MS3
£3.00

GENERAL CERTIFICATE OF EDUCATION
TYSTYSGRIF ADDYSG GYFFREDINOL

MARKING SCHEME

MATHEMATICS
AS/Advanced

JANUARY 2009

INTRODUCTION

The marking schemes which follow were those used by WJEC for the January 2009 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics C1 January 2009

Solutions and Mark Scheme

Final Version

1. (*a*) Gradient of $BC = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $BC = {}^1/_4$ (or equivalent) A1

A correct method for finding the equation of $BC(AD)$ using candidate's gradient for BC M1

Equation of BC : $y - 4 = {}^1/_4\,(x - 5)$ (or equivalent)

(f.t. candidate's gradient for BC) A1

Equation of BC : $x - 4y + 11 = 0$ (convincing) A1

Use of $m_{AB} \times m_{CD} = -1$ M1

Equation of AD : $y - (-1) = -4(x - 2)$ (or equivalent)

(f.t. candidate's gradient of BC) A1

Special case:

Verification of equation of BC by substituting coordinates of **both** B and C into the given equation M1

Making an appropriate statement A1

(*b*) An attempt to solve equations of BC and AD simultaneously M1

$x = 1$, $y = 3$ (convincing) (c.a.o.) A1

Special case

Substituting (1, 3) in equations of **both** BC and AD M1

Convincing argument that coordinates of D are (1, 3) A1

(*c*) A correct method for finding the length of CD M1

$CD = \sqrt{17}$ A1

(*d*) A correct method for finding E M1

$E(0, 7)$ A1

2. (*a*) $\frac{10\sqrt{3} - 1}{4 - \sqrt{3}} = \frac{(10\sqrt{3} - 1)(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})}$ M1

Numerator: $40\sqrt{3} + 10 \times 3 - 4 - \sqrt{3}$ A1

Denominator: $16 - 3$ A1

$\frac{10\sqrt{3} - 1}{4 - \sqrt{3}} = \frac{39\sqrt{3} + 26}{13} = 3\sqrt{3} + 2$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $4 - \sqrt{3}$

(*b*) $(2 + \sqrt{5})(5 - \sqrt{20}) = 10 - 2\sqrt{20} + 5\sqrt{5} - \sqrt{5} \times \sqrt{20}$

(4 terms, at least 3 correct) M1

$\sqrt{20} = 2\sqrt{5}$ B1

$\sqrt{5} \times \sqrt{20} = 10$ B1

$(2 + \sqrt{5})(5 - \sqrt{20}) = \sqrt{5}$ (c.a.o.) A1

Alternative Mark Scheme

$(2 + \sqrt{5})(5 - \sqrt{20}) = (2 + \sqrt{5})(5 - 2\sqrt{5})$ B1

$(2 + \sqrt{5})(5 - 2\sqrt{5}) = 10 - 4\sqrt{5} + 5\sqrt{5} - \sqrt{5} \times 2\sqrt{5}$

(4 terms, at least 3 correct) M1

$\sqrt{5} \times 2\sqrt{5} = 10$ B1

$(2 + \sqrt{5})(5 - \sqrt{20}) = \sqrt{5}$ (c.a.o.) A1

3. (*a*) $\frac{dy}{dx} = 2x - 9$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 6$ in candidate's expression for $\frac{dy}{dx}$ m1

Gradient of tangent at $P = 3$ (c.a.o.) A1

Equation of tangent at P: $y - (-5) = 3(x - 6)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided both M1 and m1 awarded) A1

(*b*) Use of gradient of tangent at $Q \times {}^{1}/_{7} = -1$ M1

Equating candidate's expression for $\frac{dy}{dx}$ and candidate's value for gradient of tangent at Q m1

$2x - 9 = -7 \Rightarrow x = 1$ (f.t. candidate's expression for $\frac{dy}{dx}$) A1

4. $a = 3$ B1

$b = -2$ B1

$c = 5$ B1

A positive quadratic graph M1

Minimum point $(-b, c)$ A1

5. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1
$b^2 - 4ac = 8^2 - 4 \times (3k - 2) \times k$ A1
Putting $b^2 - 4ac < 0$ m1
$3k^2 - 2k - 16 > 0$ (convincing) A1
Finding critical points $k = -2$, $k = {}^8/_3$ B1
A statement (mathematical or otherwise) to the effect that
$k < -2$ or ${}^8/_3 < k$ (or equivalent) (f.t. candidate's critical points) B2
Deduct 1 mark for each of the following errors
the use of non-strict inequalities
the use of the word 'and' instead of the word 'or'

6. (*a*) $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ (–1 for each error)
(–1 for any subsequent 'simplification') B2

(*b*) An expression containing $k \times (1/4)^2 \times (2x)^3$, where k is an integer $\neq 1$ and is either the candidate's coefficient for the a^2b^3 term in (*a*) or is derived from first principles M1
Coefficient of $x^3 = 5$ (c.a.o.) A1

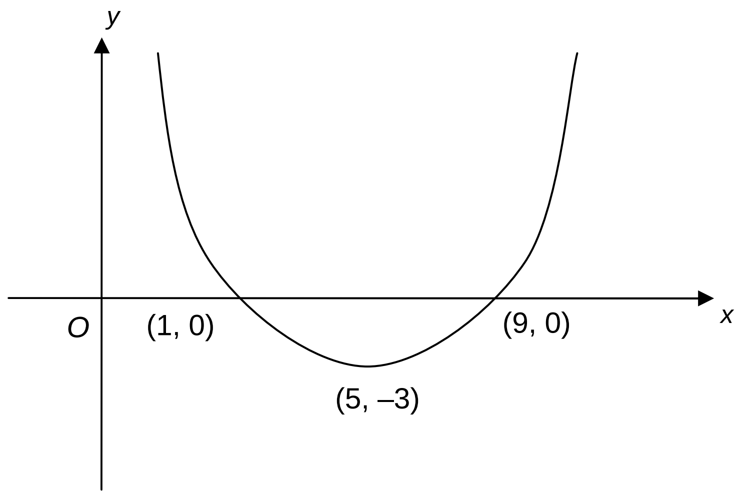
7. (*a*) An attempt to calculate $3^3 - 17$ M1
Remainder = 10 A1

