GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C1
0973/01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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Mathematics C1 May 2017

Solutions and Mark Scheme

1.	(*a*)	(i)	Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$	M1
			Gradient of $AB = {}^1/_3$ (or equivalent)	A1
		(ii)	Use of gradient $L_1 \times$ gradient $AB = -1$ (or equivalent)	M1
			A correct method for finding the equation of L_1 using candidate's gradient for L_1	M1
			Equation of L_1: $y - 5 = -3(x - 4)$ (or equivalent) (f.t. candidate's gradient for AB provided that both the 3rd and 4th marks (M1, M1) have been awarded)	A1
	(*b*)	(i)	An attempt to solve equations of L_1 and L_2 simultaneously	M1
			$x = 7$, $y = -4$ (convincing)	A1
		(ii)	A correct method for finding the length of $AC(BC)$	M1
			$AC = \sqrt{130}$	A1
			$BC = \sqrt{90}$	A1
			$\text{Cos } BCA = \dfrac{BC}{CA} = \dfrac{\sqrt{90}}{\sqrt{130}}$ (f.t. candidate's derived values for AC and BC)	M1
			$\text{Cos } BCA = \dfrac{3}{\sqrt{13}}$ (c.a.o.)	A1
	(*c*)	(i)	A correct method for finding D	M1
			$D(1, 14)$	A1
		(ii)	Isosceles	E1

2.	(*a*)	$\dfrac{5\sqrt{5} - 9}{3 + 2\sqrt{5}} = \dfrac{(5\sqrt{5} - 9)(3 - 2\sqrt{5})}{(3 + 2\sqrt{5})(3 - 2\sqrt{5})}$	M1
		Numerator: $15 \times \sqrt{5} - 10 \times 5 - 9 \times 3 + 18 \times \sqrt{5}$	A1
		Denominator: $9 - 20$	A1
		$\dfrac{5\sqrt{5} - 9}{3 + 2\sqrt{5}} = 7 - 3\sqrt{5}$ (c.a.o.)	A1
		Special case If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $3 + 2\sqrt{5}$	
	(*b*)	$(2\sqrt{13})^2 = 52$	B1
		$3\sqrt{7} \times \sqrt{28} = 42$	B1
		$\dfrac{5\sqrt{99}}{\sqrt{11}} = 15$	B1
		$(2\sqrt{13})^2 - 3\sqrt{7} \times \sqrt{28} - \dfrac{5\sqrt{99}}{11} = -5$ (c.a.o.)	B1

3. (*a*) $\frac{dy}{dx} = \frac{3}{2}x - 4$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 6$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 5$ (c.a.o.) A1

Equation of tangent at P: $y - (-7) = 5(x - 6)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided M1 and m1 both awarded) A1

(*b*) Use of gradient of tangent = $\frac{-1}{\text{gradient of normal}}$ (o.e.) M1

An attempt to put candidate's expression for $\frac{dy}{dx} = \frac{1}{2}$

(f.t candidate's derived value for gradient of tangent) m1

x-coordinate of $Q = 3$ (c.a.o.) A1

4. (*a*) $a = -2$ B1

$b = 5$ B1

$c = 85$ B1

(*b*) Stationary value = 85 (f.t. candidate's value for c) B1

This is a maximum B1

5. (*a*) $\left(x + \frac{2}{x}\right)^4 = x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$

(4 or 5 terms correct) B2

(**If B2 not awarded**, award B1 for 3 correct terms)

$\left(x + \frac{2}{x}\right)^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (5 terms correct) B2

(**If B2 not awarded**, award B1 for 3 or 4 correct terms)

(– 1 for further incorrect simplification)

(*b*) Coefficient of $x = {}^6C_1 \times a^5 \times 2(x)$ B1

Coefficient of $x^2 = {}^6C_2 \times a^4 \times 2^2(x^2)$ B1

$15 \times a^4 \times m = 6 \times a^5 \times 2$ (m = **4** or 2) M1

$a = 5$ (c.a.o.) A1

6. Finding critical values $x = -{}^3/_2$, $x = -4$ B1

A statement (mathematical or otherwise) to the effect that $x \le -4$ or $-{}^3/_2 \le x$

(or equivalent, f.t. candidate's derived critical values) B2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*) Use of $f(2) = 0$ M1

$8k + 8 - 82 + 10 = 0 \Rightarrow k = 8$ (convincing) A1

Special case

Candidates who assume $k = 8$ and then either show that $f(2) = 0$ or that $x - 2$ is a factor by long division are awarded B1

(*b*) $f(x) = (x - 2)(8x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x - 2)(8x^2 + 18x - 5)$ A1

$f(x) = (x - 2)(4x - 1)(2x + 5)$ (f.t. only $8x^2 - 18x - 5$ in above line) A1

Special case

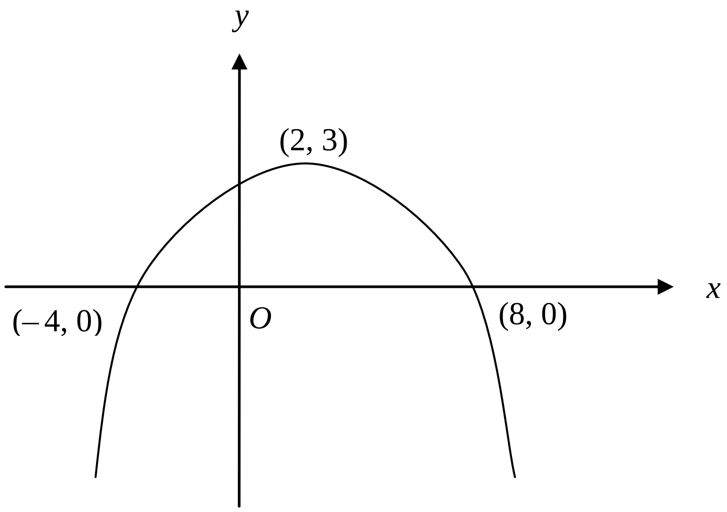
Candidates who find one of the remaining factors, $(4x - 1)$ or $(2x + 5)$, using e.g. factor theorem, are awarded B1

(*c*) Attempting to find $f(-1/2)$ M1

Remainder = 30 A1

If a candidate tries to solve (*c*) by using the answer to part (*b*), f.t. for M1 and A1 when candidate's expression is of the form $(x - 2) \times$ two linear factors

8. (*a*)



Concave down curve with maximum at (2, *a*)	B1
Maximum at (2, 3)	B1
Both points of intersection with *x*-axis	B1

(*b*) The stationary point will always be a minimum — E1

The *y*-coordinate of the stationary point will always be – 6 — E1

9. (*a*)

$y + \delta y = -5(x + \delta x)^2 - 7(x + \delta x) + 13$		B1
Subtracting y from above to find δy		M1
$\delta y = -10x\delta x - 5(\delta x)^2 - 7\delta x$		A1
Dividing by δx and letting $\delta x \to 0$		M1
$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -10x - 7$	(c.a.o.)	A1

(*b*) $\frac{dy}{dx} = 6 \times \frac{3}{4} \times x^{-1/4} + 5 \times -3 \times x^{-4}$ (completely correct answer) B2

(**If B2 not awarded**, award B1 for at least one correct non-zero term)

10. (*a*) (i) $\dfrac{dy}{dx} = 3x^2 - 18x + 15$ B1

Putting candidate's derived $\dfrac{dy}{dx} = 0$ M1

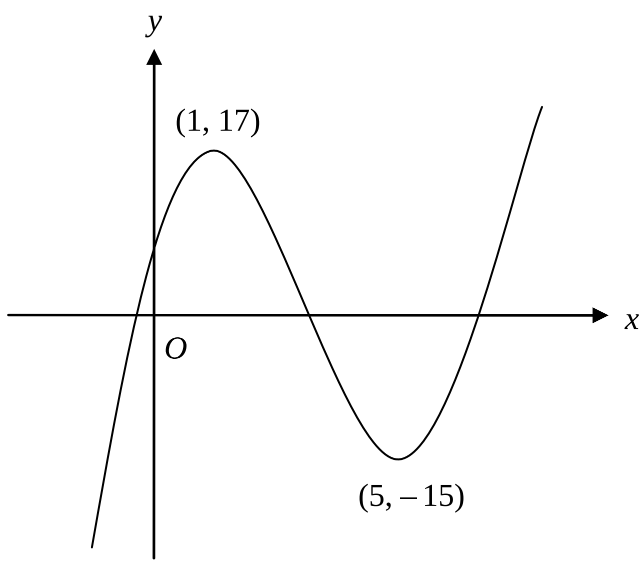
$x = 1, 5$ (both correct) (f.t. candidate's $\dfrac{dy}{dx}$) A1

Stationary points are (1, 17) and (5, – 15)
(both correct) (c.a.o) A1

(ii) A correct method for finding nature of stationary points yielding
either (1, 17) is a maximum point
or (5, – 15) is a minimum point
(f.t. candidate's derived values) M1

Correct conclusion for other point
(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1
Correct marking of both stationary points
(f.t. candidate's derived maximum and minimum points) A1

(*c*) Use of both $k = -15$, $k = 17$ to find the range of values for k
(f.t. candidate's y-values at stationary points) M1

$k < -15$ or $17 < k$ (f.t. candidate's y-values at stationary points) A1

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C2
0974/01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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Mathematics C2 May 2017

Solutions and Mark Scheme

1.

0	2·645751311		
0·5	2·598076211		
1	2·449489743		
1·5	2·179449472		
2	1·732050808	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$$I \approx \frac{0·5}{2} \times \{2·645751311 + 1·732050808 + 2(2·598076211 + 2·449489743 + 2·179449472)\}$$

$I \approx 18·83183297 \times 0·5 \div 2$

$I \approx 4·707958243$

$I \approx 4·708$ (f.t. one slip) A1

Special case for candidates who put $h = 0·4$

0	2·645751311		
0·4	2·615339366		
0·8	2·521904043		
1·2	2·357965225		
1·6	2·107130751		
2	1·732050808	(all values correct)	(B1)

Correct formula with $h = 0·4$ (M1)

$$I \approx \frac{0·4}{2} \times \{2·645751311 + 1·732050808 + 2(2·615339366 + 2·521904043 + 2·357965225 + 2·107130751)\}$$

$I \approx 23·58248089 \times 0·4 \div 2$

$I \approx 4·716496178$

$I \approx 4·716$ (f.t. one slip) (A1)

Note: Answer only with no working shown earns 0 marks

2. (*a*) $\sin^2\theta + 6(1 - \sin^2\theta) + 13\sin\theta = 0,$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$5\sin^2\theta - 13\sin\theta - 6 = 0 \Rightarrow (5\sin\theta + 2)(\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\frac{2}{5}$, ($\sin\theta = 3$) (c.a.o.) A1

$\theta = 203{\cdot}58°, 336{\cdot}42°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $5\sin\theta + 2 = 0$, ignore roots outside range.
$\sin\theta = -$, f.t. for 2 marks, $\sin\theta = +$, f.t. for 1 mark

(*b*) $A = 110°$ B1

$B - C = 22°$ B1

$110° + B + C = 180°$ (f.t. candidate's value for A) M1

$B = 46°, C = 24°$ (f.t. one error) A1

3. (*a*) $(2x + 1)^2 = x^2 + (x + 5)^2 - 2 \times x \times (x + 5) \times \cos 60°$ (o.e.)

(correct use of cos rule) M1

$3x^2 - x - 24 = 0$ (convincing) A1

An attempt to solve the given quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c = 3$ and $b \times d = -24$ M1

$(3x + 8)(x - 3) = 0 \Rightarrow x = 3$ A1

(*b*) $\frac{\sin ACB}{3} = \frac{\sin 60°}{7}$

(substituting the correct values in the correct places in the sin rule) M1

$ACB = 21{\cdot}8°$ A1

(Allow ft for x>0 obtained in (a) for M1A1)

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + a$

In order to make further progress, the two expressions for S_n must contain at least three pairs of terms, including the first pair, the last pair and one other pair of terms

Either:

$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$

Or:

$2S_n = [a + a + (n-1)d]$ n times M1

$2S_n = n[2a + (n-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $\frac{8}{2} \times (2a + 7d) = 156$ B1

$2a + 7d = 39$

$\frac{16}{2} \times (2a + 15d) = 760$ B1

$2a + 15d = 95$

An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1

$d = 7,\ a = -5$ (c.a.o.) A1

(*c*) $d = 9$ B1

A correct method for finding $(p + 8)$ th term M1

$(p + 8)$ th term $= 2129$ (c.a.o.) A1

5. (*a*) $a = 100,\ r = 1{\cdot}2$

Value of donation in 12th year $= 100 \times 1{\cdot}2^{11}$ M1

Value of donation in 12th year = £743 A1

(*b*) $100 \times \frac{(1 - 1{\cdot}2^n)}{1 - 1{\cdot}2} = 15474$ M1

$1 - 1{\cdot}2^n = 154{\cdot}74 \times (-0{\cdot}2)$ m1

$1{\cdot}2^n = 31{\cdot}948$ A1

$n = \frac{\log 31{\cdot}948}{\log 1{\cdot}2}$ m1

$n = 19$ cao A1

6. (*a*) $2 \times \frac{x^{-4}}{-4} - 6 \times \frac{x^{7/4}}{7/4} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $16 - a^2 = 0 \Rightarrow -4$ B1

(ii) $\frac{dy}{dx} = -2x$ M1

Gradient of tangent = 8 (f.t. candidate's value for *a*) A1

$b = 32$ (convincing) A1

(iii) Use of integration to find the area under the curve M1

$\int (16 - x^2)\,dx = 16x - (1/3)x^3$ (correct integration) A1

Correct method of substitution of candidate's limits m1

$$[16x - (1/3)x^3]_{-4}^{0} = 0 - [-64 - (-64/3)] = 128/3$$

Area of the triangle = 64 (f.t. candidate's value for *a*) B1

Use of candidate's value for *a* and 0 as limits and trying to find total area by subtracting area under curve from area of triangle m1

Shaded area = 64 – 128/3 = 64/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$\frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}$ (the laws of indicies) B1

