = 3$	A1	ft similar expressions
	$x = (2 + 3t)e^{-3t} + 3t - 2$		
	When $t = 2$		
	$x = 8e^{-6} + 4$	A1	cao
	$x = \underline{4.(02)\ (4.01983)}$		

Q	Solution	Mark	Notes
4(a).	Use of N2L $8g - 0.4v^2 = 8a$	M1	
	$196 - v^2 = 20v\dfrac{dv}{dx}$	A1	use of $a = v\dfrac{dv}{dx}$,convincing
4(b)	$\int dx = \int \dfrac{20v dv}{196 - v^2}$	M1	correct sep variables
	x (+C) = $20 \times -\dfrac{1}{2}\ln\lvert 196 - v^2 \rvert$	A1A1	A1 for $\ln\lvert 196 - v^2 \rvert$, A1 all correct
	x (+C) = $-10\ln\lvert 196 - v^2 \rvert$		
	When $x = 0$, $v = 0$	m1	
	C = -10ln196		
	$x = 10\ln\left\lvert \dfrac{196}{196 - v^2} \right\rvert$	A1	cao
	When $v = 10$		
	$x = 10\ln\dfrac{196}{96}$ = <u>7.14 (m)</u>	A1	cao
4(c)	$196 - v^2 = 20\dfrac{dv}{dt}$		
	$\int dt = \int \dfrac{20dv}{14^2 - v^2}$	M1	correct sep variables
	$t = \dfrac{20}{2 \times 14}\ln\left\lvert \dfrac{14 + v}{14 - v} \right\rvert$ +(C)	A1A1	A1 for $\ln\left\lvert \dfrac{14 + v}{14 - v} \right\rvert$, A1 all correct
	When $t = 0$, $v = 0$	m1	used
	C = 0	A1	
	$t = \dfrac{5}{7}\ln\left\lvert \dfrac{14 + v}{14 - v} \right\rvert$		
	$e^{1.4t} = \dfrac{14 + v}{14 - v}$	m1	inversion
	$v = 14\left(\dfrac{e^{1.4t} - 1}{e^{1.4t} + 1}\right)$	A1	cao any correct expres.
	When $t = 2$		
	v = <u>12.39</u>	A1	cao

Q	Solution	Mark	Notes
5	Speed of *A* just before string becomes taut is given by		
	$v^2 = u^2+2as$, a=(±)9.8, s=(1.8-0.2)	M1	
	v^2 = 0+2×9.8×1.6		
	v = 5.6 (ms^{-1})	A1	
	Impulse = change in momentum	M1	used
	Apply to *A*		
	J = 2 × 5.6 - 2v	A1	ft answer in (a)
	Apply to *B*		
	J = 5v	B1	
	Solving simultaneously	m1	
	2 × 5.6 - 2v = 5v		
	7v = 11.2		
	Speed of *B* = <u>1.6 (ms^{-1})</u>	A1	cao
	J = 5v = <u>8 (Ns)</u>	A1	ft speed of *B*

Q	Solution	Mark	Notes
6(a)			
		A2	-1 each error
6(b)	Resolve vertically	M1	equation, no missing, no extra force. sin/cos
	$S\cos60° + R = 25g$	A1	
	Resolve horizontally	M1	equation, no missing no extra force. sin/cos
	$F = S\sin60°$	A1	
	$F = 0.3R$	B1	used
	$0.3R = S\sin60°$		
	$R = \dfrac{\sqrt{3}}{2\times 0\cdot 3}S$		
	$0.5S + R = 25g$		
	$0.5S + \dfrac{\sqrt{3}}{2\times 0\cdot 3}S = 25 \times 9.8$	m1	eliminating one variable
			Depends on both M's
	$S = \underline{72.34\ (\text{N})}$	A1	cao
	$R = \underline{208.83\ (\text{N})}$	A1	cao
6(c)	Moments about A	M1	equation, no missing, no extra force. dim correct
	$Sx = 25g \times 5\cos60°$	A1	LHS correct
		A1	RHS correct
	$x = \dfrac{25\times 9\cdot 8\times 5\times \cos 60^\circ}{72\cdot 340711}$		
	$x = \underline{8.46(69)}$	A1	cao