(*b*) Attempting to find $f(r) = 0$ for some value of r M1
$f(2) = 0 \Rightarrow x - 2$ is a factor A1
$f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct M1
$f(x) = (x - 2)(6x^2 + 5x - 4)$ A1
$f(x) = (x - 2)(3x + 4)(2x - 1)$ (f.t. only $6x^2 - 5x - 4$ in above line) A1
Roots are $x = 2, -{}^4/_3, {}^1/_2$ (f.t. for factors $3x \pm 4$, $2x \pm 1$) A1
Special case
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1

8. (*a*) $y + \delta y = 7(x + \delta x)^2 + 5(x + \delta x) - 2$ B1
Subtracting y from above to find δy M1
$\delta y = 14x\delta x + 7(\delta x)^2 + 5\delta x$ A1
Dividing by δx and letting $\delta x \to 0$ M1
$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 14x + 5$ (c.a.o.) A1

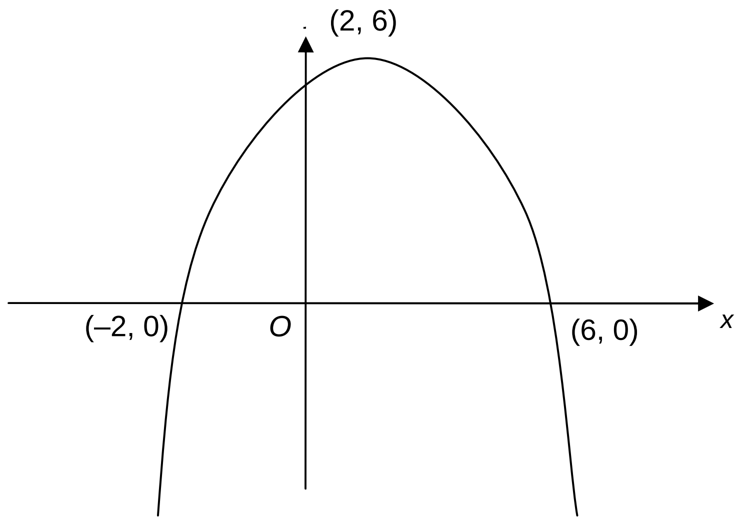
(*b*) Required derivative = $2 \times (-3) \times x^{-4} + 5 \times ({}^2/_3) \times x^{-1/3}$ B1, B1

9. (*a*)



Concave up curve and *y*-coordinate of minimum = –3	B1
x-coordinate of minimum = 5	B1
Both points of intersection with *x*-axis	B1

(*b*)



Concave down curve and *x*-coordinate of maximum = 2	B1
y-coordinate of maximum = 6	B1
Both points of intersection with *x*-axis	B1

10. (*a*) $\frac{dy}{dx} = 3x^2 + 6x - 9$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

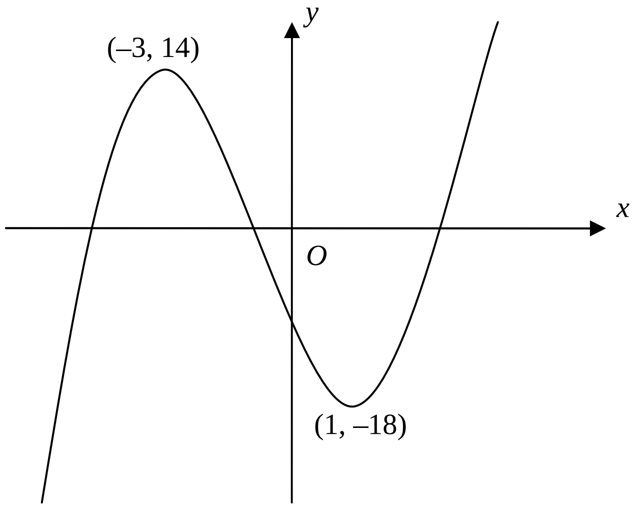
$x = -3, 1$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(-3, 14)$ and $(1, -18)$ (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding
either $(-3, 14)$ is a maximum point
or $(1, -18)$ is a minimum point (f.t. candidate's derived values) M1

Correct conclusion for other point
(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points
(f.t. candidate's derived maximum and minimum points) A1

(*c*) A statement identifying the number of roots as the number of times the curve crosses the *x*-axis (any curve) M1

Correct interpretation of the number of roots from the candidate's **cubic** graph. A1

Mathematics C2 January 2009

Solutions and Mark Scheme

Final Version

1.

0	1·0		
0·25	0·996108949		
0·5	0·94117647		
0·75	0·759643916	(3 values correct)	B1
1	0·5	(5 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$I \approx \frac{0{\cdot}25}{2} \times \{1{\cdot}0 + 0{\cdot}5 + 2(0{\cdot}996108949 + 0{\cdot}94117647 + 0{\cdot}759643916)\}$

$I \approx 0{\cdot}861732333$

$I \approx 0{\cdot}862$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}2$

0	1·0		
0·2	0·998402555		
0·4	0·975039001		
0·6	0·885269121		
0·8	0·709421112		
1	0·5	(all values correct)	B1

Correct formula with $h = 0{\cdot}2$ M1

$I \approx \frac{0{\cdot}2}{2} \times \{1{\cdot}0 + 0{\cdot}5 + 2(0{\cdot}998402555 + 0{\cdot}975039001 + 0{\cdot}885269121 + 0{\cdot}709421112)\}$

$I \approx 0{\cdot}863626357$

$I \approx 0{\cdot}864$ (f.t. one slip) A1

2. (*a*) $6(1 - \sin^2\theta) + \sin\theta = 4$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$6\sin^2\theta - \sin\theta - 2 = 0 \Rightarrow (3\sin\theta - 2)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{2}{3}, -\frac{1}{2}$ A1

$\theta = 41{\cdot}81°, 138{\cdot}19°, 210°, 330°$ (41·81°, 138·19°) B1

(210°) B1

(330°) B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) $3x = 123{\cdot}00°, 303{\cdot}00°, 483{\cdot}00°,$ (one value) B1