$\log_a x/y = p - q$ (the relationship between log and power)

$\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) $\frac{1}{3}\log_b x^{15} = \log_b x^5$, $4\log_b 3/x = \log_b 3^4/x^4$

(one correct use of power law) B1

$\frac{1}{3}\log_b x^{15} - \log_b 27x + 4\log_b 3/x = \log_b \frac{x^5 \times 3^4}{27x \times x^4}$ (addition law) B1

(subtraction law) B1

$\frac{1}{3}\log_b x^{15} - \log_b 27x + 4\log_b 3/x = \log_b 3$ (c.a.o.) B1

(*c*) $\log_d 5 = \frac{1}{3} \Rightarrow 5 = d^{1/3}$

(rewriting log equation as power equation) M1

$d = 125$ A1

8. (*a*) (i) $A(-5, 4)$ B1

A correct method for finding radius M1

Radius = $\sqrt{20}$ A1

(ii) **Either:**

A correct method for finding AP^2 M1

$AP^2 = 25\ (> 20) \Rightarrow P$ is outside C

(f.t. candidate's coordinates for A) A1

Or:

An attempt to substitute $x = -2$, $y = 0$ in the equation of C (M1)

$(-2)^2 + 0^2 + 10 \times (-2) - 8 \times 0 + 21 = 5\ (> 0)$

$\Rightarrow P$ is outside C (A1)

(*b*) An attempt to substitute $(2x + 4)$ for y in the equation of the circle

M1

$5x^2 + 10x + 5 = 0$ A1

Either: Use of $b^2 - 4ac$ m1

Discriminant = 0, $\Rightarrow y = 2x + 4$ is a tangent to the circle

A1

$x = -1$, $y = 2$ A1

Or: An attempt to factorise candidate's quadratic (m1)

Repeated (single) root, $\Rightarrow y = 2x + 4$ is a tangent to the circle (A1)

$x = -1$, $y = 2$ (A1)

9. (*a*) (i) $L = R\theta + r\theta$ B1

(ii) $K = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$ B1

(*b*) $K = \frac{1}{2}\theta(R + r)(R - r)$ M1

$L = \theta(R + r)$, $R - r = x$ (both expressions) m1

$K = \frac{1}{2}Lx$ A1

<u>Alternative solution</u>

$K = \dfrac{1}{2}\theta(R^2 - r^2)$

$K = \dfrac{1}{2}\theta((r+x)^2 - r^2)$ (M1)

$K = \dfrac{1}{2}\theta(2rx + x^2)$

$K = \dfrac{1}{2}x\theta(2r + x)$ (m1)

$K = \dfrac{1}{2}x\theta(R + r)$

$K = \dfrac{1}{2}Lx$ (A1)

10.	(*a*)	$t_3 = 67$		B1
		$t_1 = 7$	(f.t. candidate's value for t_3)	B1
	(*b*)	29999999 is of the form $3k - 1$ (not $3k + 1$)	(o.e.)	
		OR		
		The number does not end in a 2 or a 7		E1

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C3
0975/01

INTRODUCTION

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Mathematics C3 June 2017

Solutions and Mark Scheme

1. (*a*)

5	3·258096538		
5·5	3·442019376		
6	3·610917913		
6·5	3·766997233		
7	3·912023005	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}5$ M1

$I \approx \frac{0{\cdot}5}{3} \times \{3{\cdot}258096538 + 3{\cdot}912023005 + 4(3{\cdot}442019376 + 3{\cdot}766997233) + 2(3{\cdot}610917913)\}$

$I \approx 43{\cdot}22802181 \times 0{\cdot}5 \div 3$

$I \approx 7{\cdot}204670301$

$I \approx 7{\cdot}2$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*)

$\int_5^7 \ln\left(\frac{3}{\sqrt{(1 + x^2)}}\right) dx = \int_5^7 \ln 3 \, dx - \frac{1}{2}\int_5^7 \ln(1 + x^2)\, dx$ M1

$\frac{1}{2}\int_5^7 \ln(1 + x^2)\, dx \approx 3{\cdot}6$ (f.t. candidate's answer to (*a*)) B1

$\int_5^7 \ln\left(\frac{3}{\sqrt{(1 + x^2)}}\right) dx \approx 2{\cdot}2 - 3{\cdot}6 = -1{\cdot}4$ (f.t. candidate's answer to (*a*)) A1

2. (*a*) $6(\sec^2\theta - 1) - 6 = 4\sec^2\theta + 5\sec\theta$.

(correct use of $\tan^2\theta = \sec^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant m1

$2\sec^2\theta - 5\sec\theta - 12 = 0 \Rightarrow (2\sec\theta + 3)(\sec\theta - 4) = 0$

$\Rightarrow \sec\theta = -\frac{3}{2}, \sec\theta = 4$

$\Rightarrow \cos\theta = -\frac{2}{3}, \cos\theta = \frac{1}{4}$ (c.a.o.) A1

$\theta = 131{\cdot}81°, 228{\cdot}19°$ B1 B1

$\theta = 75{\cdot}52°, 284{\cdot}48°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) Correct use of $\sec\phi = \frac{1}{\cos\phi}$ and $\tan\phi = \frac{\sin\phi}{\cos\phi}$ (o.e.) M1

$\sin\phi = -\frac{3}{5}$ A1

$\phi = 323{\cdot}13°, 216{\cdot}87°$ (f.t. for $\sin\phi = -a$) A1

3. (*a*) $\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$\frac{d}{dx}(-3x^2y) = -3x^2\frac{dy}{dx} - 6xy$ B1

$\frac{d}{dx}(x^4) = 4x^3$, $\frac{d}{dx}(-4x) = -4$, $\frac{d}{dx}(7) = 0$ B1

$\frac{dy}{dx} = \frac{4 - 4x^3 + 6xy}{6y^2 - 3x^2}$ (o.e.) (c.a.o.) B1

(*b*) (i) candidate's *x*-derivative = $7 + 4t$ B1

candidate's *y*-derivative = $\frac{(7 + 4t)r - (4 + 3t)m}{(7 + 4t)^2}$,

where *r*, *m* are integers M1

candidate's *y*-derivative = $\frac{(7 + 4t)3 - (4 + 3t)4}{(7 + 4t)^2}$ A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{5}{(7 + 4t)^3}$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{-3 \times 5 \times 4}{(7 + 4t)^4}$ (o.e.)

(f.t. candidate's expression of correct given form for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$

(f.t. candidate's expression for $\frac{d}{dt}\left(\frac{dy}{dx}\right)$) M1

$\frac{d^2y}{dx^2} = \frac{-60}{(7 + 4t)^5}$ (c.a.o.) A1

4. (*a*) (i) $V(x) = 150 \Rightarrow x \times (x + 4) \times (x - 2) = 150$ M1

$x^3 + 2x^2 - 8x - 150 = 0$ (convincing) A1

(ii) Let $f(x) = x^3 + 2x^2 - 8x - 150$

Use of a correct method to find $f(x)$ when $x = 5$ and $x = 6$ M1

$f(5) = -15\ (< 0)$, $f(6) = 90\ (> 0)$

Change of sign $\Rightarrow 5 < x < 6$ A1

(*b*) $x_0 = 6$

$x_1 = 5{\cdot}013297935$ (x_1 correct, at least 2 places after the point) B1

$x_2 = 5{\cdot}190516135$

$x_3 = 5{\cdot}163166906$

$x_4 = 5{\cdot}167508826 = 5{\cdot}17$ (x_4 correct to 2 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 5{\cdot}165$ and $x = 5{\cdot}175$ M1

$f(5{\cdot}165) = -0{\cdot}178\ (< 0)$, $f(5{\cdot}175) = 0{\cdot}751\ (> 0)$ A1

Change of sign $\Rightarrow x = 5{\cdot}17$ correct to two decimal places A1

5. (*a*) (i) $\frac{dy}{dx} = \frac{1}{2} \times (3x^2 + 5x)^{-1/2} \times f(x)$ ($f(x) \neq 1$) M1

$\frac{dy}{dx} = \frac{1}{2} \times (3x^2 + 5x)^{-1/2} \times (6x + 5)$ A1

(ii) $\frac{dy}{dx} = \boldsymbol{\frac{3}{\sqrt{(1 - (3x)^2)}}}$ **or** $\frac{1}{\sqrt{(1 - (3x)^2)}}$ **or** $\frac{3}{\sqrt{(1 - 3x^2)}}$ M1

$\frac{dy}{dx} = \frac{3}{\sqrt{(1 - 9x^2)}}$ A1

(*b*) $x = \cot y \Rightarrow \frac{dx}{dy} = -\operatorname{cosec}^2 y$ B1

$\frac{dx}{dy} = -(1 + \cot^2 y)$ B1

$\frac{dx}{dy} = -(1 + x^2)$ B1

$\frac{dy}{dx} = -\frac{1}{1 + x^2}$ (c.a.o.) B1

6. (*a*) (i) $\int 8e^{2-5x}\,dx = k \times 8 \times e^{2-5x} + c$ $\quad (k = 1, -5, {}^{1}/_{5}, -{}^{\mathbf{1}}/_{\mathbf{5}})$ M1

$\int 8e^{2-5x}\,dx = -\frac{8}{5} \times e^{2-5x} + c$ A1

(ii) $\int 6(4x-7)^{-1/3} = \frac{6 \times k \times (4x-7)^{2/3}}{2/3} + c$ $\quad (k = 1, 4, {}^{\mathbf{1}}/_{\mathbf{4}})$ M1

$\int 6(4x-7)^{-1/3} = \frac{6 \times 1/4 \times (4x-7)^{2/3}}{2/3} + c$

$\int 6(4x-7)^{-1/3} = \frac{9}{4} \times (4x-7)^{2/3} + c$ A1

(iii) $\int \cos\left(\frac{7x-9}{3}\right) dx = k \times \sin\left(\frac{7x-9}{3}\right) + c$ $\quad (k = 1, {}^{7}/_{3}, {}^{\mathbf{3}}/_{\mathbf{7}}, -{}^{3}/_{7}, {}^{1}/_{7})$ M1

$\int \cos\left(\frac{7x-9}{3}\right) dx = \frac{3}{7} \times \sin\left(\frac{7x-9}{3}\right) + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) (i) $\frac{dy}{dx} = \frac{a + bx}{3x^2 - 8}$ (including $a = 1$, $b = 0$) M1

$\frac{dy}{dx} = \frac{6x}{3x^2 - 8}$ A1

(ii) $\int_2^6 \frac{3x}{3x^2-8}\,dx = r\left[\ln(3x^2-8)\right]_2^6$

where r is a constant M1

$\int_2^6 \frac{3x}{3x^2-8}\,dx = \frac{1}{2}\left[\ln(3x^2-8)\right]_2^6$ A1

$\int_2^6 \frac{3x}{3x^2-8}\,dx = r\{\ln(108-8)) - \ln(12-8)\}$ m1

$\int_2^6 \frac{3x}{3x^2-8}\,dx = \ln(5)$ (c.a.o.) A1

7. (*a*) Choice of negative x M1

Correct verification that L.H.S. of inequality > 5 and a statement to the effect that this is in fact the case A1

(*b*) $a = -\frac{1}{2}$ B1

$b = -6$ B1

8. (*a*) $y - 2 = \frac{3}{\sqrt{(5x-4)}}$ B1

An attempt to isolate $5x - 4$ by crossmultiplying and squaring M1

$x = \frac{1}{5}\left(4 + \frac{9}{(y-2)^2}\right)$ (c.a.o.) A1

$f^{-1}(x) = \frac{1}{5}\left(4 + \frac{9}{(x-2)^2}\right)$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = (2, 2{\cdot}5]$ B1 B1

9. (*a*) $R(f) = [8 + k, \infty)$ B1

(*b*) $8 + k \geq -3$ M1

$k \geq -11$ ($\Rightarrow$ least value of k is -11)

(f.t. candidate's $R(f)$ provided it is of form $[a, \infty)$) A1

(*c*) (i) $gf(x) = (4x + k)^2 - 9$ B1

(ii) $(4 \times 2 + k)^2 - 9 = 7$

(substituting 2 for x in candidate's expression for $gf(x)$ and putting equal to 7) M1

Either $k^2 + 16k + 48 = 0$ or $(8 + k)^2 = 16$ (c.a.o.) A1

$k = -4, -12$ (f.t. candidate's quadratic in k) A1

$k = -4$ (c.a.o.) A1

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C4
0976/01

INTRODUCTION

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Mathematics C4 June 2017

Solutions and Mark Scheme

1. (*a*) $f(x) \equiv \dfrac{A}{(x-1)^2} + \dfrac{B}{(x-1)} + \dfrac{C}{(x+4)}$ (correct form) M1

$8x^2 + 7x - 25 \equiv A(x+4) + B(x-1)(x+4) + C(x-1)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -2,\ C = 3,\ B = 5$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\dfrac{9x^2 + 5x - 24}{(x-1)^2(x+4)} = \dfrac{8x^2 + 7x - 25}{(x-1)^2(x+4)} + \dfrac{x^2 - 2x + 1}{(x-1)^2(x+4)}$ M1

$\dfrac{x^2 - 2x + 1}{(x-1)^2(x+4)} = \dfrac{1}{x+4}$ A1

$\dfrac{9x^2 + 5x - 24}{(x-1)^2(x+4)} = \dfrac{-2}{(x-1)^2} + \dfrac{5}{(x-1)} + \dfrac{4}{(x+4)}$

(f.t. candidate's values for *A*, *B*, *C*) A1

2. (*a*) $6y^5\dfrac{dy}{dx} - 12x^3 - 9x^2\dfrac{dy}{dx} - 18xy = 0$

$\left(6y^5\dfrac{dy}{dx} - 12x^3\right)$ B1

$\left(-9x^2\dfrac{dy}{dx} - 18xy\right)$ B1

$\dfrac{dy}{dx} = \dfrac{6xy + 4x^3}{2y^5 - 3x^2}$

(convincing i.e intermediary line required) B1

(*b*) $y = 0 \Rightarrow x = 2$ or $x = -2$ B1

At $(2, 0)$, $\dfrac{dy}{dx} = -\dfrac{8}{3}$ B1

At $(-2, 0)$, $\dfrac{dy}{dx} = \dfrac{8}{3}$ B1

3. (*a*) $5\cos^2\theta + 7 \times 2\sin\theta\cos\theta = 3\sin^2\theta$

(correct use of $\sin 2\theta = 2\sin\theta\cos\theta$) M1

An attempt to form a quadratic equation in $\tan\theta$ by dividing throughout by $\cos^2\theta$ and then using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ m1

$3\tan^2\theta - 14\tan\theta - 5 = 0$ (c.a.o.) A1

$\tan\theta = -\dfrac{1}{3}$, $\tan\theta = 5$ (c.a.o.) A1

$\theta = 161{\cdot}57°$ B1

$\theta = 78{\cdot}69°$ B1

Note: F.t. candidate's derived quadratic equation in $\tan\theta$.
Do not award the corresponding B1 if the candidate gives more than one root in that particular branch. Ignore roots outside range.