GCE MARKING SCHEME

SUMMER 2016

Mathematics – S1
0983/01

INTRODUCTION

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GCE Mathematics - S1
Summer 2016 Mark Scheme

Ques	Solution	Mark	Notes
1(a)	$P(A\cup B) = P(A) + P(B)$	M1	Award M1 for the **use** of the formulae in all three parts
	$= 0.7$	A1	
(b)	$P(A\cap B) = 0.12$	B1	
	$P(A\cup B) = P(A) + P(B) - P(A\cap B)$	M1	
	$= 0.58$	A1	
(c)	$P(A\cap B) = P(A\mid B)P(B)$	M1	
	$= 0.1$	A1	
	$P(A\cup B) = P(A) + P(B) - P(A\cap B)$	m1	
	$= 0.6$	A1	
2(a)	$P(\text{red}) = 0.45\times 0.03 + 0.55\times 0.05$	M1A1	
	$= 0.041$	A1	
(b)	$P(\text{female}\mid\text{red}) = \dfrac{0.55\times 0.05}{0.041}$	B1B1	B1 num, B1 denom FT denominator from (a)
	$= 0.671$ cao (55/82)	B1	
3(a)	$E(Y) = 2a + b = 8$	M1A1	
	$\text{Var}(Y) = 2a^2 = 8$	M1A1	Award SC2 for correct answer unsupported
	$a = 2$; $b = 4$	A1A1	
(b)	Any statement which mentions that certain values, eg 0, cannot be taken by Y.	B1	
4(a)(i)	$P(\text{no Welsh}) = \dfrac{4}{8}\times\dfrac{3}{7}\times\dfrac{2}{6}$ or $\dfrac{\binom{4}{3}}{\binom{8}{3}}$	M1	
	$= \dfrac{1}{14}$ (0.071)	A1	
(ii)	$P(\text{1 of each}) = \dfrac{4}{8}\times\dfrac{2}{7}\times\dfrac{2}{6}\times 6$ or $\dfrac{\binom{4}{1}\times\binom{2}{1}\times\binom{2}{1}}{\binom{8}{3}}$	M1A1	M1A0 if 6 omitted
	$= \dfrac{2}{7}$ (0.286)	A1	
(b)	$P(\text{Jack selected}) = \dfrac{1}{8}+\dfrac{7}{8}\times\dfrac{1}{7}+\dfrac{7}{8}\times\dfrac{6}{7}\times\dfrac{1}{6}$ or $\dfrac{\binom{7}{2}}{\binom{8}{3}}$	M1	
	$= \dfrac{3}{8}$ (0.375)	A1	Accept answer with no working

Ques	Solution	Mark	Notes
5(a)(i)	X is Poi(6) si $\text{P}(X = 5) = \dfrac{\text{e}^{-6} \times 6^5}{5!}$ $= 0.161$	**B1** **M1** **A1**	Award M0 if no working seen or if tables used
(ii)	$\text{P}(X > 3) = 1 - \text{e}^{-6}\left(1 + 6 + \dfrac{36}{2} + \dfrac{216}{6}\right)$ $= 0.849$	**M1A1** **A1**	Award M1A0A0 if one of the four terms is missing
(b)	Looking at the appropriate section of the table, Mean = 2.4 $t = \dfrac{2.4}{0.2} = 12$	**M1A1** **A1**	Award M1 for evidence of sensible use of table Accept 12 with no working
6(a)(i)	X is B(8,0.12) si $\text{P}(X < 2) = 0.88^8 + 8 \times 0.88^7 \times 0.12$ $= 0.752$	**B1** **M1** **A1**	Award the first M1 in (iii) if not awarded in (i) for adding the six probabilities
(ii)	$\text{P}(X = 2) = 28 \times 0.88^6 \times 0.12^2$ $= 0.187$	**B1**	
(iii)	$\text{P}(X > 2) = 1 - 0.752 - 0.187$ $= 0.061$	**B1**	FT from two other calculated probabilities
(b)	E(Profit) $= 0.187 \times 10 + 0.061 \times 25 - 5$ $= -£1.61$ (Accept 1.6)	**M1** **A1**	M1A0 if – 5 omitted FT from (a) Allow $0.187 \times 5 + 0.061 \times 20 - 0.752 \times 5$
7(a)(i)	$0.3 + 0.2 + 0.1 + a + b = 1$ $a + b = 0.4$	**B1**	.
(ii)	$\text{E}(X) = 0.3 \times 1 + 0.2 \times 2 + 0.1 \times 3 + 4a + 5b = 2.85$ $4a + 5b = 1.85$ Solving, $a = 0.15,\ b = 0.25$	**M1** **A1** **m1** **A1**	
(b)	The possible pairs are (1,1), (1,2), (1,3),(2,2) P= $0.3 \times 0.3 + 2 \times 0.3 \times 0.2 + 2 \times 0.3 \times 0.1 + 0.2 \times 0.2$ $= 0.31$	**B1** **M1A1** **A1**	Award M1A0A0 if one of the terms is missing or if (1,1) or (2,2) is double counted Award SC1 for Prob < 4 (0.21) or Prob = 4 (0.1)