$x = 41{\cdot}00°, 101{\cdot}00°, 161{\cdot}00°,$ (one value) B1

(three values) B1

Note: Subtract 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) $9^2 = 7^2 + x^2 - 2 \times 7 \times x \times {}^2/_7$ (correct substitution in cos rule) M1

$x^2 - 4x - 32 = 0$ A1

$x = 8$ (f.t. one slip in simplified quadratic) A1

(*b*) (i) Use of $\sin^2 B\hat{A}C = 1 - \cos^2 B\hat{A}C$ M1

$\sin B\hat{A}C = \frac{\sqrt{45}}{7}$ A1

(ii) $\frac{\sin ACB}{7} = \frac{\sin B\hat{A}C}{9}$ (correct use of sin rule) m1

$\sin ACB = \frac{\sqrt{45}}{9} = \frac{\sqrt{5}}{3}$ (c.a.o.) A1

4. (*a*) $a + 12d = 51$ B1

$a + 8d = k \times (a + d)$ ($k = 5, {}^1/_5$) M1

$a + 8d = 5(a + d)$ A1

$3d = 4a$

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$d = 4$, $a = 3$ (both values) (c.a.o.) A1

(*b*) $S_{20} = \frac{20}{2} \times (5 + 62)$ (substitution of values in formula for sum of A.P.) M1

$S_{20} = 670$ A1

5. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by *r* and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) $r = 0{\cdot}9$ B1

$S_{18} = \frac{10(1 - 0{\cdot}9^{18})}{1 - 0{\cdot}9}$ (f.t. candidate's numerical value for *r*) M1

$S_{18} \approx 84{\cdot}990 = 85$ (c.a.o.) A1

(*c*) (i) $ar = -4$ B1

$\frac{a}{1 - r} = 9$ B1

An attempt to solve these equations simultaneously by eliminating *a* M1

$9r^2 - 9r - 4 = 0$ (convincing) A1

(ii) $r = -{}^1/_3$ (c.a.o.) B1

$|r| < 1$ E1

6. (*a*) $3 \times \frac{x^{-1}}{-1} - 2 \times \frac{x^{3/2}}{3/2} + c$ (Deduct 1 mark if no c present) B1,B1

(*b*) (i) $5x - 4 - x^2 = 0$ M1

An attempt to solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 4$ (o.e.) m1

$(x - 1)(x - 4) = 0 \Rightarrow x = 1, x = 4$ (both values, c.a.o.) A1

(ii) Total area $= \int_1^4 (5x - 4 - x^2)\,dx - \int_4^5 (5x - 4 - x^2)\,dx$

(use of integration) M1

(subtraction of integrals with correct use of candidate's x_A, x_B and 5 as limits) m1

$= [(5/2)x^2 - 4x - (1/3)x^3]_1^4 - [(5/2)x^2 - 4x - (1/3)x^3]_4^5$

(correct integration) B3

$= \{[40 - 16 - 64/3] - [5/2 - 4 - 1/3)]\}$
$\quad - \{[125/2 - 20 - 125/3] - [40 - 16 - 64/3]\}$

(substitution of candidate's limits in at least one integral) m1

$= 19/3$ (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (relationship between log and power) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indicies) B1

$\log_a xy = p + q$ (relationship between log and power)

$\log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) $\log_9 x = -\frac{1}{2} \Rightarrow x = 9^{-1/2}$ (rewriting log equation as power equation) M1

$x = 9^{-1/2} \Rightarrow x = \frac{1}{3}$ A1

(*c*) $2\log_a 3 = \log_a 3^2$ (power law) B1

$\log_a x + 2\log_a 3 = \log_a (3^2 \times x)$ (addition law) B1

$4x + 7 = 3^2 \times x$ (removing logs) M1

$x = 1{\cdot}4$ (c.a.o.) A1

8. (*a*) $A(-2, 1)$ B1
A correct method for finding the radius M1
Radius = 5 A1

(*b*) An attempt to substitute $(6 - x)$ for y in the equation of the circle M1
$x^2 - 3x + 2 = 0$ (or $2x^2 - 6x + 4 = 0$) A1
$x = 1$, $x = 2$ (correctly solving candidate's quadratic, both values) A1
Points of intersection are (1, 5), (2, 4) (c.a.o.) A1

(*c*) Distance between centres of C_1 and $C_2 = 13$ B1
Use of the fact that distance between centres = sum of the radii M1
$r = 8$ (c.a.o.) A1

9. (*a*) Substitution of values in area formula for triangle M1
Area = $\frac{1}{2} \times 4{\cdot}8^2 \times \sin 0{\cdot}7 = 7{\cdot}42$ cm^2. A1

(*b*) Let $R\hat{O}Q = \varphi$ radians
$4{\cdot}8 \times \varphi = L$, $\frac{1}{2} \times 4{\cdot}8^2 \times \varphi = A$ (at least one correct equation) B1
An attempt to eliminate φ M1
$k = 2{\cdot}4$ A1

A Level Mathematics C3
January 2009
Marking Scheme

1.		$h = \dfrac{2\pi}{\frac{9}{4}} = \dfrac{\pi}{18}$	M1 (correct formula with $h = \pi/18$)
		Integral $= \dfrac{\pi}{3 \times 18}$ [0 + (–0.26651509) + 4 (–0.01530883 – 0.14384104) + 2(–0.06220246)]	B1 (3 values) B1 (other 2 values)
		≈ -0.0598	A1 (F.T. one slip)
		$\int_0^{\frac{2\pi}{9}} \ln(\cos^2 x)\,dx \approx 2(-0.0598) = -0.1196$	B1
			(5)
2.	(a)	$\theta = 0$, $\cos 2\theta = 1$, for example	B1 (choice of θ and one correct evaluation)
		$2\cos^2\theta - \sin^2\theta = 2$	B1
		(statement is false)	
	(b)	$3(\sec^2\theta - 1) = 7 + \sec\theta$	M1 (use of correct formula)
		$3\sec^2\theta - \sec\theta - 10 = 0$	M1 (attempt to solve quadratic, or correct formula or
		$(3\sec\theta + 5)(\sec\theta - 2) = 0$	$(a\sec\theta + b)(c\sec\theta + d)$ with $ac = 3$ $bd = -10$)
		$\sec\theta = -\dfrac{5}{3}, 2$	
		$\cos\theta = -\dfrac{3}{5}, \dfrac{1}{2}$	A1 (values of $\cos\theta$)
		$\theta = 126.9^\circ, 233.1^\circ, \quad 60^\circ, 300^\circ$ (allow to nearest degree)	B1 (126.9°) B1 (233.1°) B1 (60°, 300°)
			(8)