(*b*) (i) $R = 4$ B1

Correctly expanding $\cos(\phi - \alpha)$ and using either $4\cos\alpha = \sqrt{5}$ **or** $4\sin\alpha = \sqrt{11}$ **or** $\tan\alpha = \dfrac{\sqrt{11}}{\sqrt{5}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 56°$ (c.a.o) A1

(ii) Least value of $\dfrac{1}{\sqrt{5}\cos\phi + \sqrt{11}\sin\phi + 6} = \dfrac{1}{4 \times k + 6}$.

($k = \mathbf{1}$ or -1)

(f.t. candidate's value for R) M1

Least value $= \dfrac{1}{10}$ (f.t. candidate's value for R) A1

Corresponding value for $\phi = 56°$ (o.e.)

(f.t. candidate's value for α) A1

4.

Volume = $\pi\int_{\pi/6}^{\pi/3}(\cos x + \sec x)^2\,dx$ B1

Correct use of $\cos^2 x = \dfrac{(1 + \cos 2x)}{2}$ M1

Integrand = $\dfrac{(1 + \cos 2x)}{2} + 2 + \sec^2 x$ (c.a.o.) A1

$\int a\cos 2x\,dx = \dfrac{a}{2}\sin 2x$ $(a \neq 0)$ B1

$\int b\,dx = bx$ and $\int \sec^2 x\,dx = \tan x$ $(b \neq 0)$ B1

Correct substitution of correct limits in candidate's integrated expression of the form

$px + q\sin 2x + \tan x$ $(p \neq 0, q \neq 0)$ M1

Volume = $\pi \times (4{\cdot}566551037 - 2{\cdot}102853559) = 7{\cdot}74$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. (*a*) $(1 + 4x)^{-1/2} = 1 - 2x + 6x^2 + \ldots$ $(1 - 2x)$ B1

$(6x^2)$ B1

$|x| < \tfrac{1}{4}$ or $-\tfrac{1}{4} < x < \tfrac{1}{4}$ B1

(*b*) $1 + 4y + 8y^2 = 1 + 4(y + 2y^2)$ M1

$(1 + 4y + 8y^2)^{-1/2} = 1 - 2(y + 2y^2) + 6(y + 2y^2)^2 + \ldots$

(f.t. candidate's expression from part (*a*)) m1

$(1 + 4y + 8y^2)^{-1/2} = 1 - 2y + 2y^2 + \ldots$

(f.t. candidate's expression from part (*a*)) A1

6. (*a*) candidate's x-derivative $= 2at$

candidate's y-derivative $= 3bt^2$ (at least one term correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{3bt}{2a}$ (o.e.) (c.a.o.) A1

Equation of tangent at P: $y - bp^3 = \frac{3bp}{2a}(x - ap^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$2ay = 3bpx - abp^3$ (convincing) A1

(*b*) Substituting $4a$ for x and $8b$ for y in equation of tangent M1

$16ab = 12abp - abp^3 \Rightarrow p^3 - 12p + 16 = 0$ (convincing) A1

$(p - 2)(p^2 + 2p - 8) = 0$ M1

$(p - 2)(p - 2)(p + 4) = 0$ A1

$p = 2$ corresponds to $(4a, 8b) \Rightarrow p = -4$ (c.a.o.) A1

7. (*a*) $u = \ln x \Rightarrow du = \frac{1}{x}dx$ B1

$dv = x^{-4}\,dx \Rightarrow v = \frac{1}{-3}x^{-3}$ (o.e.) B1

$\int \frac{\ln x}{x^4}dx = \frac{1}{-3}x^{-3} \times \ln x - \int \frac{1}{-3}x^{-3} \times \frac{1}{x}dx$ (o.e.) M1

$\int \frac{\ln x}{x^4}dx = \frac{1}{-3}x^{-3} \times \ln x - \frac{1}{9}x^{-3} + c$ (c.a.o.) A1

(*b*) $\int x^3(x^2 + 1)^4\,dx = \int f(u) \times u^4 \times du$ $(f(u) = pu + q, p \neq 0, q \neq 0)$ M1

$\int x^3(x^2 + 1)^4\,dx = \int \frac{(u - 1)}{2} \times u^4 \times du$ A1

$\int (pu^5 + qu^4)\,du = \frac{pu^6}{6} + \frac{qu^5}{5}$ B1

Either: Correctly inserting limits of 1, 2 in candidate's $\frac{pu^6}{6} + \frac{qu^5}{5}$

or: Correctly inserting limits of 0, 1 in candidate's

$\frac{p(x^2 + 1)^6}{6} + \frac{q(x^2 + 1)^5}{5}$ m1

$\int_0^1 x^3(x^2 + 1)^4\,dx = \frac{43}{20} = 2{\cdot}15$ (c.a.o.) A1

8. (*a*) $\frac{dN}{dt} = k\sqrt{N}$ B1

(*b*) $\int \frac{dN}{\sqrt{N}} = \int k\,dt$ M1

$\frac{N^{1/2}}{1/2} = kt + c$ A1

Substituting 256 for N and 5 for t and 400 for N and 7 for t in candidate's derived equation m1

$32 = 5k + c$, $40 = 7k + c$ (both equations) (c.a.o.) A1

Attempting to solve candidate's derived simultaneous linear equations in k and c $(k = 4, c = 12)$ m1

$N = (2t + 6)^2$ (o.e.) (c.a.o.) A1

9. (*a*) $\mathbf{AD} = \mathbf{AO} + \mathbf{OD} = -\mathbf{a} + 2\mathbf{b}$ B1

Use of $\mathbf{a} + \lambda\mathbf{AD}$ (o.e.) to find vector equation of AD M1

Vector equation of AD: $\mathbf{r} = \mathbf{a} + \lambda(-\mathbf{a} + 2\mathbf{b})$

$\mathbf{r} = (1 - \lambda)\mathbf{a} + 2\lambda\mathbf{b}$ (convincing) A1

(*b*) $\mathbf{BC} = \mathbf{BO} + \mathbf{OC} = 5\mathbf{a} - \mathbf{b}$ B1

Vector equation of BC: $\mathbf{r} = \mathbf{b} + \mu(5\mathbf{a} - \mathbf{b})$

$\mathbf{r} = 5\mu\mathbf{a} + (1 - \mu)\mathbf{b}$ (o.e.) B1

(*c*) $1 - \lambda = 5\mu$

$2\lambda = 1 - \mu$

(comparing candidate's coefficients of **a** and **b** and an attempt to solve) M1

$\lambda = \frac{4}{9}$ or $\mu = \frac{1}{9}$ (f.t. candidate's derived vector equation of BC) A1

$\mathbf{OE} = \frac{5}{9}\mathbf{a} + \frac{8}{9}\mathbf{b}$ (f.t. candidate's derived vector equation of BC) A1

10. $a^2 = 7b^2 \Rightarrow (7k)^2 = 7b^2 \Rightarrow b^2 = 7k^2$ B1

$\therefore$ 7 is a factor of b^2 and hence 7 is a factor of b B1

$\therefore$ a and b have a common factor, which is a contradiction to the original assumption B1

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - FP1
0977-01

INTRODUCTION

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FP1 – June 2017 – Markscheme

Ques	Solution	Mark	Notes
1(a)	det(**M**) = 6 – 4 + 2(3 – 4) +3(8 – 9)	M1	
	= – 3	A1	
(b)(i)	adj(**M**) = $\begin{bmatrix} 2 & 8 & -7 \\ -1 & -7 & 5 \\ -1 & 2 & -1 \end{bmatrix}$	M1A1	Award M1 if at least 5 correct elements
(ii)	$\mathbf{M}^{-1} = -\frac{1}{3}\begin{bmatrix} 2 & 8 & -7 \\ -1 & -7 & 5 \\ -1 & 2 & -1 \end{bmatrix}$	B1	FT if at least one M1 awarded
(c)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 2 & 8 & -7 \\ -1 & -7 & 5 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 11 \\ 11 \\ 17 \end{bmatrix}$	M1	FT inverse in (b)(ii)
	$= \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$	A1	
2	$S_n = \sum_{r=1}^{n}(3r-2)^2$	M1	
	$S_n = 9\sum_{r=1}^{n} r^2 - 12\sum_{r=1}^{n} r + 4\sum_{r=1}^{n} 1$	A1	
	$= \frac{9n(n+1)(2n+1)}{6} - \frac{12n(n+1)}{2} + 4n$	A1	
	$= \frac{n\left(9(n+1)(2n+1) - 36(n+1) + 24\right)}{6}$	A1	
	$= \frac{n\left(18n^2 + 27n + 9 - 36n - 36 + 24\right)}{6}$	A1	
	$= 3n^3 - \frac{3}{2}n^2 - \frac{1}{2}n$	A1	
3	EITHER		
	$\lvert 1+2\mathrm{i}\rvert = \sqrt{5}\,;\lvert -3+\mathrm{i}\rvert = \sqrt{10};\lvert 1+3\mathrm{i}\rvert = \sqrt{10}$	B2	For both moduli and arguments, B1 for 2 correct values
	$\arg(1+2\mathrm{i}) = 1.107; \arg(-3+\mathrm{i}) = 2.820;$ $\lvert \arg(1+3\mathrm{i}) = 1.249$	B2	Accept 63.43°, 161.56°,71.56°
	$\lvert z\rvert = \frac{\sqrt{5}\times\sqrt{10}}{\sqrt{10}} = \sqrt{5}$ cao	M1A1	
	$\arg(z) = 1.107 + 2.820 - 1.249 = 2.68$ cao	M1A1	Accept 153°

Ques	Solution	Mark	Notes
	OR $\frac{(1+2i)(-3+i)}{(1+3i)} = \frac{(-5-5i)}{(1+3i)}$ $= \frac{(-5-5i)(1-3i)}{(1+3i)(1-3i)}$ $= \frac{(-20+10i)}{10}$ $= -2 + i$ $\lvert z \rvert = \sqrt{5};\ \arg(z) = 153^\circ$ or 2.68 rad	**(M1A1)** **(M1)** **(A1)** **(A1)** **(A1)** **(B1B1)**	FT from line above provided both M marks awarded and arg is not in the 1st quadrant
4(a)	Reflection matrix = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Translation matrix = $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Rotation matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$	**B1** **B1** **B1** **M1** **A1**	Convincing, answer given
(b)	Fixed points satisfy $\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ $x = y - 1$ $y = x - 2$ These equations have no solution because, for example, $x = y - 1 = y + 2$ therefore no fixed points or algebra leading to $0 = 3$ or equivalent	**M1** **A1** **A1**	A1 both equations Convincing FT from line above provided it leads to no fixed point

Ques	Solution	Mark	Notes
5(a)	Using row operations, $x + 3y - z = 1$ $7y - 4z = -1$ $14y - 8z = 3 - \lambda$ It follows that $3 - \lambda = -2$ $\lambda = 5$	**M1** **A1** **A1** **A1**	
(b)	Let $z = \alpha$ $y = \dfrac{4\alpha - 1}{7}$ $x = \dfrac{10 - 5\alpha}{7}$	**M1** **A1** **A1**	FT from (a)
6	Putting n = 1 states that 8 is divisible by 8 which is correct so true for n = 1. Let the result be true for $n = k$, ie $9^k - 1$ is divisible by 8 or $9^k = 8N + 1$ Consider (for $n = k + 1$) $9^{k+1} - 1 = 9 \times 9^k - 1$ $= 9(8N + 1) - 1$ $= 72N + 8$ Both terms are divisible by 8 Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	**B1** **M1** **M1** **A1** **A1** **A1** **A1**	 Only award if all previous marks awarded
7(a)	Taking logs, $\ln f(x) = \tan x \ln \tan x$ Differentiating, $\dfrac{f'(x)}{f(x)} = \sec^2 x \ln \tan x + \dfrac{\tan x \sec^2 x}{\tan x}$ $f'(x) = (\tan x)^{\tan x} \sec^2 x (1 + \ln \tan x)$	**M1** **A1A1** **A1**	 A1 for LHS, A1 for RHS
(b)	Stationary points satisfy $1 + \ln \tan x = 0$ $\tan x = \dfrac{1}{e}$ $x = 0.35$	**M1** **A1** **A1**	

Ques	Solution	Mark	Notes
8(a)	$x+\mathrm{i}y=\frac{1}{u+\mathrm{i}v}\times\frac{u-\mathrm{i}v}{u-\mathrm{i}v}$	M1	
	$=\frac{u-iv}{u^2+v^2}$	A1	
	$x=\frac{u}{u^2+v^2}; y=\frac{-v}{u^2+v^2}$	A1A1	
(b)(i)	Putting $x+y=1$ gives	M1	FT from (a)
	$\frac{u-v}{u^2+v^2}=1$	A1	
	$u^2+v^2-u+v=0$	A1	
	This is the equation of a circle		
(ii)	Completing the square,		
	$\left(u-\frac{1}{2}\right)^2+\left(v+\frac{1}{2}\right)^2=\frac{1}{2}$	M1	
	The centre is $\left(\frac{1}{2},-\frac{1}{2}\right)$	A1	
	The radius is $\frac{1}{\sqrt{2}}$	A1	
(c)	Putting $w = z$,	M1	Allow working in terms of x,y,u,v
	$z^2=1$ giving $z=\pm 1$	m1	
	The two possible positions are (1,0) and (– 1,0)	A1	