Ques	Solution	Mark	Notes
8(a)	$np = 3$ giving $p = 0.06$	**M1A1**	
(b)	$P(X = 2) = \binom{50}{2} \times 0.06^2 \times 0.94^{48}$	**M1**	
	$= 0.2262$	**A1**	
(c)	Using the Poisson table,		
	$P(X = 2) = 0.4232 - 0.1991$ or $0.8009 - 0.5768$	**M1**	Award M0A0 for 0.2240 from formula
	$= 0.2241$	**A1**	
	Percentage error = $\frac{0.0021}{0.2241} \times 100 < 1\%$	**B1**	Allow 0.2240 for this B1
9(a)(i)	$F(x) = k\int_1^x (2u-1)\mathrm{d}u$	**M1**	M1 for the integral of $f(x)$ limits may be left until 2nd line.
	$= k\left[u^2 - u\right]_1^x$	**A1**	
	$= kx(x-1)$	**A1**	
(ii)	F(2) = 1	**M1**	Allow integration of $f(x)$ from 1 to 2.
	$2k = 1$	**A1**	
	$k = \frac{1}{2}$		
(b)(i)	$E(X) = \int_1^2 \frac{1}{2}x(2x-1)\mathrm{d}x$	**M1**	M1 for the integral of $xf(x)$, limits may be left until 2nd line.
	$= \frac{1}{2}\left[\frac{2x^3}{3} - \frac{x^2}{2}\right]_1^2$	**A1**	
	$= 1.58$ (19/12)	**A1**	
(ii)	The median m satisfies		
	$F(m) = \frac{m(m-1)}{2} = \frac{1}{2}$	**M1**	Accept a geometrical argument FT $F(m)$ from (a) if it gives a quadratic equation and an answer in [1,2]
	$m^2 - m - 1 = 0$	**A1**	
	$m = \frac{1 \pm \sqrt{1+4}}{2}$	**M1**	Condone the absence of ±
	$m = 1.62$	**A1**	
(iii)	$P(X > 1.5) = 1 - F(1.5)$	**M1**	FT F from (a) if possible
	$= 0.625$	**A1**	