3. (a) $2x + 3x\dfrac{dy}{dx} + 3y + 4y\dfrac{dy}{dx} - 2 = 0$ — B1 $\left(3x\dfrac{dy}{dx} + 3y\right)$ (*o.e*)

$2 + 3\dfrac{dy}{dx} + 6 + 8\dfrac{dy}{dx} - 2 = 0$ — B1 $\left(4y\dfrac{dy}{dx}\right)$ (*o.e*)

B1 (correct diff ^{n} of $x^2, -2x$ and 13)

$\dfrac{dy}{dx} = -\dfrac{6}{11}$ — B1 (F.T. one slip)

(b) $\dfrac{dy}{dx} = \dfrac{8e^{2t} + 3e^{t}}{2e^{t}}$ — M1

B1 (numerator $ke^{2t} + 3e^{t}$, $k = 4, 8$)

B1 (k = 8)
B1 (denominator)

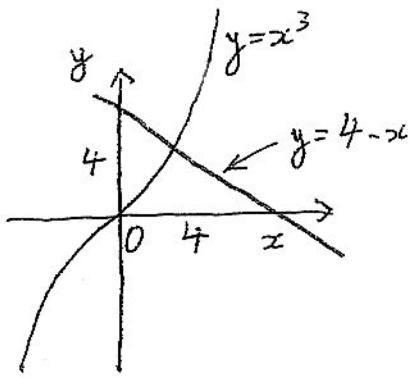
$\dfrac{8e^{2t} + 3e^{t}}{2e^{t}} = 6$ — M1

$8e^{t} = 9$ — M1

$t = ln\left(\dfrac{9}{8}\right) \approx 0.118$ — A1 (C.A.O)

(11)

4. (a)



B1 $(y = x^3)$
B1 $(y = 4 - x)$

B1 one real root
∵ one intersection

(b) $x_0 = 1.4$, $x_1 = 1.37506.....$, $x_2 = 1.37945........$ — B1 (x_1)

$x_3 = 1.37868....$, $x_4 = 1.37881.... \approx 1.3788$ — B1 (x_4 4 places)

Check 1.37875, 1.375885

<u>x</u>	<u>f(x)</u>
1.37875	– 0.00031
1.37885	·0.0036

M1 (attempt to find signs or values)

A1 (correct)

Changes of sign indicates presence of root which is 1.3788 correct to 4 dec. places — A1 (conclusion)

(8)

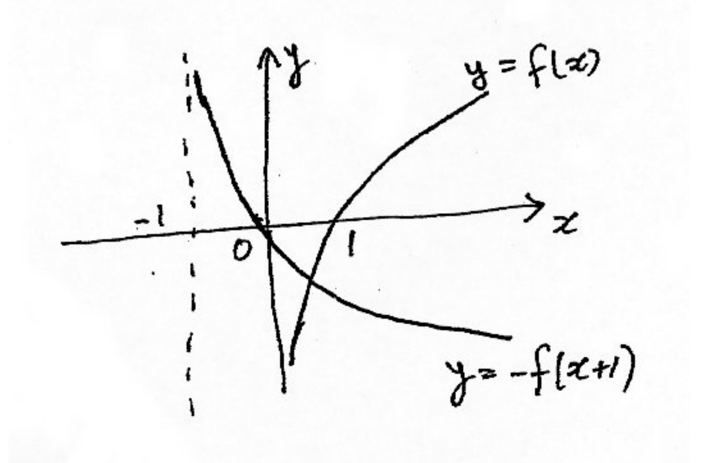
5.	(a)	(i)	$\dfrac{1}{sin\ x}$ x $cos\,x$	M1 $\left(\dfrac{f\,(x)}{sin\,x},\ f\,(x) = \pm\ cos\ x\right)$
			$= cot\ x$	A1 $(\,f\,(x) = cos\ x)$ A1 $(cot\ x)$
				$\left(accept\ \dfrac{1}{tan\ x}\right)$
		(ii)	$\dfrac{4}{\sqrt{1-(4x)^2}}$ (o.e.)	M1 $\dfrac{k}{\sqrt{1-(4x)^2}}$
				A1 $(k = 4)$
		(iii)	$\dfrac{(x^2+5)(6x)-(3x^2+2)(2x)}{(x^2+5)^2}$	M1 $\left(\dfrac{(x^2+5)\ f\,(x)-(3x^2+2)\ g(x)}{(x^2+5)^2}\right)$
				A1 $(f(x) = 6x,\ g(x) = 2x)$
			$= \dfrac{26x}{(x^2+5)^2}$	A1
	(b)	$x = \tan y$		
		$1 = \sec\ y^2\ \dfrac{dy}{dx}$		M1 $(l = f(y)\ \dfrac{dy}{dx},\ f\,(y) \neq k)$
				A1 $(f\,(y) = \sec^2\ y)$
		$\dfrac{dy}{dx} = \dfrac{1}{\sec^2\ y}$		
		$= \dfrac{1}{1+\tan^2\ y}$		A1
		$= \dfrac{1}{1+x^2}$		A1 (C.A.O)

(12)

6.	(a)	$2\|x\| + 9 = 5\|x\| + 5$	
		$3\|x\| = 4$	B1 $\begin{pmatrix} a\|x\| = b \\ a = 3,\ b = 4 \end{pmatrix}$
		$x = \pm 4/3$	B1 (both values) (F.T. a, b)
	(b)	$5x + 7 \le -4,\ x \le -\frac{3}{5}$	B1
and		$5x + 7 \ge -4$	M1
		$x \ge -\frac{11}{5}$	
		$-\frac{11}{5} \le x \le -\frac{3}{5}$	A1
			(5)