Ques	Solution	Mark	Notes
9(a)(i)	$\alpha+\beta+\gamma=-2$ $\beta\gamma+\gamma\alpha+\alpha\beta=3$ $\alpha\beta\gamma=-4$	**B1**	
	$\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}+\dfrac{1}{\gamma^2}=\dfrac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$	**M1**	
	$=\dfrac{(\beta\gamma+\gamma\alpha+\alpha\beta)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha^2\beta^2\gamma^2}$	**A1**	
	$=\dfrac{3^2-2\times(-4)\times(-2)}{(-4)^2}$	**A1**	
	$=-\dfrac{7}{16}$		
(ii)	There are two complex roots and one real root	**B1**	Allow a less specific correct comment, eg not all the roots are real
(b)	Let the roots be a,b,c. $a+b+c=\dfrac{\alpha}{\beta\gamma}+\dfrac{\beta}{\gamma\alpha}+\dfrac{\gamma}{\alpha\beta}$		
	$=\dfrac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta\gamma}$	**M1**	
	$=\dfrac{(\alpha+\beta+\gamma)^2-2(\beta\gamma+\gamma\alpha+\alpha\beta)}{\alpha\beta\gamma}$	**A1**	
	$=\dfrac{(-2)^2-2\times3}{(-4)}$		
	$=\dfrac{1}{2}$	**A1**	
	$bc+ca+ab=\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}+\dfrac{1}{\gamma^2}=-\dfrac{7}{16}$	**B1**	Can be implied by final answer
	$abc=\dfrac{1}{\alpha\beta\gamma}=-\dfrac{1}{4}$	**B1**	
	The required equation is $x^3-\dfrac{1}{2}x^2-\dfrac{7}{16}x+\dfrac{1}{4}=0$ (or equivalent)	**M1A1**	FT their previous values Award M1 for correct numbers irrespective of signs

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - FP2
0978-01

INTRODUCTION

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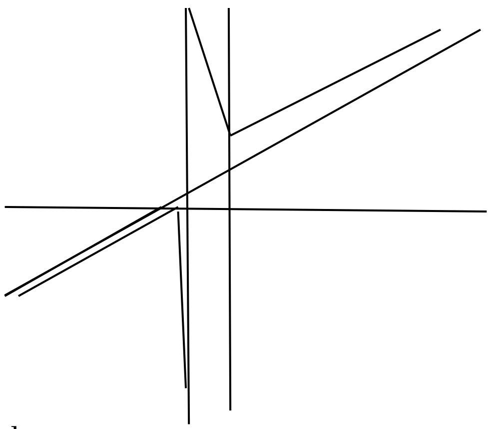
FP2 – June 2017 - Mark Scheme

Ques	Solution	Mark	Notes
1	Consider $f(-x) = \sec(-x) + (-x)\tan(-x)$ $= \sec x + x\tan x \quad (= f(x))$ Therefore f is even.	M1 A1 A1	M0 if particular value used This line must be seen
2	$\int_0^2 \frac{2x^2+5}{x^2+4}\,dx = \int_0^2 \frac{2x^2+8}{x^2+4}dx - \int_0^2 \frac{3}{x^2+4}dx$ $= [2x]_0^2 - \frac{3}{2}\left[\tan^{-1}\frac{x}{2}\right]_0^2$ $= 4 - \frac{3}{8}\pi$	M1A1 A1B1 A1	Award the B1 for a correct integration of $\frac{k}{x^2+4}$
3	$-8\mathrm{i} = 8(\cos 270^\circ + \mathrm{i}\sin 270^\circ)$ Root1 = $2(\cos 90^\circ + \mathrm{i}\sin 90^\circ)$ $= 2\mathrm{i}$ Root2 = $2(\cos 210^\circ + \mathrm{i}\sin 210^\circ)$ $= -\sqrt{3} - \mathrm{i}$ Root3 = $2(\cos 330^\circ + \mathrm{i}\sin 330^\circ)$ $= \sqrt{3} - \mathrm{i}$	B1B1 M1M1 A1 M1 A1 A1	B1 modulus, B1 argument M1for $\sqrt[3]{\text{mod}}$, M1 for arg/3 Special case – B1 for spotting 2i
4(a) (b)	Using deMoivre's Theorem, $z^n + z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ $= \cos n\theta + \mathrm{i}\sin n\theta + \cos(n\theta) - \mathrm{i}\sin(n\theta)$ $= 2\cos n\theta$ $z^n - z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta - \cos(-n\theta) - \mathrm{i}\sin(-n\theta)$ $= 2\mathrm{i}\sin n\theta$ $(z + z^{-1})^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$ oe $= (z^5 + z^{-5}) + 5(z^3 + z^{-3}) + 10(z + z^{-1})$ $= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ $32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ $\cos^5\theta = \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$	M1 A1 M1 A1 M1A1 A1 A1 A1	

Ques	Solution	Mark	Notes
(c)	$\int_0^{\pi/2}\cos^5\theta\,d\theta = \int_0^{\pi/2}\left(\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta\right)d\theta$	**M1**	FT from (b)
	$=\left[\frac{1}{80}\sin 5\theta+\frac{5}{48}\sin 3\theta+\frac{5}{8}\sin\theta\right]_0^{\pi/2}$	**A1**	No A marks if no working
	$=\frac{1}{80}-\frac{5}{48}+\frac{5}{8}$	**A1**	
	$=\frac{8}{15}$	**A1**	Award FT mark only if answer less that 1
5	Rewrite the equation in the form $2\sin 2\theta\sin 3\theta = \sin 3\theta$ $\sin 3\theta(2\sin 2\theta - 1) = 0$	**M1A1** **A1**	Accept answers in degrees
	Either $\sin 3\theta = 0$	**M1**	
	$3\theta = n\pi$ giving $\theta=\frac{n\pi}{3}$	**A1**	
	Or $\sin 2\theta = \frac{1}{2}$	**M1**	
	$2\theta=\left(2n+\frac{1}{2}\pm\frac{1}{3}\right)\pi$	**A1**	
	giving $\theta=\left(n+\frac{1}{4}\pm\frac{1}{6}\right)\pi$	**A1**	Accept equivalent forms
6(a)	Let $\frac{24x^2+31x+9}{(x+1)(2x+1)(3x+1)}=\frac{A}{x+1}+\frac{B}{2x+1}+\frac{C}{3x+1}$ $=\frac{A(2x+1)(3x+1)+B(x+1)(3x+1)+C(x+1)(2x+1)}{(x+1)(2x+1)(3x+1)}$	**M1**	
	$x = -1$ gives $A = 1$ $x = -½$ gives $B = 2$ $x = -1/3$ gives $C = 6$	**A1** **A1** **A1**	
(b)(i)	$\int_0^2 f(x)\,dx=\int_0^2\frac{1}{x+1}dx+\int_0^2\frac{2}{2x+1}dx+\int_0^2\frac{6}{3x+1}dx$	**M1**	FT their A,B,C if possible Their answer should be $\ln(3^A5^{B/2}7^{C/3})$ but only FT if this gives $\ln N$
	$=[\ln(x+1)]_0^2+[\ln(2x+1)]_0^2+2[\ln(3x+1)]_0^2$ $(=\ln 3+\ln 5+2\ln 7)$	**A2**	Award A1 for 2 correct integrals
	$=\ln 735$ cao	**A1**	
(ii)	The integral cannot be evaluated because the interval of integration contains points at which the integrand is not defined.	**B1**	

7(a)	$\sqrt{(x-a)^2+y^2} = x+a$ $(x-a)^2+y^2=(x+a)^2$ $x^2-2ax+a^2+y^2=x^2+2ax+a^2$ $y^2=4ax$	**M1** **A1** **A1**	Convincing
(b)	EITHER $\frac{\mathrm{d}x}{\mathrm{d}t}=2at, \frac{\mathrm{d}y}{\mathrm{d}t}=2a$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2a}{2at}=\frac{1}{t}$ OR $2y\frac{\mathrm{d}y}{\mathrm{d}x}=4a$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2a}{y}=\frac{1}{t}$ Gradient of normal = $-t$ The equation of the normal is $y-2at=-t(x-at^2)$	**M1** **A1** **(M1)** **(A1)** **A1** **A1**	 $y = -tx + at^3 + 2at$
(c)	EITHER The normal intersects the parabola again where $2as-2at=-t(as^2-at^2)$ $=-at(s-t)(s+t)$ Cancelling $a(s-t)$ both sides because $s \neq t$, $2 = -t(s+t)$ $s=-\frac{2}{t}-t$ OR The normal intersects the parabola again where $2as = -ats^2 + at^3 + 2at$ $ts^2 + 2s - 2t - t^3 = 0$ Solving, $s=\frac{-2\pm\sqrt{4+8t^2+4t^4}}{2t}$ $= -\frac{2}{t}-t, t$ (Rejecting t), $s = -\frac{2}{t}-t$	 **M1** **A1** **A1** **A1** **A1** **(M1)** **(A1)** **(M1)** **(A1)** **(A1)**	

8(a)(i)	$x = -1$	**B1**	
(ii)	$y = x + 3$	**B1**	
(b)	$f'(x) = 1 - \dfrac{1}{(x+1)^2}$	**B1**	
	Stationary points occur where $f'(x) = 0$	**M1**	
	$(x+1)^2 = 1$	**A1**	
	Giving (0,4) and (– 2,0) cao	**A1A1**	
(c)(i)	$f''(x) = \dfrac{2}{(x+1)^3}$	**B1**	
(ii)	$f''(0) = 2$ therefore (0,4) is a minimum	**B1**	B1 FT for deriv = $\dfrac{k}{(x+1)^3}$
	$f''(-2) = -2$ therefore (– 2,0) is a maximum	**B1**	
(d)		**G1** **G1** **G1**	G1 each branch, G1 asymptotes correctly positioned cao
(e)	Consider		
	$x + 3 + \dfrac{1}{x+1} = 5$	**M1**	
	$x^2 - x - 1 = 0$	**A1**	
	x = 1.618, – 0.618	**A1**	
	f^{-1}(S) = [– 0.618, 1.618]	**A1**	Accept $\left[\dfrac{1-\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}\right]$

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - FP3
0979-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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FP3 – June 2017 - Mark Scheme

Ques	Solution	Mark	Notes
1	EITHER Rewrite the equation in the form $2\left(\frac{e^{\theta}-e^{-\theta}}{2}\right)+\frac{e^{\theta}+e^{-\theta}}{2}=2$ $3e^{\theta}-4-e^{-\theta}=0$ $3e^{2\theta}-4e^{\theta}-1=0$ $e^{\theta}=\frac{4\pm\sqrt{16+12}}{6}$ = 1.548...,(–0.215...) $\theta = 0.437$	 M1A1 A1 A1 M1 A1 A1	
	OR Let $2\sinh\theta + \cosh\theta = r\sinh(\theta + \alpha)$ $= r\sinh\theta\cosh\alpha + r\cosh\theta\sinh\alpha$ Equating coefficients, $r\cosh\alpha = 2$; $r\sinh\alpha = 1$ Solving, $r=\sqrt{3}$; $\alpha = \tanh^{-1}(0.5)$ $(= 0.54930..)$ Consider $\sqrt{3}\sinh(\theta + \alpha) = 2$ $\theta + \alpha = \sinh^{-1}(2/\sqrt{3})$ $(= 0.98664...)$ $\theta = 0.98664 - 0.54930 = 0.437$	 (M1) (A1) (M1) (A1) (M1) (A1) (A1)	
2	Putting $t=\tan\left(\frac{x}{2}\right)$ $[0,\pi/2]$ becomes $[0,1]$ $dx=\frac{2dt}{1+t^2}$ $I=2\int_0^1\frac{2dt/(1+t^2)}{1+2t/(1+t^2)+2(1-t^2)/(1+t^2)}$ $=4\int_0^1\frac{dt}{3+2t-t^2}$ $=4\int_0^1\frac{dt}{4-(t-1)^2}$ $=\left[\ln\left(\frac{2+t-1}{2-t+1}\right)\right]_0^1$ $=\ln 3$	 B1 B1 M1A1 A1 m1 A1 A1	 M0 no working Accept $=\int_0^1\left(\frac{1}{3-t}+\frac{1}{1+t}\right)dt$ $=\left[-\ln(3-t)+\ln(1+t)\right]_0^1$ $=\ln 3$

Ques	Solution	Mark	Notes
3	$y = x^3, \dfrac{dy}{dx} = 3x^2$	B1	
	CSA $= 2\pi\int_0^1 y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}dx$	M1	
	$= 2\pi\int_0^1 x^3\sqrt{1+9x^4}dx$	A1	
	Put $u = 1+9x^4$	M1	
	$du = 36x^3dx, [0,1] \to [1,10]$	A1	
	CSA $= 2\pi\int_1^{10} u^{1/2}\dfrac{du}{36}$	M1	
	$= \left[2\pi\times\dfrac{u^{3/2}}{54}\right]_1^{10}$	A1	
	$= \dfrac{\pi}{27}\left(10^{3/2}-1\right)$	A1	
	$= 3.56$	A1	