GCE MARKING SCHEME

SUMMER 2016

Mathematics – S2
0984/01

INTRODUCTION

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Ques	Solution	Mark	Notes
1(a)	E(W) = 6 $E(X^2) = \text{Var}(X) + [E(X)]^2 = 6$ $E(Y^2) = \text{Var}(Y) + [E(Y)]^2 = 12$ Var(W) = $E(X^2)E(Y^2) - [E(X)E(Y)]^2$ = 36	**B1** **M1A1** **A1** **M1A1**	
(b)	The possibilities are (1,4); (2,2); (4,1) si Pr = $2e^{-2} \times \frac{3^4}{4!}e^{-3} + \frac{2^2}{2!}e^{-2} \times \frac{3^2}{2!}e^{-3} + \frac{2^4}{4!}e^{-2} \times 3e^{-3}$ = 0.12	**B1** **M1A1** **A1**	Award the M1 for multiplying and adding Poisson probabilities. Accept use of tables
2(a)	$\bar{x} = \frac{637.2}{10} = 63.7(2)$ SE of $\bar{x} = \frac{1.9}{\sqrt{10}}$ (0.6008….) 95% confidence interval limits are $63.7(2) \pm 1.96 \times 0.6008...$ giving [62.5,64.9]	**B1** **M1A1** **M1A1** **A1**	M0 no working
(b)	Width of 95% CI = $2 \times 1.96 \times \frac{1.9}{\sqrt{n}} = 1$ n = 55.47… Minimum n = 56 cao	**M1A1** **A1** **A1**	FT their z from (a)
3(a)	Upper quartile = 40 + 0.674(5) × 2.5 = 41.7	**M1** **A1**	M0 no working
(b)(i)	Let X=weight of a male, Y=weight of a female Let $U = X_1 + X_2 + X_3 + Y_1 + Y_2$ E(U) = 3 × 40 + 2 × 32 = 184 Var(U) = 3 × 2.5^2 + 2 × 1.5^2 = 23.25 $z = \frac{185-184}{\sqrt{23.25}} = 0.21$ Prob = 0.4168	**B1** **B1** **M1A1** **A1**	Accept 0.417
(ii)	Let $W = X_1 + X_2 + X_3 - 2(Y_1 + Y_2)$ E(W) = 3 × 40 – 4 × 32 = – 8 Var(W) = 3 × 2.5^2 + 8 × 1.5^2 = 36.75 $z = \frac{8}{\sqrt{36.75}} = 1.32$ Prob = 0.9066	**M1** **A1** **M1A1** **m1A1** **A1**	Accept 0.907

Ques	Solution	Mark	Notes
4(a)	Under H_0, $E(\overline{X}-\overline{Y})=0$	B1	
	$\text{Var}(\overline{X}-\overline{Y})=\frac{1.5^2}{8}+\frac{2.5^2}{12}(=0.802...)$ (77/96)	B1	
	H_1 is accepted if		
	$\frac{\lvert\overline{X}-\overline{Y}\rvert}{\sqrt{0.802...}}>1.645$	M1A1	
	$\lvert\overline{X}-\overline{Y}\rvert>1.473$	A1	Accept 1.47
	So k = 1.473		
(b)(i)			
	Now, $E(\overline{X}-\overline{Y})=0.5$ si	B1	FT k and variance
	H_0 is accepted if $\lvert\overline{X}-\overline{Y}\rvert\le 1.473$, ie	M1	
	$-1.473\le\overline{X}-\overline{Y}\le 1.473$	A1	
	$z_1=\frac{1.473-0.5}{\sqrt{0.802}}=1.09$	M1A1	Accept 1.08
	$z_2=\frac{-1.473-0.5}{\sqrt{0.802}}=-2.20$	A1	
	Required probability = 0.8621 – 0.0139	m1	Accept 0.8599 – 0.0139
	= 0.848	A1	Accept 0.846
(ii)	An appropriate comment, eg The test is unlikely to detect small differences. This is a very high error probability.	B1	FT probabilities greater than 0.5
5(a)(i)	$H_0: p=0.7; H_1: p<0.7$	B1	
(ii)	Let X denote number of seeds which germinate.		
	Under H_0, X is B(50,0.7) si	B1	
	p-value = P($X \le 32$)	B1	
	Let Y denote number of non-germinating seeds.		
	Under H_0, Y is B(50,0.3) si	B1	
	p-value = P($Y \ge 18$)	M1	
	= 0.2178	A1	
	Insufficient evidence to reject the seed manufacturer's claim.	B1	FT the p-value if > 0.05
(b)	Under H_0, X is now B(500,0.7) ≈ N(350,105) si	B1B1	B1 mean, B1 variance
	Test statistic = $\frac{329.5-350}{\sqrt{105}}$	M1A1	Award M1A0 for incorrect or no continuity correction but FT for following marks
	= – 2.00	A1	No cc, z = – 2.05, p = 0.0202
	p-value = 0.0227 or 0.0228	A1	Wrong cc, z = – 2.10, p = 0.0179
	Strong evidence to conclude that the germination rate is less than 0.7	B1	FT the p-value if < 0.05