7.	(a)	(i)	$\frac{7}{6}\ln\|6x+5\| + c$	M1 $(k\ln\|6x+5\|, k = 7, \frac{7}{6})$
				A1 $\left(k = \frac{7}{6}\right)$
		(ii)	$\frac{1}{5}\sin 5x + c$	M1 $\left(k\sin 5x,\ k = \pm\frac{1}{5}, 5, 1\right)$
				A1 $\left(k = \frac{1}{5}\right)$
	(b)		$\left[-\frac{9}{2(2x+1)}\right]_0^1$	M1 $\left(\frac{k}{2x+1,}\ k = -9, \pm\frac{9}{2}\right)$
				A1 $\left(k = -\frac{9}{2}\right)$
			$= -\frac{9}{2}\left[\frac{1}{3} - 1\right]$	M1 $\left(k\left(\frac{1}{3} - 1\right)\right.$ allowable k
			$= 3$	A1 $\left(\text{allow F.T for } k = \pm\frac{9}{2}\right)$
				(8)

8.

	$y = f(x)$ $y = -f(x+1)$	B1 (asymptote and shape) B1 (1,0) B1 (asymptote) B1 (0,0) B1 (behaviour, large y)

(5)

9. (a) Let $y = 5x^2 + 4$

$y - 4 = 5x^2$ M1 $(y-4=5x^2)$

$x = \pm\sqrt{\dfrac{y-4}{5}}$ A2 (±) OR A1 (+) A1 $\left(\pm\dfrac{\sqrt{y-4}}{5}\right)$

$x = -\sqrt{\dfrac{y-4}{5}}$ since domain $x \le 0$ A1

$f^{-1}(x) = -\sqrt{\dfrac{x-4}{5}}$ (F.T $x = f(y)$) A1

(b) domain $x \ge 4$, Range $x \le 0$ (o.e) B1

(6)

10. (a) Range of $f(x) \ge 2 - k$ (o.e) B1

(b) $2 - k \ge 0$ B1

$k \le 2$ B1

(Greatest value of k is 2)

(c) $3(4-k)^2 + 4 = 31$ M1 (attempt to form equation, correct order, un......)

$(4-k)^2 = 9$ A1

$k = 1, 7$ A1

$\therefore k = 1$ A1 (F.T max value of k from (b))

(since $k \le 2$)

(7)

Mathematics FP1 January 2009

Solutions and Mark Scheme

Final Version

1	(a)	$\ln y = x \ln 2$	B1
		$\frac{1}{y}\frac{dy}{dx} = \ln 2$	B1
		$\frac{dy}{dx} = y \ln 2 = 2^x \ln 2$	B1
	(b)	$f(x+h) - f(x) = \frac{x+h}{x+h+1} - \frac{x}{x+1}$	M1
		$= \frac{(x+1)(x+h) - x(x+h+1)}{(x+h+1)(x+1)}$	A1
		$= \frac{x^2 + x + hx + h - x^2 - hx - x}{(x+h+1)(x+1)}$	A1
		$= \frac{h}{(x+h+1)(x+1)}$	A1
		$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
		$= \lim_{h \to 0} \frac{h}{h(x+h+1)(x+1)}$	M1
		$= \frac{1}{(x+1)^2}$	A1
2		$S_n = \sum_{r=1}^{n} (2r-1)^2$	M1
		$= 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$	A1
		$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$	A1A1A1
		$= \frac{n}{3}\left[4n^2 + 6n + 2 - 6n - 6 + 3\right]$	
		$= \frac{n(4n^2-1)}{3}$	
		$= \frac{n(2n+1)(2n-1)}{3}$ cao	A1

3 $\alpha + \beta + \gamma = -4$

$\beta\gamma + \gamma\alpha + \alpha\beta = 3$

$\alpha\beta\gamma = -2$ B1

Consider

` $\beta\gamma + \gamma\alpha + \alpha\beta = 3$ B1

$\beta\gamma^2\alpha + \gamma\alpha^2\beta\gamma + \alpha\beta^2\gamma = \alpha\beta\gamma(\alpha + \beta + \gamma)$ M1

$= 8$ A1

$\beta\gamma.\gamma\alpha.\alpha\beta = \alpha^2\beta^2\gamma^2$ M1

$= 4$ A1

The required equation is

$x^3 - 3x^2 + 8x - 4 = 0$ B1

[FT on candidates earlier results]

4 (a) $2(x + iy) - i(x - iy) = 1 + 4i$ B1

$2x - y + i(2y - x) = 1 + 4i$ B1B1

Equating real and imaginary parts, M1

$2x - y = 1$

$2y - x = 4$ A1

The solution is $x = 2$, $y = 3$ ($z = 2 + 3i$) A1A1

(b) $\frac{1+3i}{2-i} = \frac{(1+3i)(2+i)}{(2-i)(2+i)}$ M1

$= \frac{-1+7i}{5}$ A1A1

$|z| = \frac{\sqrt{1+49}}{5} = \sqrt{2}$ B1

Arg(z) = 1.71 rad (98.1°) M1A1

5 (a) Fixed points satisfy

$$\begin{bmatrix} 0.6 & 0.8 & 2 \\ -0.8 & 0.6 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ M1

giving

$0.4x - 0.8y = 2$

$0.8x + 0.4y = 3$ A1

The solution is $(x, y) = \left(4, -\frac{1}{2}\right)$. cao A1A1

(b) The centre is $\left(4, -\frac{1}{2}\right)$ [FT from (a)] B1

The angle of rotation satisfies M1

$\sin\theta = -\frac{4}{5}, \cos\theta = \frac{3}{5}$ A1

$\theta = -53.1^\circ$ or 306.9° A1

6 The statement is true for $n = 1$ since putting $n = 1$, we obtain

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

which is correct. B1

Let the statement be true for $n = k$, ie

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & 2k & 2k^2 \\ 0 & 1 & 2k \\ 0 & 0 & 1 \end{bmatrix}$$ M1