Ques	Solution	Mark	Notes
4(a)	EITHER		
	$f(x) = \cos\ln(1+x)$		
	$f'(x) = -\sin\ln(1+x) \times \dfrac{1}{1+x}$	B1	
	$(1+x)f'(x) = -\sin\ln(1+x)$	B1	
	$(1+x)f''(x) + f'(x) = -\cos\ln(1+x) \times \dfrac{1}{1+x}$	M1	Convincing
	$(1+x)^2 f''(x) + (1+x)f'(x) + f(x) = 0$	A1	
	OR		
	$f(x) = \cos\ln(1+x)$		
	$f'(x) = -\sin\ln(1+x) \times \dfrac{1}{1+x}$	(B1)	
	$f''(x) = -\cos\ln(1+x) \times \dfrac{1}{(1+x)^2} + \sin\ln(1+x) \times \dfrac{1}{(1+x)^2}$	(B1)	
	$(1+x)^2 f''(x) + (1+x)f'(x) + f(x)$	(M1)	
	$= -\cos\ln(1+x) + \sin\ln(1+x) - \sin\ln(1+x) + \cos\ln(1+x) = 0$	(A1)	Convincing
(b)	Using the above results, $f(0) = 1,\ f'(0) = 0,\ f''(0) = -1$	B2	Award B1 for two correct values
	Differentiating again,		
	$2(1+x)f''(x) + (1+x)^2 f'''(x) + f'(x)$	M1	
	$+ (1+x)f''(x) + f'(x) = 0$		
	Therefore $f'''(0) = 3$	A1	
	The Maclaurin series is		
	$1 - \dfrac{1}{2}x^2 + \dfrac{3}{6}x^3 + ..$ giving	A1	
	$1 - \dfrac{1}{2}x^2 + \dfrac{1}{2}x^3 + ...$		convincing
(c)	Differentiating,		
	$-\sin\ln(1+x) \times \dfrac{1}{1+x} = -x + \dfrac{3}{2}x^2 + \ldots$	M1	
	$\sin\ln(1+x) = -(1+x)(-x + \dfrac{3}{2}x^2 + \ldots))$	A1	
	$= x - \dfrac{3}{2}x^2 + x^2 + \ldots$	M1	
	$= x - \dfrac{1}{2}x^2 + \ldots$	A1	

Ques	Solution	Mark	Notes
5(a)	tan(0.9)tanh(0.9) – 1 = – 0.0973... tan(1.1)tanh(1.1) – 1 = 0.572... The change of sign indicates a root between 0.9 and 1.1	B1 B1 B1	
(b)(i)	$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\tan^{-1}\left(\frac{1}{\tanh\theta}\right)\right)=\frac{1}{1+\frac{1}{\tanh^2\theta}}\times-\frac{1}{\tanh^2\theta}\times\mathrm{sech}^2\theta$ $= -\frac{1-\tanh^2\theta}{1+\tanh^2\theta}$	**M1A1A1**	Do not award the second A1 if the required result is not derived
(ii)	EITHER For $\theta > 0$, tanhθ lies between 0 and 1. Therefore $1-\tanh^2\theta<1+\tanh^2\theta$ so that the modulus of the above derivative is less than 1 therefore convergent. OR For $\theta = 1$, $\left\lvert-\left(\frac{1-\tanh^2\theta}{1+\tanh^2\theta}\right)\right\rvert=0.266$ This is less than 1 therefore convergent.	B1 B1 (B1) (B1)	
(c)(i)	Successive iterations give 1 0.9199161588 ... etc	**M1A1**	
(ii)	The value of α is 0.938 correct to 3 decimal places.	A1	

Ques	Solution	Mark	Notes
6(a)	$I_n = \int_0^{\pi/4} \tan^{n-2}x\tan^2 x\mathrm{d}x$	M1	
	$I_n = \int_0^{\pi/4} \tan^{n-2}x(\sec^2 x - 1)\mathrm{d}x$	A1	
	$= \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\pi/4} - I_{n-2}$	M1A1A1	convincing
	$= \frac{1}{n-1} - I_{n-2}$		
(b)	$\int_0^{\pi/4}\left(3+\tan^2 x\right)^2\mathrm{d}x = \int_0^{\pi/4} 9\mathrm{d}x + \int_0^{\pi/4} 6\tan^2 x\mathrm{d}x + \int_0^{\pi/4}\tan^4 x\mathrm{d}x$		
	$= 9I_0 + 6I_2 + I_4$	M1	
		A1	
	$I_0 = \frac{\pi}{4}$	B1	
	$I_2 = 1 - I_0 = 1 - \frac{\pi}{4}$	B1	
	$I_4 = \frac{1}{3} - I_2 = \frac{\pi}{4} - \frac{2}{3}$	B1	
	Substituting above,		
	$\int_0^{\pi/4}(3+\tan^2 x)^2\mathrm{d}\theta = 9\frac{\pi}{4} + 6(1-\frac{\pi}{4}) + \left(\frac{\pi}{4} - \frac{2}{3}\right)$	M1	
	$= \frac{16}{3} + \pi$	A1	

Ques	Solution	Mark	Notes
7(a)	For C_1 consider $x = r\cos\theta = \sqrt{3}\sin\theta\cos\theta$ $= \dfrac{\sqrt{3}}{2}\sin 2\theta$	M1	
	It follows that x is maximised at P when $\theta = \dfrac{\pi}{4}$.	A1	
	For C_2 consider $y = r\sin\theta = \sin\theta\cos\theta$ $= \dfrac{1}{2}\sin 2\theta$	M1	
	It follows that y is maximised at Q when $\theta = \dfrac{\pi}{4}$	A1	
	Therefore O, P and Q lie on the line $\theta = \dfrac{\pi}{4}$. oe	A1	
(b)(i)	The graphs intersect where $\sqrt{3}\sin\theta = \cos\theta$	M1	
	$\tan\theta = \dfrac{1}{\sqrt{3}}$	A1	
	$\theta = \dfrac{\pi}{6}, r = \sqrt{3}\sin\left(\dfrac{\pi}{6}\right)$ or $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$	A1	Convincing
(ii)	Area of region = $\dfrac{1}{2}\displaystyle\int_0^{\pi/6} 3\sin^2\theta \mathrm{d}\theta + \dfrac{1}{2}\int_{\pi/6}^{\pi/2}\cos^2\theta \mathrm{d}\theta$	M1M1	M1 the integrals, M1 for addition
	$= \dfrac{3}{4}\displaystyle\int_0^{\pi/6}(1-\cos 2\theta)\mathrm{d}\theta + \dfrac{1}{4}\int_{\pi/6}^{\pi/2}(1+\cos 2\theta)\mathrm{d}\theta$ oe	A1A1	Limits si
	$= \dfrac{3}{4}\left[\theta - \dfrac{1}{2}\sin 2\theta\right]_0^{\pi/6} + \dfrac{1}{4}\left[\theta + \dfrac{1}{2}\sin 2\theta\right]_{\pi/6}^{\pi/2}$	A1A1	Award A1 for one correct integration, A1 for fully correct line
	$= 0.221 \quad \left(\dfrac{5\pi}{24} - \dfrac{\sqrt{3}}{4}\right)$	A1	

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - M1
0980-01

INTRODUCTION

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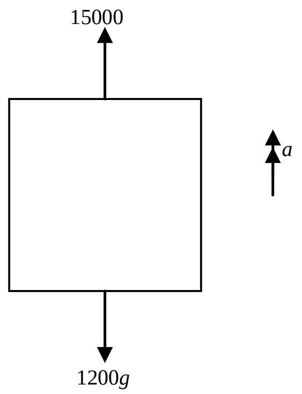
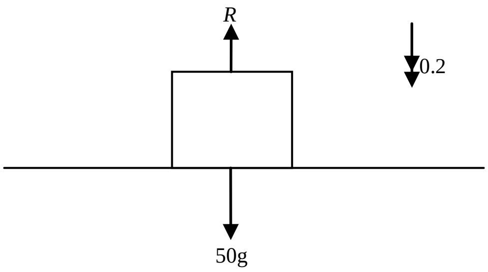
MATHEMATICS M1 (June 2017)
Markscheme

Q	Solution	Mark	Notes
1(a)			



Q	Solution	Mark	Notes
	N2L applied to lift,upwards +ve	M1	dimensionally correct 15000, 1200g opposing No extra forces.
	$15000 - 1200g = 1200a$	A1	
	$15000 - 1200 \times 9.8 = 1200a$		
	$a = \underline{2.7}$	A1	
1(b)			

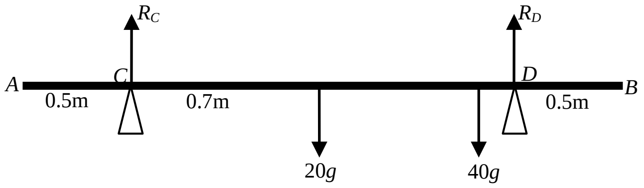


Q	Solution	Mark	Notes
	N2L applied to crate, down +ve	M1	dimensionally correct R and $50g$ opposing. No extra forces.
	$50g - R = 50a$	A1	
	$R = 50(9.8 - 0.2)$		
	$R = \underline{480\ (\text{N})}$	A1	

Q	Solution	Mark	Notes
2(a)	Impulse on Q = 2(7.5 – (-3))	M1	
	I = <u>21 (Ns)</u>	A1	magnitude required.
2(b)	Conservation of momentum	M1	equation required. Allow 1 sign error
	$6 \times 5 + 2 \times (-3) = 6v + 2 \times 7.5$	A1	
	v = <u>1.5 (ms^{-1})</u>	A1	cao speed required
2(c)	Restitution equation	M1	allow one sign error Ft v
	$7.5 - 1.5 = -e(-3 - 5)$	A1	Ft v
	e = <u>0.75</u>	A1	cao
2(d)	speed after rebound = 7.5×0.6	M1	
	= <u>4.5 (ms^{-1})</u>	A1	cao allow -4.5

Q	Solution	Mark	Notes

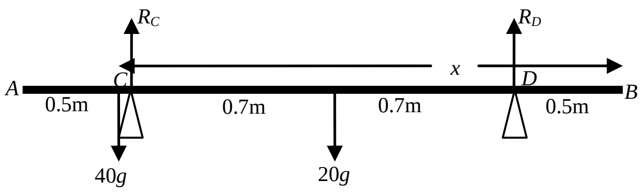
3.



3(a)	Moments about D	M1	dimen correct equation All forces, no extra
	$40g\times0.1 + 20g\times0.7 = R_C\times1.4$	B1	any correct moment
		A1	correct equation
	$R_C = \underline{126\text{(N)}}$	A1	cao
	Resolve vertically	M1	dimen correct equation All forces, no extra
	$R_C + R_D = 40g + 20g$	A1	
	$R_D = \underline{462\text{(N)}}$	A1	cao

<u>Alternative method</u>
Two simultaneous equations award B1 M1 A1 M1 A1 A1cao A1cao

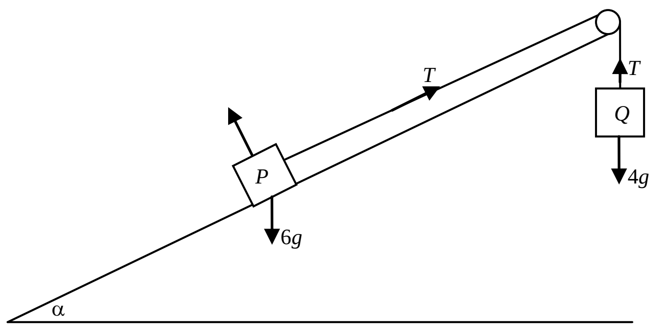
3(b)



	Moments about C	M1	dimen correct equation All forces, no extra oe
	$40g(x - 1.9) + R_D\times1.4 = 20g\times0.7$ Equilibrium on point of collapse when R_D=0. or if moments about point not C R_C=60g, (and R_D=0 implied).	M1	
	$40g(x - 1.9) = 20g\times0.7$ $x = \underline{2.25\text{(m)}}$	A1	cao

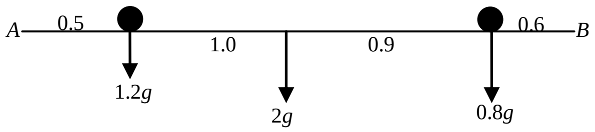
Q	Solution	Mark	Notes
4(a)	using $v=u+at$, $u=0$, $v=15$, $t=50$	M1	
	$15 = 0 + 50a$	A1	
	$a = \underline{0.3\ (\text{ms}^{-2})}$	A1	cao
4(b)	N2L $T - R = ma$	M1	dim correct equation
	$300 - R = 800 \times 0.3$	A1	Ft a
	$R = 300 - 240$		
	$R = \underline{60\ (\text{N})}$	A1	cao
4(c)	using $s=ut+0.5at^2$, $u=0$, $a=0.3$(c), $t=50$	M1	oe
	$s = 0.5 \times 0.3 \times 50^2$	A1	FT a
	$s = 375$		
	Distance used in braking = 500 – 375 = 125		
	Using $v^2=u^2+2as$, $u=15$, $v=0$, $s=125$(c)	M1	oe
	$0 = 15^2 + 2 \times a \times 125$	A1	
	$a = -\dfrac{15^2}{2\times125}$		
	$a = -0.9$		
	$800 \times (-)(0.9) = (-)720$	B1	ft a
	N2L		
	$-B - R = ma$	M1	dim correct equation
	$B = \underline{660\ (\text{N})}$	A1	cao
	<u>Alternative</u>		
	$(-)F = 800 \times (-)(0.9)$	(B1)	
	$F = 720$		
	Force exerted by brakes = 720 – 60	(M1)	
	= $\underline{660\ (\text{N})}$	(A1)	cao

Q	Solution	Mark	Notes
5			



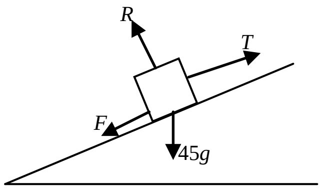
Q	Solution	Mark	Notes
5(a)	$\sin\alpha = \frac{3}{5}$		
	$4g - T = 4a$	B1	
	N2L applied to second particle	M1	Dim correct equation. T and weight opposing sin/cos required.
	$T - 6g\sin\alpha = 6a$	A1	
	Adding $4g - 6g \times \frac{3}{5} = 10a$	m1	
	$a = \underline{0.04g} = \underline{0.392(\text{ms}^{-2})}$	A1	cao mag req. accept 0.4
	$T = \underline{3.84g} = \underline{37.632(\text{N})}$	A1	cao accept 37.6/7
5(b)	Using $v^2=u^2+2as$, $u=0$, $a=0.392$(c), $s=1.5$	M1	oe
	$v^2 = 2 \times 0.04g \times 1.5$	A1	Ft a
	$v = \frac{\sqrt{3g}}{5} = \underline{1.0844(\text{ms}^{-1})}$	A1	cao
5(c)	Using $v=u+at$, $v=0$, $u=\frac{\sqrt{3g}}{5}$ (c), $a=(\pm)0.6g$	M1	oe
	$0 = \frac{\sqrt{3g}}{5} - 0.6gt$	A1	Ft v from (b)
	$t = 0.1844$	A1	cao
	Required time = $\underline{0.37(\text{s})}$	A1	Ft t, 2dp required.