Ques	Solution	Mark	Notes
6(a)	$P(Y < 8) = P(X > 12)$ $= 0.8$	M1 A1	Award the M1 for stating that Y is uniform on [0,10]
(b)(i)	$Y = 20 - X$	B1	
(ii)	$P(XY > 64) = P[X(20 - X) > 64]$ $= P(X^2 - 20X + 64 < 0)$ The critical values are 4 and 16 OR $P[(X-4)(X-16)] < 0$ The required region is $X < 16$ Prob = 0.6	M1 A1 A1 A1 A1	
(c)	EITHER Prob density of X is $f(x) = 0.1$ $(10 < x < 20)$ si $E(XY) = \int_{10}^{20}(20x - x^2)\times\frac{1}{10}\,dx$ $= \frac{1}{10}\left[10x^2 - \frac{x^3}{3}\right]_{10}^{20}$ $= 66.7$ (200/3) OR $E(XY) = 20E(X) - E(X^2)$ $E(X) = 15$ $E(X^2) = \mathrm{Var}(X) + [E(X)]^2$ $= 100/12 + 225$ (700/3) $E(XY) = 66.7$ (200/3)	B1 M1A1 A1 A1 (M1) (B1) (M1) (A1) (A1)	Limits may be left until the next line

GCE MARKING SCHEME

SUMMER 2016

Mathematics – S3
0985/01

INTRODUCTION

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Ques	**Solution**	**Mark**	**Notes**
1	The sample space and corresponding probabilities are as follows. Sample / Max / Prob: 2,2,2 / 2 / 1/20 2,2,10 / 10 / 6/20 2,2,50 / 50 / 3/20 2,10,10 / 10 / 3/20 2,10,50 / 50 / 6/20 10,10,50 / 50 / 1/20	**B3** **B3**	B3 for correct samples and max B3 for correct probabilities – 1 each error or omission
	$E(M) = 2\times\frac{1}{20} + 10\times\frac{9}{20} + 50\times\frac{10}{20}$ -= 29.6 (p)	**M1** **A1**	
2(a)	$H_0 : \mu = 61; H_1 : \mu < 61$	**B1**	
(b)	$\sum x = 603.4$ si; $\sum x^2 = 36419.5$ UE of μ = 60.34	**B1B1** **B1**	No working need be seen
	UE of $\sigma^2 = \frac{36419.5}{9} - \frac{603.4^2}{90}$ = 1.149 (431/375)	**M1** **A1**	M0 division by 10 Answer only no marks
(c)	$\text{Test stat} = \frac{60.34 - 61}{\sqrt{\frac{1.149}{10}}}$ = – 1.947	**M1A1** **A1**	M0 for no working Note that p-value = 0.0417
	DF = 9 si Crit t value = 1.833	**B1** **B1**	
	This result suggests that we should reject H_0, ie that the average miles per gallon is less than 61 because 1.947 > 1.833 oe	**B1** **B1**	FT the conclusion No FT for reason if z-value used