Consider

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^k \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ M1

$$= \begin{bmatrix} 1 & 2k & 2k^2 \\ 0 & 1 & 2k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ A1

$$= \begin{bmatrix} 1 & 2+2k & 2+4k+2k^2 \\ 0 & 1 & 2+2k \\ 0 & 0 & 1 \end{bmatrix}$$ M1A1

$$= \begin{bmatrix} 1 & 2(k+1) & 2(k+1)^2 \\ 0 & 1 & 2(k+1) \\ 0 & 0 & 1 \end{bmatrix}$$ A1

Thus true for $n = k \Rightarrow$ true for $n = k + 1$, hence proved by induction. A1

7 Let

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ M1

$\det(\mathbf{A}) = ad - bc$ A1

$$k\mathbf{A} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$ A1

Consider

$$\det(\text{k}\mathbf{A}) = k^2ad - k^2bc$$

$$= k^2(ad - bc)$$ A1

$$= k^2 \det(\mathbf{A})$$

8 (a) $u + \mathrm{i}v = x + \mathrm{i}y - (x + \mathrm{i}y)^2$ or $(x + \mathrm{i}y)(1 - x - \mathrm{i}y)$ M1

$= x + \mathrm{i}y - (x^2 + 2\mathrm{i}xy - y^2)$ A1

Equating real and imaginary parts, M1

$v = y(1 - 2x)$

$u = x - x^2 + y^2$ A1

(b) Putting $y = x$, M1

$v = x(1 - 2x)$

$u = x$ A1

Eliminating x, the equation of the locus of Q is M1

$v = u(1 - 2u)$ A1

9 (a)(i) $\det(\mathbf{A}) = (\lambda + 1)(2 - \lambda^2) + 1(2\lambda - 1) + \lambda(\lambda - 4)$ M1A1

$= 1 - \lambda^3$ A1

(ii) When $\lambda = 1$, $\det(\mathbf{A}) = 0$ so $\mathbf{A}$ is singular. B1

Since 1 is known to have only one real cube root, we know that $\lambda = 1$ is the only value of λ for which $\mathbf{A}$ is singular (or by factorising the cubic and showing the other roots are complex). B1

(b)(i) The equations are

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

The equations are consistent because the first and third rows are identical. B1

Put $z = \alpha$. M1

Then $y = \dfrac{4 - \alpha}{3}$, $x = \dfrac{1 - \alpha}{3}$ A1

(ii) $$\mathbf{A} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

Cofactor matrix = $\begin{bmatrix} 1 & -3 & -5 \\ 0 & 2 & 2 \\ 1 & -1 & -1 \end{bmatrix}$ B1

Adjugate matrix = $\begin{bmatrix} 1 & 0 & 1 \\ -3 & 2 & -1 \\ -5 & 2 & -1 \end{bmatrix}$ B1

From (a), determinant = 2

$\mathbf{A}^{-1} = \dfrac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ -3 & 2 & -1 \\ -5 & 2 & -1 \end{bmatrix}$ B1

$\mathbf{X} = \dfrac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ -3 & 2 & -1 \\ -5 & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$ B1

1. (a) Using $s = \frac{1}{2}(u+v)t$ with $s = 1200$, $v = 26$, $t = 60$. (2500) oe M1

$1200 = \frac{1}{2}(u + 26)\,60$ A1

$u = \underline{14 \text{ ms}^{-1}}$ cao A1

(b) Using $v = u + at$ with $v = 26$, $u = 14$(c), $t = 60$. oe M1

$26 = 14 + 60a$ ft u A1

$a = \underline{0.2 \text{ ms}^{-2}}$ ft u A1

(c) Using $v^2 = u^2 + 2as$ with $u = 26$, $a = 0.2$(c), $s = 2500$. (u = 14c, 3700) oe M1

$v^2 = 26^2 + 2 \times 0.2 \times 2500$ ft a A1

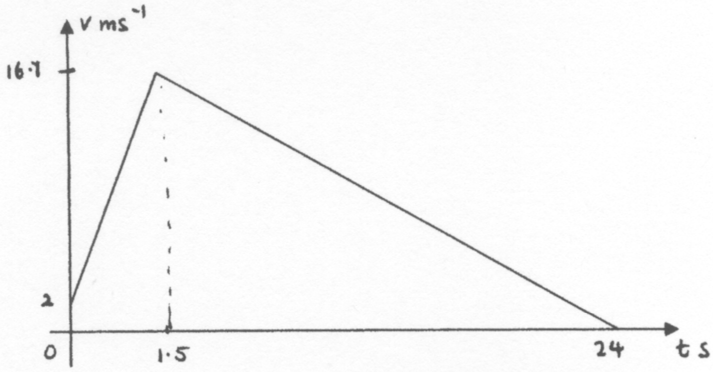
$v = \underline{40.9 \text{ ms}^{-1}}$ ft a A1

2. (a) Using $v = u + at$ with $u = 2$, $a = (\pm)9.8$, $t = 1.5$. M1

$v = 2 + 9.8 \times 1.5$ A1

$v = \underline{16.7 \text{ ms}^{-1}}$ A1

(b)



(0,2) to (1.5,16.7c) M1

(0,2) to (1.5,16.7) to (24,0) A1

axes, labels and units A1

(c) Height = distance travelled used M1

$= 0.5(2 + 16.7) \times 1.5 + 0.5 \times 16.7 \times 22.5$ ft v B1

$= \underline{201.9 \text{ m}}$ ~~cao~~ ft ✓ A1

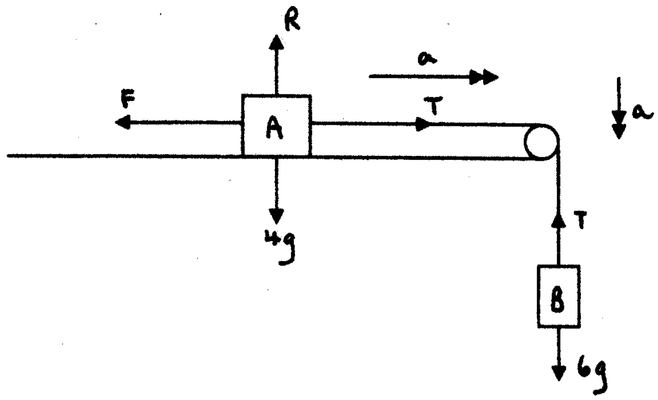
3. (a) N2L $15g - R = 15a$ dim correct, 15g andR opposing M1 A1