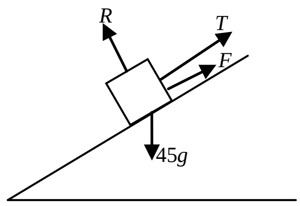
Q	Solution	Mark	Notes
6.			



Take moments about B	M1	dimensionally correct 4 terms equation, condone no g throughout.
$(1.2g + 2g + 0.8g)x$ $= 1.2g\times2.5 + 2g\times1.5 + 0.8g\times0.6$	B1 A1	any correct moment correct equation
$x = \underline{1.62 \text{ (m)}}$	A1	

Q	Solution	Mark	Notes
7			





Resolve perpendicular to plane	M1	accept $\sin\alpha$
$R = 45g\cos\alpha = (36g = 352.8)$	A1	
$F = 0.5 \times R = (18g = 176.4)$	m1	
N2L parallel to plane	M1	or N2L with a=0 Dimensionally correct All forces, T and wt opp.
For greatest T		
$T = 45g\sin\alpha + F$	A1	a=0
$T = 27g + 18g$		
$T = \underline{45g = 441\text{(N)}}$	A1	cao
N2L parallel to plane	M1	or N2L with a=0 Dimensionally correct All forces, T and wt opp. F in opposite direction to previous N2L.
For least T		
$45g\sin\alpha = T + F$	A1	a=0
$T = 45g\sin\alpha - F$		
$T = 27g - 18g$		
$T = \underline{9g = 88.2\text{(N)}}$	A1	cao

Condone absence of ‘greatest/least’ but if present must be correct for A1.

Q	Solution				Mark	Notes
8(a).		Area	from $AF(x)$	from $AB(y)$		
	ABEF	180	5	9	B1	
	BCD	90	15	6	B1	
	Lamina	270	x	y	B1	areas correct, allow areas in proportion 2:1:3.

Q	Solution	Mark	Notes
	Moments about *AF*		
	$270x = 180 \times 5 + 90 \times 15$	M1	
	$270x = 2250$		
	$x = \dfrac{25}{3} = 8.3$	A1	cao
	Moments about *AB*		
	$270y = 180 \times 9 + 90 \times 6$	M1	
	$270y = 2160$		
	$y = \underline{8}$	A1	cao
8(b)	Identification of correct triangle	M1	
	$\tan\theta = \left(\dfrac{10 - 25/3}{18 - 8}\right)$	A1	Ft x, y
	$\theta = \tan^{-1}\left(\dfrac{5}{30}\right)$		
	$\theta = \underline{9.5^{(o)}}$ or $\theta = \underline{0.165^{(c)}}$	A1	FT x, y units not required but if present must be correct.

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - M2
0981-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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Mathematics M2 (June2017)
Markscheme

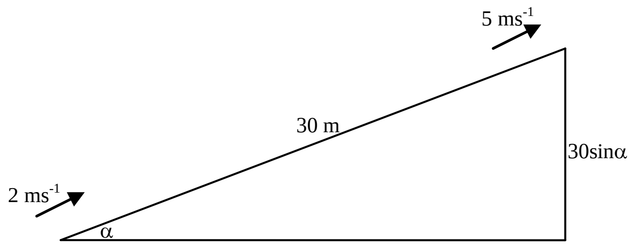
Q	Solution	Mark	Notes
1(a)(i)	$\mathbf{v} = \dfrac{\mathrm{d}}{\mathrm{d}t}\mathbf{r}$	M1	differentiation attempted
			Vector required
	$\mathbf{v} = (\sin t + t\cos t)\mathbf{i} + (\cos t - t\sin t)\mathbf{j}$	A1	
	$(\text{mod } \mathbf{v})^2 = (\sin t + t\cos t)^2 + (\cos t - t\sin t)^2$	M1	
	$= \sin^2 t + 2t\sin t\cos t + t^2\cos^2 t$ $+ \cos^2 t - 2t\sin t\cos t + t^2\sin^2 t$	A1	Ft similar expressions
	$= 1 + t^2$		
	Speed of $P = \sqrt{1+t^2}$	A1	cao
1(a)(ii)	Momentum vector = $m\mathbf{v}$		
	$= 3[(\sin t + t\cos t)\mathbf{i} + (\cos t - t\sin t)\mathbf{j}]$	B1	ft $\mathbf{v}$(c)
	$= 3(\sin t + t\cos t)\mathbf{i} + 3(\cos t - t\sin t)\mathbf{j}$		
1(b)	At $t = \dfrac{\pi}{6}$,		
	$\mathbf{r} = \dfrac{\pi}{6}\sin\dfrac{\pi}{6}\mathbf{i} + \dfrac{\pi}{6}\cos\dfrac{\pi}{6}\mathbf{j}$	B1	
	$\mathbf{r} = \dfrac{\pi}{12}\mathbf{i} + \dfrac{\pi\sqrt{3}}{12}\mathbf{j}$		
	If perpendicular, $\mathbf{r}.(b\,\mathbf{i} + \sqrt{3}\,\mathbf{j}) = 0$	M1	
	$(\dfrac{\pi}{12}\mathbf{i} + \dfrac{\pi\sqrt{3}}{12}\mathbf{j}).(b\,\mathbf{i} + \sqrt{3}\,\mathbf{j})$		
	$= \dfrac{\pi}{12}b + \dfrac{\pi\sqrt{3}}{12} \times \sqrt{3}$	M1A1	method correct, no **i**, **j**
	$\dfrac{\pi}{12}b + \dfrac{3\pi}{12} = 0$		
	$b + 3 = 0$		
	$b = \underline{-3}$	A1	cao

Q	Solution	Mark	Notes
2(a)	$x = \int 4t^3 - 6t + 7\,\mathrm{d}t$	M1	at least one power increased.
	$x = t^4 - 3t^2 + 7t + (C)$	A1	correct integration
	When $t = 0$, $x = 5$	m1	initial conditions used
	$C = 5$		
	$x = t^4 - 3t^2 + 7t + 5$		
	When $t = 2$	m1	used
	$x = 2^4 - 3\times2^2 + 7\times2 + 5$		
	$x = 16 - 12 + 14 + 5$		
	$x = \underline{23\ (\mathrm{m})}$	A1	cao
2(b)	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	at least one power decreased.
	$a = 12t^2 - 6$	A1	
	$F = ma = 0.8(12t^2 - 6)$	M1	Ft a
	When $t = 3$		
	$F = 0.8(12\times3^2 - 6)$		
	$F = \underline{81.6\ (\mathrm{N})}$	A1	cao

Q	Solution	Mark	Notes
3(a).	$T = \dfrac{P}{v}$		
	$T = \dfrac{12000}{3} = (4000)$	B1	
	N2L	M1	dimensionally correct 4 terms, allow sin/cos
	$T - mg\sin\alpha - R = ma$	A1	
	$4000 - 3000\times9.8\times0.1 - 460 = 3000a$		
	$a = \underline{0.2\ (\text{ms}^{-2})}$	A1	cao
3(b)	N2L	M1	dimensionally correct 4 terms, allow sin/cos
	$a = 0$	M1	
	$T - 10v - mg\sin\alpha - R = 0$		
	$\dfrac{12000}{v} - 10v - 3000\times9.8\times0.1 - 460 = 0$	A1	
	$\dfrac{12000}{v} - 10v - 3400 = 0$		
	$12000 - 10v^2 - 3400v = 0$		
	$v^2 + 340v - 1200 = 0$		
	$v = \dfrac{-340 \pm \sqrt{340^2 + 4\times1200}}{2}$	m1	dep on both M
	$v = \underline{3.49}$	A1	cao answer rounding to 3.5.

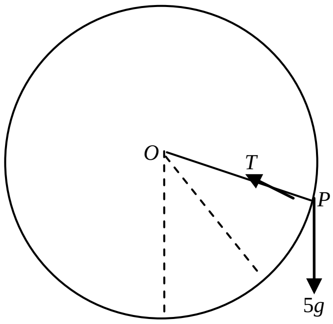
Q	Solution	Mark	Notes
4(a)	initial vertical vel of P = 15sin60º		
	$= \dfrac{15\sqrt{3}}{2} = 12.99$		
	initial vertical vel of Q = $v$sin30º	B1	either correct expression
	use of $s = ut + 0.5gt^2$	M1	
	height of P at time $t = \dfrac{15\sqrt{3}}{2}t - 0.5gt^2$		
	height of Q at time $t = 0.5vt - 0.5gt^2$	A1	either
	For collision		
	$\dfrac{15\sqrt{3}}{2}t - 0.5gt^2 = 0.5vt - 0.5gt^2$	m1	
	$v = 15\sqrt{3} = 25.98$	A1	accept 26
4(b)	initial horiz vel of P = 15cos60º		
	= 7.5		
	initial horiz vel of Q = 15√3cos30º		
	= 22.5	B1	either
	For collision,		
	$7.5t + 22.5t = 18$	M1	
	$t = 0.6$ (s)	A1	convincing
4(c)	use of $v=u+at$, $u=\dfrac{15\sqrt{3}}{2}$ (c), $a=\pm 9.8$, $t=0.6$	M1	
	$v = \dfrac{15\sqrt{3}}{2} - 9.8\times 0.6$	A1	Ft u
	$v = 7.1$		
	speed = $\sqrt{7.1^2 + 7.5^2}$	M1	accept candidate's values
	= $\underline{10.3(\text{ms}^{-1})}$	A1	cao

Q	Solution	Mark	Notes
5			



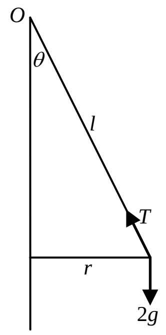
KE at t=0 = 0.5 × 4000 × 2^2	M1A1	v=2 or 5
KE at t=0 = 8000 (J)		
PE at t =0 = 0		
KE at t=8 = 0.5 × 4000 × 5^2		
KE at t=8 = 50000 (J)		
PE at t =8 = 4000 × 9.8 × h	M1	
PE at t =8 = 4000 × 9.8 × 30sinα	A1	
PE at t =8 = 58800 (J)		
WD by engine = 43000 × 8	B1	
WD by engine = 344000 (J)		
Work-energy principle	M1	KE, PE and WD(2 terms)
8000 + 344000 = WD + 50000 + 58800	A1	correct equation
WD = <u>243200 (J)</u>	A1	cao

Q	Solution	Mark	Notes
6			



6(a)	conservation of energy	M1	KE and PE in equation
	$0.5mu^2 - mgl\cos 60^o = 0.5mv^2 - mgl\cos\theta$	A1A1	
	$v^2 = u^2 - 0.8g + 1.6g\cos\theta$	A1	cao
	$v^2 = u^2 - 7.84 + 15.68\cos\theta$		
6(b)	N2L towards centre	M1	dim correct equation T and $5g\cos\theta$ opposing
	$T - 5g\cos\theta = \dfrac{5v^2}{0.8}$	A1	
	$T = 5g\cos\theta + \dfrac{5}{0.8}(u^2 - 0.8g + 1.6g\cos\theta)$	m1	subt v^2 equivalent expressions
	$T = 6.25u^2 - 5g + 15g\cos\theta$	A1	cao, any correct expression
	$T = 6.25u^2 - 49 + 147\cos\theta$		
6(c)	For complete circles,		
	$T \geq 0$ when $\theta = 180^o$, $(\cos\theta = -1)$.	M1	
	$6.25u^2 \geq 49 + 147$		
	$u^2 \geq 31.36$		
	$u \geq 5.6$	A1	cao
6(d)	For complete circles,		
	$v^2 \geq 0$ when $\theta = 180^o$, $(\cos\theta = -1)$.	M1	
	$u^2 \geq 7.84 + 15.68$		
	$u^2 \geq 23.52$		
	$u \geq 4.85$	A1	cao

Q	Solution	Mark	Notes
7.			



7(a)	Resolve vertically	M1	
	$T\cos\theta = 2g$	A1	allow m
	N2L towards centre of motion	M1	
	$T\sin\theta = 2r\omega^2$	A1	
	$T\sin\theta = 2l\sin\theta\ \omega^2$	A1	use of $r=l\sin\theta$
	$T = 2l\omega^2$		
	$2l\ \omega^2 \cos\theta = 2g$		
	$\cos\theta = \dfrac{g}{l\omega^2}$	A1	convincing
7(b)(i)	$T\cos\theta = 2g$, $T = 20g$		
	$\cos\theta = \underline{0.1}$	B1	
7(b)(ii)	$\cos\theta = 0.1$ and $\omega^2 = 3g$, $\cos\theta = \dfrac{g}{l\omega^2}$		
	$0.1 = \dfrac{g}{l \times 3g}$	M1	or $20g=2l\text{x}3g$
	$l = \dfrac{10}{3}$	A1	convincing
7(b)(iii)	Hooke's Law		
	$T = \dfrac{\lambda x}{natural\ length}$	M1	used, condone natural length=10/3, but x not10/3or 3
	$20g = \dfrac{\lambda(\frac{10}{3} - 3)}{3}$	A1	one of 10/3-3 or 3 correct
	$\lambda = \underline{180g} = \underline{1764}$	A1	cao

7(b)(iv)EE $= \dfrac{\lambda x^2}{2(nat\ len)}$	M1	used
EE $= \dfrac{1764}{2\times 3\times 3^2}$		
EE $= \dfrac{98}{3}$ = <u>32.67 (J)</u>	A1	cao

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - M3
0982-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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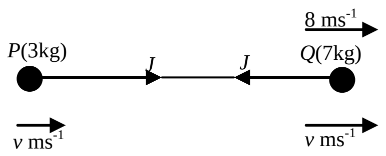
Mathematics M3 (June2017)
Markscheme

Q	Solution	Mark	Notes
1(a)	$\frac{dx}{dt} = 2 - x$		
	$\int \frac{dx}{2-x} = \int dt$	M1	sep variables, (2-x) required
	$-\ln\lvert 2-x\rvert = t + (C)$	A1	correct integration ft x-2
	When $t = 0$, $x = 0$	m1	use of initial conditions
	C = -ln 2	A1	ft if ln present.
	$t = \ln \left\lvert \frac{2}{2-x} \right\rvert$		
	When $x = 1$		
	$t = \ln 2 = (0.693)$	A1	cao
	$e^{-t} = \frac{2-x}{2}$	m1	correct method inversion
	$x = 2(1 - e^{-t})$	A1	any correct exp. cao
1(b)	$\frac{d^2x}{dt^2} = -\frac{dx}{dt}$	M1	
	$\frac{d^2x}{dt^2} = -(2-x) = x - 2$		
	$\frac{d^2x}{dt^2} = 2(1 - e^{-t}) - 2$	m1	substitute for x
	$\frac{d^2x}{dt^2} = -2e^{-t}$	A1	
	<u>Alternative</u>		
	$x = 2(1 - e^{-t})$		
	$\frac{dx}{dt} = 2e^{-t}$	(M1)(A1)	ft similar expressions
	$\frac{d^2x}{dt^2} = -2e^{-t}$	(A1)	ft $\frac{dx}{dt} = -2e^{-t}$ only.