Ques	Solution	Mark	Notes
3(a)	$\hat{p} = \frac{44}{80} = 0.55$ si	**B1**	
	ESE = $\sqrt{\frac{0.55 \times 0.45}{80}}$ (= 0.0556..) si	**M1A1**	M1A0 if √ omitted
	90% confidence limits are $0.55 \pm 1.645 \times 0.0556..$	**M1A1**	M1 correct form, A1 correct z
	giving [0.459,0.641]	**A1**	
(b)(i)	$\hat{q} = \frac{0.555 + 0.705}{2} = 0.63$	**B1**	
	Games won = 0.63 × 100 = 63	**B1**	
(ii)	$0.705 - 0.555 = 2 \times z\sqrt{\frac{0.63 \times 0.37}{100}}$ or equiv	**M1A1**	
	z = 1.55	**A1**	
	Prob from tables = 0.0606 (0.9394)	**A1**	
	Confidence level = 88%	**A1**	
4(a)	$H_0 : \mu_A = \mu_B; H_1 : \mu_A \neq \mu_B$	**B1**	
(b)	$\bar{x} = 251.6; \bar{y} = 251.4$ or $\bar{x} - \bar{y} = 0.2$	**B1**	
	$s_x^2 = \frac{5064256}{79} - \frac{20128^2}{79 \times 80} = 0.648...(256/395)$	**M1A1**	
	$s_y^2 = \frac{5056222}{79} - \frac{20112^2}{79 \times 80} = 0.825...(326/395)$	**A1**	
	[Accept division by 80 giving 0.64 and 0.815..]		
	SE = $\sqrt{\frac{0.648..}{80} + \frac{0.825..}{80}}$ = 0.135.. (0.1348…)	**M1A1**	
	$z = \frac{251.6 - 251.4}{0.135..}$	**m1**	
	=1.47 or 1.48	**A1**	
	Prob from tables = 0.071or 0.069	**A1**	
	p-value = 0.14	**B1**	FT from line above
	Insufficient evidence to reject H_0	**B1**	FT the p-value
(c)	The CLT allows us to assume that the distributions of the sample means are (approximately) normal	**B1**	

Ques	Solution	Mark	Notes
5(a)	$\sum x = 210, \sum x^2 = 9100,$ $\sum y = 1286, \sum xy = 48730$	B2	Minus 1 each error
	$S_{xy} = 48730 - 210 \times 1286/6 = 3720$	B1	
	$S_{xx} = 9100 - 210^2/6 = 1750$	B1	
	$b = \dfrac{3720}{1750} = 2.13$ (372/175)	M1A1	M0 no working
	$a = \dfrac{1286 - 2.13 \times 210}{6} = 140$ (2099/15)	M1A1	M0 no working
(b)(i)	SE of $b = \dfrac{1.5}{\sqrt{1750}}$ (0.0358..)	M1A1	FT from (a)
	95% confidence limits are 2.1257.. ± 1.96 × 0.0358.. [2.06, 2.20]	m1A1 A1	
(ii)	$x_0 = 35$	B1	
	Because the SE of y or the width of the interval is minimum when $x_0 = \bar{x}$	B1	

Ques	Solution	Mark	Notes
6(a)(i)	$E(\overline{X}) = \dfrac{\sum_{i=1}^{n} E(X_i)}{n}$	M1	
	$= \dfrac{n\mu}{n} = \mu$	A1	
	(Therefore $\overline{X}$ is an unbiased estimator)		
(ii)	$\text{Var}(\overline{X}) = \dfrac{\sum_{i=1}^{n} \text{Var}(X_i)}{n^2}$	M1	
	$= \dfrac{n\sigma^2}{n^2} = \dfrac{\sigma^2}{n}$	A1	
	SE of $\overline{X} = \dfrac{\sigma}{\sqrt{n}}$		
(b)(i)	$\text{Var}(X_i) = E(X_i^{\,2}) - [E(X_i)]^2$	M1	
	$\sigma^2 = E(X_i^{\,2}) - \mu^2$	A1	
	$E(X_i^{\,2}) = \mu^2 + \sigma^2$		
(ii)	$E(S^2) = \dfrac{\sum_{i=1}^{n} E(X_i^{\,2}) - nE(\overline{X}^2)}{n-1}$	M1	
	$= \dfrac{n(\mu^2 + \sigma^2) - n\left(\mu^2 + \dfrac{\sigma^2}{n}\right)}{n-1}$	A1A1	
	$= \sigma^2$		
(c)	$\text{Var}(S) = E(S^2) - [E(S)]^2$	M1	
	$[E(S)]^2 = \sigma^2 - \text{Var}(S)$	M1	
	$< \sigma^2$ (since $\text{Var}(S) > 0$)	A1	
	Therefore $E(S) < \sigma$ so $E(S) \neq \sigma$	A1	FT above line if both M marks awarded
	(Therefore S is not an unbiased estimator for σ)		