$a = -2$ $R = 15 \times 9.8 - 15 \times (-2)$

$R = \underline{177 \text{ N}}$ A1

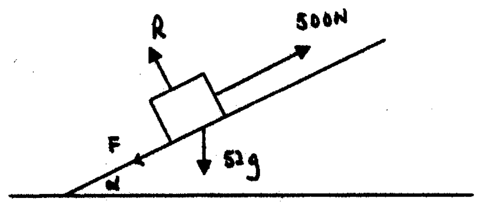
(b) $R = 15g = \underline{147 \text{ N}}$ B1

4.



$R = 4g$	si	B1
$F = \mu R = 0.3 \times 4g = 1.2g$	ft R	B1
N2L applied to each mass		
$6g - T = 6a$		M1 A1
$T - F = 4a$		M1 A1
Adding $6g - 1.2g = 10a$	both Ms	m1
$a = \dfrac{(6-1.2)\times 9.8}{10}$		
$= \underline{4.704\text{ ms}^{-2}}$	ft slip in R	A1
$T = 4 \times 4{\cdot}704 + 1{\cdot}2 \times 9{\cdot}8$		
$= \underline{30.576\text{ N}}$	ft slip in R	A1

5.



$R = 52g\cos\alpha$		M1 A1
$= 470.4\text{ N}$		
$F = \mu R$		M1
$= 188.16\text{ N}$		
N2L	dim correct, all forces	M1
$500 - F - mg\sin\alpha = ma$		A1
$500 - 188.16 - 196 = 52a$		
$a = \underline{2.23\text{ms}^{-2}}$	cao	A1

6. (a) $e = \dfrac{2.8}{4}$ M1

$e = \underline{0.7}$ A1

(b) $I = 3(4 + 2.8)$ M1

$I = \underline{20.4\text{ Ns}}$ A1

(c) Conservation of momentum M1

$3 \times 2.8 + 5 \times 1.5 = 3v_A + 5v_B$ A1

$3v_A + 5v_B = 15.9$

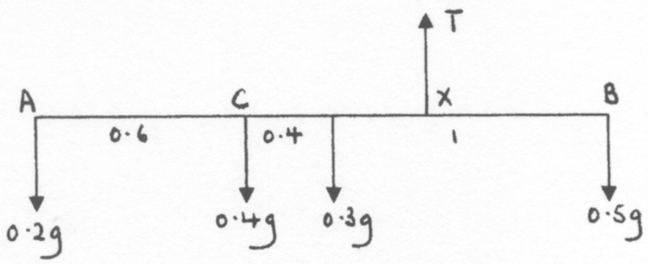
Restitution M1

$v_B - v_A = -0.6(1.5 - 2.8)$ = 0.78 A1

$-3v_A + 3v_B = 2.34$

Adding	$8v_B = 18.24$	dep on both Ms	m1
	$v_B = \underline{2.28\text{ ms}^{-1}}$	cao	A1
	$v_A = \underline{1.5\text{ ms}^{-1}}$	cao	A1

7.



(a) $T = (0.2 + 0.4 + 0.3 + 0.5)g$ M1 A1

$= 1.4g$

$= \underline{13.72\text{ N}}$ A1

(b) Moments about A — dim correct to obtain equation — M1

$Tx = 0.4g \times 0.6 + 0.3g \times 1 + 0.5g \times 2$ ft T B1 A1

$1.4x = 0.24 + 0.3 + 1 = 1.54$

$x = \underline{1.1\text{ m}}$ cao A1

8. Resolve horizontally — M1

$T_X \cos 23° = T_Y \cos 30°$ — A1

$$T_Y = \frac{2\cos 23^\circ}{\sqrt{3}} T_X$$

Resolve vertically — M1

$T_X \sin 23° + T_Y \cos 60° = 12g$ — A1

$$T_X\left(\sin 23^\circ + \frac{\cos 23^\circ}{\sqrt{3}}\right) = 12g$$ dep on both Ms — m1

$T_X = \underline{127.52 \text{ N}}$ cao — A1

$T_Y = \underline{135.55 \text{ N}}$ cao — A1

9. (a)

	Area	from AD	from AB
$ABCD$	30	2.5	3
XYZ	3	3	2
Lamina	27	x	y

Moments about AD — M1

$30 \times 2.5 = 3 \times 3 + 27x$ — A1

$$x = \frac{66}{27} = \frac{22}{9}$$

$$x = 2\frac{4}{9}$$ cao — A1

Moments about AB — M1

$30 \times 3 = 3 \times 2 + 27y$ — A1

$$y = \frac{84}{27} = \frac{28}{9}$$

$$y = 3\frac{1}{9}$$ cao — A1

(b) $$\theta = \tan^{-1}\left(\frac{5 - \frac{22}{9}}{\frac{28}{9}}\right)$$ — M1 A1