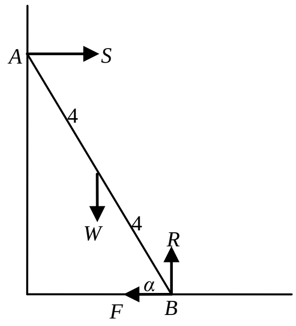
Q	Solution	Mark	Notes
2			
	Impulse = change in momentum Applied to Q $J = 7 \times 8 - 7v$	 M1 A1	 allow +/-J
	Applied to P $J = 3v$	 B1	
	$3v = 56 - 7v$ $v = \underline{5.6\ (\text{ms}^{-1})}$	m1 A1	 cao
	$J = \underline{16.8\ (\text{Ns})}$	A1	cao

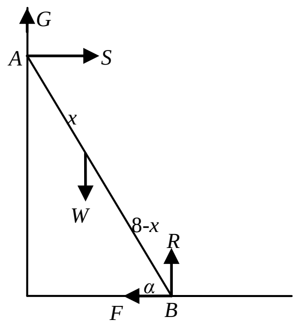
Q	Solution	Mark	Notes
3(a)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 5x = 0$		
	Auxilliary equation $m^2 – 6m + 5 = 0$	M1	
	$(m – 1)(m – 5) = 0$, $m = 1, 5$		
	G.S. is $x = Ae^t + Be^{5t}$	A1	ft 2 real roots
	When $t = 0$, $x = 8$ and $\frac{dx}{dt} = 16$	m1	used both
	$A + B = 8$		
	$\frac{dx}{dt} = Ae^t + 5Be^{5t}$	B1	ft similar expressions
	$A + 5B = 16$		
	Solving, $A = 6$, $B = 2$	A1	both values cao
	$x = 6e^t + 2e^{5t}$		
3(b)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 10x = 0$		
	Auxilliary equation $m^2 – 6m + 10 = 0$	M1	
	$m = 3 \pm i$		
	C.F. is $x = e^{3t}(A\sin t + B\cos t)$	A1	ft complex roots
	Using initial conditions	m1	used both
	$B = 8$		
	$\frac{dx}{dt} = 3e^{3t}(A\sin t + B\cos t) + e^{3t}(A\cos t - B\sin t)$	B1	ft similar expression
	$16 = 24 + A$, $A = -8$	A1	both values cao
	$x = 8e^{3t}(-\sin t + \cos t)$		
3(c)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} = (12t - 26)$,		
	Auxilliary equation $m^2 – 6m = 0$	M1	
	$m = 0, 6$		
	C.F. is $x = A + Be^{6t}$	A1	ft 0, another real root
	For P.I. try $x = at^2 + bt$	M1	allow $at+b$
	$2a – 6(2at + b) = 12t – 26$	A1	correct LHS
	$a = -1$	m1	comparing coefficients
	$2a – 6b = -26$, $b = 4$	A1	both values cao
	$x = A + Be^{6t} – t^2 + 4t$		
	$8 = A + B$		
	$\frac{dx}{dt} = 6Be^{6t} – 2t + 4$	B1	ft similar CF+PI
	$16 = 6B + 4$		
	$B = 2$, $A = 6$	A1	both values cao
	$x = 2e^{6t} – t^2 + 4t + 6$		

Q	Solution	Mark	Notes
4(a)	N2L applied to P	M1	Dimensionally correct All forces
	$-3v^2 = 0.5\ \frac{\mathrm{d}v}{\mathrm{d}t}$		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = -6v^2$	A1	convincing
4(b)	$-\int \frac{dv}{v^2} = 6\int dt$	M1	separating variables
	$\frac{1}{v} = 6t + (C)$	A1	correct integration
	When t=0, v=2	m1	use of initial conditions
	$C = \frac{1}{2}$		
	$\frac{1}{v} = 6t + \frac{1}{2}$		
	$v = \frac{2}{12t+1}$	A1	cao, any correct exp.
4(c)	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -6v^2$	M1	
	$\frac{\mathrm{d}v}{\mathrm{d}x} = -6v$		
	$\int \frac{dv}{v} = -6\int dx$	m1	separating variables
	$\ln v = -6x + (C)$	A1	correct integration
	when $x = 0$, $v = 2$	m1	use of initial conditions
	$C = \ln 2$		
	$-6x = \ln v - \ln 2$		
	$v = 2e^{-6x}$	A1	cao, any correct exp.
4(d)	Rate of work = $F.v$	M1	used
	Rate of work = $3v^2 \times v$	A1	
	Rate of work = $3(2e^{-6x})^3$		
	Rate of work = $24e^{-18x}$	A1	cao, any correct exp.

Q	Solution	Mark	Notes
5(a)	$v^2 = -4x^2 + 8x + 21$	M1	attempt to differentiate
	$2v\frac{\mathrm{d}v}{\mathrm{d}x} = -8x + 8$	A1	or dv/dx=
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -4(x-1)$		
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4(x-1)$	A1	
	Let $y = x - 1$, $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$,		
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -4\mathrm{y} = -2^2y$		
	Hence motion is simple harmonic	A1	convincing
	Centre of motion is $x = 1$	B1	
5(b)	$\omega = 2$	B1	
	Period = $\frac{2\pi}{2} = \pi$	B1	convincing
	Amplitude is given by x-1 when $v = 0$ $-4x^2 + 8x + 21 = -4(x-1)^2 + 25 = 0$ $(x-1) = \pm 2.5$	M1	v=0
	Amplitude = a = 2.5	A1	cao
	<u>Alternative solution</u> $v^2 = \omega^2[a^2 - y^2]$	(M1)	attempt to write equation in correct form
	$v^2 = 2^2[2.5^2-(x-1)^2]$		
	Hence $\omega = 2$	(B1)	
	Period = $\frac{2\pi}{2} = \pi$	(B1)	
	Amplitude = a = 2.5	(A1)	cao
	<u>Alternative solution</u> Amplitude is given when $v = 0$	(M1)	used
	$-4x^2 + 8x + 21 = 0$ $(2x + 3)(2x - 7) = 0$ $x = -1.5, 3.5$		
	amplitude = 3.5 – 1 = 2.5	(A1)	cao

5(c)	$(x - 1) = 2.5 \sin(2t)$ $x = 2.5 \sin(2t) + 1$	M1	
	$3 - 1 = 2.5 \sin(2t)$	m1	use of 3-centre
	$2t = \sin^{-1}\left(\dfrac{2}{2.5}\right)$	m1	inversion ft a,ω,centre
	$2t = 0.927295$ $t = \underline{0.4636\ (\text{s})}$	A1	cao

Q	Solution	Mark	Notes
6(a)			
	Resolve vertically $R = W$	B1	
	Resolve horizontally $S = F = \mu R = \mu W$	B1	
	Moments about B $W\times 4\cos\alpha \;=\; S\times 8\sin\alpha$	M1	dim correct, all forces no extra except friction A
	$16W \;=\; \mu W\times 8\times 3$ $\mu \;=\; \dfrac{2}{3}$	A1	cao

Q	Solution	Mark	Notes
6(b)			
	$F = 0.6R$ $G = 0.6S$	B1	both
	Resolve vertically	M1	dimensionally correct All forces, no extra
	$G + R = W$ $0.6S + R = W$	A1	
	Resolve horizontally	M1	dimensionally correct All forces, no extra
	$S = F$ $S = 0.6R$	A1	
	$0.6 \times 0.6R + R = W$ $1.36R = W$		
	Moments about A	M1	dimensionally correct All forces, no extra
	$Wx\cos\alpha + 0.6R\times8\sin\alpha = R\times8\cos\alpha$	A2	-1 each error
	$1.36Rx\frac{4}{5} + 4.8R\times\frac{3}{5} = 8R\times\frac{4}{5}$	m1	substitute to obtain one common factor force
	$5.44x + 14.4 = 32$ $5.44x = 17.6$		
	$x = \frac{55}{17} = \underline{3.2353\text{ (m)}}$	A1	cao

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - S1
0983-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

S1– June 2017 – Markscheme

Ques	Solution	Mark	Notes
1(a)	(If A,B are independent,)P($A\cap B$)= 0.2 × 0.3 = 0.06 (Using P($A\cap B$) = P(A) + P(B) – P($A\cup B$)) EITHER P($A\cap B$) = 0.2 + 0.3 – 0.4 = 0.1 OR P($A\cup B$) = 0.2 + 0.3 – 0.06 = 0.44 (A and B are not independent because) EITHER 0.1≠ 0.06 OR 0.4 ≠ 0.44	B1 B1 B1	
(b)(i)	$P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$ $=\dfrac{1}{3}$ So $P(A'\mid B)=\dfrac{2}{3}$	M1 A1 A1	FT from (a) M0 if independence assumed
(ii)	$P(A\cup B')=P(A)+P(B')-P(A\cap B')$ $=P(A)+1-P(B)-\left(P(A)-P(A\cap B)\right)$ $=\dfrac{4}{5}$	M1 m1 A1	M0 if independence assumed
2(a)	$E(X^2)=\text{Var}(X)+\left(E(X)\right)^2$ = 104	M1 A1	
(b)	E(Y) = 2E(X) + 3 = 23 Var(Y) = 4Var(X) = 16	M1 A1 M1 A1	Award M0 for 2×, M1 for 4×
3(a)	P(1 each col) = $\dfrac{4}{9}\times\dfrac{3}{8}\times\dfrac{2}{7}\times 6$ or $\dfrac{\binom{4}{1}\binom{3}{1}\binom{2}{1}}{\binom{9}{3}}$ $=\dfrac{2}{7}$ (0.286)	M1A1 A1	M1A0 if 6 omitted
(b)	P(3 same col) = $\dfrac{4}{9}\times\dfrac{3}{8}\times\dfrac{2}{7}+\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}$ or $\dfrac{\binom{4}{3}+\binom{3}{3}}{\binom{9}{3}}$ $=\dfrac{5}{84}$ (0.0595)	M1A1 A1	

Ques	Solution	Mark	Notes
4(a)(i)	P(at least 1 error) = $1-e^{-0.8}$ = 0.551	**M1** **A1**	M0 exactly 1, M1 more than 1 Accept the use of tables
(ii)	P(3rd page 1st error) = $(1 – 0.551)^2 \times 0.551$ = 0.111	**M1** **A1**	FT $0.449^2 \times$ answer to (a)
(b)(i)	$p_n = (e^{-0.8})^n$ $= e^{-0.8n}$	**M1** **A1**	Accept 0.449 for $e^{-0.8}$ A1 can be earned later
(ii)	Consider $e^{-0.8n} < 0.001$ $-0.8n\log e < \log 0.001$ giving $n > 8.63...$ Therefore take $n = 9$	**M1** **A1** **A1** **A1**	Allow the use of = Accept solutions using tables or evaluating powers of $e^{-0.8}$
5(a)(i)	X is B(10,0.7)	**B1**	
(ii)	E(X) = 7 SD(X) = $\sqrt{10\times 0.7\times 0.3}$ = 1.45	**B1** **M1** **A1**	Accept $\sqrt{2.1}, \frac{\sqrt{210}}{10}$
(iii)	Let Y = Number of games won by Brian so that Y is B(10,0.3) $P(X \ge 6) = P(Y \le 4)$ = 0.8497	**M1** **m1** **A1**	M0 no working Accept summing individual probabilities
(b)	Let G = number of games lasting more than 1 hour G is B(44,0.06) which is approx Po(2.64) P(G >2) = $1-e^{-2.64}\left(1+2.64+\frac{2.64^2}{2}\right)$=0.492	**B1** **M1A1**	si M0 no working
6(a)	$E(X)=\frac{1}{54}\left(2\times 2^2+3\times 3^2+4\times 4^2+5\times 5^2\right)$ = 4.15 (112/27) E(X^2)=$\frac{1}{54}\left(2^2\times 2^2+3^2\times 3^2+4^2\times 4^2+5^2\times 5^2\right)$ (18.11.) Var(X) = 18.11.. – 4.1481...^{2} = 0.904 (659/729)	**M1** **A1** **M1A1** **A1**	Allow MR for wrong range .
(b)	The possible values are 4,5,5 P(Sum = 14) = $\frac{4^2\times 5^2\times 5^2}{54^3}\times 3$ = 0.191	**B1** **M1A1** **A1**	si Accept 0.19

Ques	Solution	Mark	Notes
7(a)(i)	P(+) $= 0.05\times0.96+0.95\times0.02$ $= 0.067$	**M1A1** **A1**	
(ii)	P(disease|+) $= \dfrac{0.05\times0.96}{0.067}$ $= 0.716$ cao	**B1B1** **B1**	FT denominator from (i)
(b)(i)	P(2nd +) = 0.716 × 0.96 + (1 – 0.716) × 0.02 = 0.693	**M1** **A1**	FT from (a)
(ii)	P(disease|2nd +) $= \dfrac{0.716\times0.96}{0.693}$ = 0.992 cao (2304/2323)	**M1** **A1**	Accept $\dfrac{0.05\times0.96^2}{0.05\times0.96^2+0.95\times0.02^2}$
8(a)(i)	F(2) = 1 so $12k = 1$ giving $k=\dfrac{1}{12}$	**M1** **A1**	Convincing
(ii)	Use of $F(x) = 0.95$ $x^4 - x^2 - 11.4 = 0$ $x^2 = 3.913...$ $x = 1.98$	**M1** **A1** **A1** **A1**	
(iii)	$P(X < 1.25 \| X < 1.75) = \dfrac{F(1.25)}{F(1.75)}$ $= \dfrac{1.25^4-1.25^2}{1.75^4-1.75^2}$ = 0.14	**M1** **A1** **A1**	
(b)(i)	$f(x) = F'(x)$ $= \dfrac{1}{6}(2x^3 - x)$	**M1** **A1**	M1 for knowing you have to differentiate
(ii)	Use of $E(\sqrt{X}) = \int\sqrt{x}f(x)\mathrm{d}x$ $= \dfrac{1}{6}\int\sqrt{x}(2x^3-x)\mathrm{d}x$ $= \dfrac{1}{6}\left[\dfrac{4x^{9/2}}{9}-\dfrac{2x^{5/2}}{5}\right]_1^2$ = 1.29	**M1** **A1** **A1** **A1**	FT from (b)(i) if answer between 1 and 2

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - S2
0984-01

INTRODUCTION

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S2 - June 2017 - Markscheme

Ques	Solution	Mark	Notes
1(a)	E(X) = 2.0, E(Y) = 1.6 E(W) = E(X)E(Y) = 3.2 Var(X) = 1.2,Var(Y) = 1.28 $E(X^2) = \text{Var}(X) + [E(X)]^2 = 5.2$ $E(Y^2) = \text{Var}(Y) + [E(Y)]^2 = 3.84$ Var(W) = $E(X^2)E(Y^2) - [E(X)E(Y)]^2$ = 9.73	**B1** **M1** **A1** **B1** **M1A1** **A1** **M1** **A1**	si si Allow
(b)	P(W = 0) = P{(X = 0)$\cup$(Y = 0)} = P(X = 0) + P(Y = 0) – P{(X = 0)$\cap$(Y = 0)} = $0.6^5 + 0.8^8 - 0.6^5 \times 0.8^8$ = 0.232	**M1** **m1** **A1** **A1**	P(W = 0) = 1 – P(X≥0)P(Y≥0) =1 – (1 – P(X = 0))(1 –P(Y = 0)) = 1 – (1 – 0.6^5)(1 – 0.8^8) = 0.232
2	Under H_0, the number, X, of breakdowns in 100 days is Poi(80) which is approx N(80,80) $z = \dfrac{64.5 - 80}{\sqrt{80}}$ = – 1.73 p-value = 0.0418 There is strong evidence to conclude that the mean number of breakdowns per day has been reduced.	**B1B1** **M1A1** **A1** **A1** **A1**	Award M1A0 for an incorrect or no continuity correction and FT for the following marks 64 → z = –1.79 → p-value = 0.0367 63.5 → z = –1.84 → p-value = 0.0329 FT the p-value
3(a)	90th percentile = $\mu + 1.282\sigma$ = 128	**M1** **A1**	
(b)	Let X = weight of an apple, Y = weight of a pear Let S denote the sum of the weights of 10 apples Then E(S) = 1100 Var(S) = 10×14^2 = 1960 $z = \dfrac{1000 - 1100}{\sqrt{1960}}$ = (–) 2.26 Prob = 0.01191	**B1** **M1** **A1** **m1** **A1** **A1**	
(c)	Let $U = X_1 + X_2 + X_3 - Y_1 - Y_2$ E(U) = 3 × 110 – 2 × 160 = 10 Var(U) = $3 \times 14^2 + 2 \times 16^2$ = 1100 We require P(U > 0) $z = \dfrac{0 - 10}{\sqrt{1100}}$ = (–) 0.30 Prob = 0.6179	**M1** **A1** **M1A1** **m1** **A1** **A1**	si, condone incorrect notation

Ques	Solution	Mark	Notes
4(a)	Let x,y denote distance travelled by models A,B respectively. $\bar{x}=166.9; \bar{y}=163.9$ Standard error = $\sqrt{\dfrac{2\times 2.5^2}{8}}$ (=1.25) 95% confidence limits are $166.9-163.9\pm 1.96\times 1.25$ giving [0.55,5.45]	**B1 B1** **M1A1** **M1A1** **A1**	
(b)	The lower end of the interval will be 0 if $1.25z=3$ $z=2.4$ Tabular value = 0.008(2) cao Smallest confidence level = 98.4%	**M1** **A1** **A1** **A1**	FT their SE and $\bar{x},\bar{y}$ (for the first two marks only)
5(a)(i)	Under H_0, X is B(50,0.75) Since p > 0.5, we consider X' which is B(50,0.25) $P(X\le 31)=P(X'\ge 19)=0.0287$ $P(X\ge 44)=P(X'\le 6)=0.0194$ Significance level = 0.0481	**B1** **M1** **A1** **A1** **A1**	si
(ii)	If p = 0.5, P(Accept H_0) = P(32 ≤ X ≤ 43) = 1 – 0.9675 = 0.0325	**M1** **A1**	
(b)(i)	Let Y now denote the number of heads so that under H_0, Y is B(200,0.75) ≅ N(150,37.5) $z=\dfrac{139.5-150}{\sqrt{37.5}}$ =(–)1.71 Tabular value = 0.0436 p-value = 0.0872 (accept 0.0873)	**B1** **M1A1** **A1** **A1** **A1**	Award M1A0 for incorrect or no continuity correction but FT for following marks 139 → z = –1.80 → p-value = 0.0359 138.5 → z = –1.88 → p-value = 0.0301 Penultimate A1 for doubling line above
(ii)	There is insufficient evidence to reject H_0.	**A1**	FT the p-value

Ques	Solution	Mark	Notes
6(a)(i)	$f(x) = \frac{1}{b-a}$, $a \le x \le b$ $= 0$ otherwise	B1	Allow <
(ii)	$E(X^2) = \frac{1}{b-a}\int x^2 \mathrm{d}x$	M1	
	$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$	A1	Condone omission of limits
	$= \frac{b^3-a^3}{3(b-a)}$	A1	
	$= \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$ $= \frac{(b^2+ab+a^2)}{3}$	A1	
(iii)	$\mathrm{Var}(X) = E(X^2) - \left(E(X)\right)^2$	M1	
	$= \frac{b^2+ab+a^2}{3} - \left(\frac{a^2+2ab+b^2}{4}\right)$	A1	
	$= \frac{4b^2+4ab+4a^2-3a^2-6ab-3b^2}{12}$ $= \frac{(b-a)^2}{12}$	A1	Convincing
(b)(i)	$E(Y) = \frac{1}{b-a}\int \frac{1}{x}\mathrm{d}x$	M1	
	$= \frac{1}{b-a}\left[\ln x\right]_a^b$	A1	Condone omission of limits
	$= \frac{\ln b - \ln a}{b-a}$	A1	
(ii)	$P(Y \le y) = P\left(\frac{1}{X} \le y\right)$	M1	
	$= P\left(X \ge \frac{1}{y}\right)$ $= \frac{b - \frac{1}{y}}{b-a}$	A1	

Ques	Solution	Mark	Notes
(iii)	PDF = derivative of above line $= \dfrac{1}{(b-a)y^2}$	**M1** **A1**	

GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - S3
0985-01

INTRODUCTION

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S3 – June 2017 – Markscheme

Ques	Solution	Mark	Notes
1	$\bar{x} = 59.1$ si	**B1**	
	Var estimate = $\dfrac{349425}{99} - \dfrac{5910^2}{100 \times 99} = 1.4545\ldots(16/11)$	**M1A1**	
	(Accept division by 100 which gives 1.44)		
	99% confidence limits are		
	$59.1 \pm 2.576\sqrt{1.4545/100}$	**M1A1**	M0 if 100 or √ omitted, A1 correct z
	giving [58.8,59.4] cao	**A1**	
2(a)	Let S denote the score on one of the dice. Then,		
	$\text{P}(S \le x) = \dfrac{x}{6}$ for x = 1,2,3,4,5,6	**M1**	
	So		
	$\text{P}(X \le x)$ = P(All three scores $\le x$)	**A1**	
	$= \left(\dfrac{x}{6}\right)^3$		Convincing
(b)	$P(X = x) = P(X \le x) - P(X \le x-1)$	**M1**	
	$= \dfrac{x^3 - (x-1)^3}{216} \quad \left(\dfrac{3x^2 - 3x + 1}{216}\right)$	**A1**	
(c)	A valid attempt at considering relevant probabilities.	**M1**	
	Most likely value = 6	**A1**	
3	$\bar{x} = 41.1; \bar{y} = 34.9$	**B1**	
	$s_x^2 = \dfrac{84773}{49} - \dfrac{2055^2}{49 \times 50} = 6.3775\ldots(625/98)$	**M1A1**	
	$s_y^2 = \dfrac{61121}{49} - \dfrac{1745^2}{49 \times 50} = 4.5$	**A1**	
	[Accept division by 50 giving 6.25 and 4.41]		
	SE = $\sqrt{\dfrac{6.3775..}{50} + \dfrac{4.5}{50}}$ = 0.4664... (0.4617…)	**M1A1**	M0 no working
	$z = \dfrac{41.1 - 34.9 - 5}{0.4664..}$	**m1A1**	
	=2.57 (2.60)	**A1**	
	p-value = 0.005	**A1**	
	Very strong evidence in support of Mair's belief (namely that the difference in the mean weights of male and female dogs is more than 5kg)	**A1**	FT the p-value if less than 0.05

Ques	Solution	Mark	Notes
4(a)	$\hat{p}=0.32$ si	B1	
	ESE = $\sqrt{\dfrac{0.32\times0.68}{75}}$ (= 0.05386..) si	M1A1	
	95% confidence limits are $0.32\pm1.96\times0.05386..$	M1A1	M0 no working A1 correct z
	giving [0.21,0.43]	A1	
(b)	The statement is incorrect because you cannot make a probability statement about a constant interval containing a constant value.	B1	
	EITHER The correct interpretation is that the calculated interval is an observed value of a random interval which contains the value of p with probability 0.95.	B1	
	OR If the process could be repeated a large number of times, then (approx) 95% of the intervals produced would contain p.	(B1)	
5(a)	$\sum x=306;\sum x^2=10407.52$	B1B1	
	UE of μ = 34	B1	No working need be seen
	UE of $\sigma^2=\dfrac{10407.52}{8}-\dfrac{306^2}{72}$	M1	M0 division by 9
	= 0.44	A1	Answer only no marks
(b)	DF = 8 si	B1	
	t-value = 2.306	B1	M0 for using Z
	95% confidence limits are		FT from (a)
	$34\pm2.306\times\sqrt{\dfrac{0.44}{9}}$	M1	
	giving [33.5,34.5] cao	A1	

Ques	Solution	Mark	Notes
6(a)	$S_{xy} = 2744 - 140 \times 107.3/6 = 240.33$	**B1**	M0 no working
	$S_{xx} = 3850 - 140^2/6 = 583.33$	**B1**	
	$b = \dfrac{240.33}{583.33} = 0.412$	**M1** **A1**	
	$a = \dfrac{107.3 - 0.412 \times 140}{6} = 8.27$	**M1** **A1**	
(b)(i)	$H_0 : \beta = 0.4$; $H_1 : \beta \neq 0.4$	**B1**	
(ii)	SE of $b = \dfrac{0.2}{\sqrt{583.33}}$ (0.00828..)	**M1A1**	
	Test statistic = $\dfrac{0.412 - 0.4}{0.00828}$	**m1A1**	
	= 1.45	**A1**	
	Tabular value = 0.0735	**A1**	
	p-value = 0.147	**A1**	Award for doubling line above
(iii)	The data support Emlyn's belief.	**A1**	FT the p-value

Ques	Solution	Mark	Notes
7(a)(i)	$E(X) = p + \frac{2(1-p)}{3} + \frac{3(1-p)}{3} + \frac{4(1-p)}{3}$	M1	
	$= \frac{3p+2-2p+3-3p+4-4p}{3}$	A1	
	$= 3-2p$	A1	
(ii)	$E(X^2) = p + (2^2+3^2+4^2)\frac{(1-p)}{3}$	M1A1	$\left(\frac{29}{3} - \frac{26}{3}p\right)$
	$\text{Var}(X) = p + (2^2+3^2+4^2)\frac{(1-p)}{3} - (3-2p)^2$	A1	
	$= \frac{2}{3} + \frac{10}{3}p - 4p^2$	A1	
	$= \frac{2}{3}(1-p)(1+6p)$		
(b)(i)	$E(U) = \frac{3-E(X)}{2}$	M1	M0 if no E
	$= \frac{3-(3-2p)}{2}$	A1	
	$= p$		
	(Therefore U is an unbiased estimator)		
(ii)	$\text{Var}(U) = \frac{1}{4}\text{Var}(\overline{X})$	M1	
	$= \frac{\frac{2}{3}(1-p)(1+6p)}{4n}$	A1	
(c)(i)	Y is $\text{B}(n,p)$	B1	
(ii)	$\text{E}(V) = \frac{E(Y)}{n}$	M1	M0 if no E
	$= \frac{np}{n} = p$	A1	
	(Therefore V is an unbiased estimator)		
(iii)	$\text{Var}(V) = \frac{\text{Var}(Y)}{n^2}$	M1	
	$= \frac{p(1-p)}{n}$ oe	A1	

Ques	Solution	Mark	Notes
(d)	$\dfrac{\text{Var}(U)}{\text{Var}(V)} = \dfrac{\frac{2}{3}(1-p)(1+6p)}{4n} \div \dfrac{p(1-p)}{n}$	**M1**	
	$= \dfrac{1+6p}{6p}$ oe cao	**A1**	
	> 1 oe	**A1**	No FT for incorrect ratio
	Therefore V is the better estimator.	**A1**	

CE Maths (S3) MS Summer 2017