$$= \tan^{-1}\left(\frac{23}{28}\right)$$

$= \underline{39.4°}$ ft x, y — A1

(c) Required distance $= \frac{22}{9} = 2\frac{4}{9}$ ft x — B1

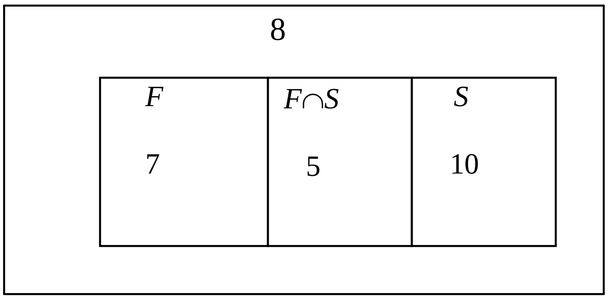
Mathematics S1 January 2009

Solutions and Mark Scheme

Final Version

1	(a)	Using $P(A \cup B) = P(A) + P(B)$	M1
		$0.93 = 0.65 + P(B)$ so $P(B) = 0.28$	A1
	(b)	Using $P(A \cup B) = P(A) + P(B) - P(A)P(B)$	M1
		$0.93 = 0.65 + P(B) - 0.65P(B)$	A1
		$0.35P(B) = 0.28$	M1
		$P(B) = 0.8$	A1
2	EITHER		
	(a)	$P(F \cup S) = 1 - P(F' \cap S')$	M1
		$= 1 - 8/30 = 22/30$	A1
		$P(F \cap S) = P(F) + P(S) - P(F \cup S)$	M1
		$= (12 + 15 - 22)/30 = 5/30$ (1/6)	A1
	(b)	$P(F \cap S') = P(F) - P(F \cap S)$	M1
		$= (12 - 5)/30 = 7/30$	A1

OR



B4

	(a)	$P(F \cap S) = \frac{5}{30}$	B1
	(b)	$P(F \cap S') = \frac{7}{30}$	B1
		[FT on minor slip]	
3	(a)(i)	Prob = $e^{-2.75} \times \frac{2.75^4}{4!} = 0.152$	M1A1
	(ii)	$P(\le 2) = e^{-2.75}\left(1 + 2.75 + \frac{2.75^2}{2}\right)$ (= 0.481)	M1A1
		Reqd prob = 0.519	A1
	(b)(i)	Reqd prob = 0.8153	M1A1
	(ii)	Reqd prob = 0.6472 – 0.4232 or 0.5768 – 0.3528	B1B1
		= 0.224	B1

4	(a)	$E(Y) = 3 \times 4 - 7 = 5$	M1A1
		$Var(X) = 4$ si	B1
		$Var(Y) = 9 \times 4 = 36$	M1A1
	(b)	$P(Y > 0) = P(3X > 7)$	M1
		$= P(X \geq 3)$	A1
		$= 0.7619$ cao	A1
5			
	(a)	EITHER	
		Total number of possibilities = $\binom{16}{3}$ (=560)	B1
		Number of possibilities = $\binom{4}{1} \times \binom{4}{3}$ (= 16)	B1
		Prob = $\frac{16}{560} (= \frac{1}{35})$ cao	B1
		OR	
		Prob = $\frac{4}{16} \times \frac{3}{15} \times \frac{2}{14} \times 4$	M1A1
		$= \frac{1}{35}$ cao	A1
	(b)	EITHER	
		Number of possibilities = $(4 \times 4 \times 4) \times 4$ (= 256)	M1A1A1
		Prob = $\frac{256}{560} (= \frac{16}{35})$ cao	A1
		OR	
		Prob = $(\frac{4}{16} \times \frac{4}{15} \times \frac{4}{14} \times 6) \times 4$	M1A1A1
		$= \frac{16}{35}$ cao	A1
6	(a)(i)	Let number of red flowers = X.	
		X is B(20,0.6) si	B1
		$P(X = 10) = \binom{20}{10} \times 0.6^{10} \times 0.4^{10}$	M1
		$= 0.117$	A1
	(ii)	Let number of non-red flowers = Y so Y is B(20,0.4) si	B1
		$P(X \geq 12) = P(Y \leq 8)$	M1
		$= 0.5956$	A1
	(b)	Let number of failures be U.	
		U is B(80,0.04) which is approx Poi(3.2). si	B1
		$P(U < 5) = 0.7806$	M1A1

7	(a)	$P(\text{Sum} = 5) = \frac{4}{36} = \frac{1}{9}$	M1A1
	(b)(i)	$\text{P(Score} = 5) = \frac{1}{2} \times P(\text{Score} = 5 \mid \text{head}) + \frac{1}{2} \times P(\text{Score} = 5 \mid \text{tail})$	M1
		$= \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{9}$	A1
		$= \frac{5}{36}$	A1
	(ii)	$P(\text{head} \mid 5) = \frac{1/12}{5/36}$ (FT denominator from (i))	B1B1
		$= \frac{3}{5}$ cao (but FT from (a))	B1
8	(a)	$E(X) = \Sigma x P(X = x)$	M1
		$= \frac{1}{20}(8 \times 2 + 6 \times 4 + 4 \times 6 + 2 \times 8))$	A1
		$= 4$	A1
		$E(X^2) = \Sigma x^2 P(X = x)$	M1
		$= \frac{1}{20}(8 \times 2^2 + 6 \times 4^2 + 4 \times 6^2 + 2 \times 8^2)$ (= 20)	A1
		Variance = 20 – 16 = 4 cao	A1
	(b)	The possibilities are 2 and 6 or 4 and 4.	M1
		$\text{Prob} = \frac{8}{20} \times \frac{4}{20} + \frac{4}{20} \times \frac{8}{20} + \frac{6}{20} \times \frac{6}{20}$ (Must have 3 terms)	M1
		$= \frac{1}{4}$	A1

9	(a)	Since $F(2) = 1$, it follows that	M1
		$k \times 2^3 = 1 \Rightarrow k = \frac{1}{8}$	A1
	(b)	Prob = F(1.5) – F(0.5)	M1
		$= \frac{1}{8}\left(1.5^3 - 0.5^3\right) = \frac{13}{32}$ (0.406)	A1
	(c)	The median m satisfies	
		$\frac{1}{8}m^3 = \frac{1}{2}$	M1
		$m = \sqrt[3]{4} (= 1.59)$	A1
	(d)	The probability density $f(x)$ is given by	
		$f(x) = F'(x) = \frac{3}{8}x^2$	M1A1
		$E(X) = \int_0^2 \frac{3}{8}x^2 \times x\,\mathrm{d}x$	M1A1
		$= \frac{3}{8}\left[\frac{x^4}{4}\right]_0^2$	A1
		$= \frac{3}{2}$	A1

WJEC
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk/exams.html