



## **GCE MARKING SCHEME**

**MATHEMATICS  
AS/Advanced**

**JANUARY 2010**

## **INTRODUCTION**

The marking schemes which follow were those used by WJEC for the January 2010 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

# Mathematics C1 January 2010

## Solutions and Mark Scheme

### Final Version

1. (a) Gradient of  $BC = \frac{\text{increase in } y}{\text{increase in } x}$  M1  
Gradient of  $BC = -1/2$  (or equivalent) A1
- (b) (i) Use of gradient  $L_1 = \text{gradient } BC$  M1  
A correct method for finding the equation of  $L_1$  using candidate's gradient for  $L_1$  M1  
Equation of  $L_1$ :  $y - 10 = -1/2 [x - (-11)]$  (or equivalent)  
(f.t. candidate's gradient for  $BC$ ) A1  
Equation of  $L_1$ :  $x + 2y - 9 = 0$  (convincing) A1
- (ii) Use of gradient  $L_2 \times \text{gradient } BC = -1$  M1  
A correct method for finding the equation of  $L_2$  using candidate's gradient for  $L_2$  M1  
**(to be awarded only if corresponding M1 is not awarded in part (b)(i))**  
Equation of  $L_2$ :  $y - 8 = 2(x - 3)$  (or equivalent)  
(f.t. candidate's gradient for  $BC$ ) A1
- (c) (i) An attempt to solve equations of  $L_1$  and  $L_2$  simultaneously M1  
 $x = 1, y = 4$  (convincing.) A1
- (ii) A correct method for finding the length of  $BD$  M1  
 $BD = 10$  A1
- (iii) A correct method for finding the coordinates of the mid-point of  $BD$  M1  
Mid-point of  $BD$  has coordinates  $(-2, 8)$  A1

2. (a) 
$$\frac{2\sqrt{11}-3}{\sqrt{11}+2} = \frac{(2\sqrt{11}-3)(\sqrt{11}-2)}{(\sqrt{11}+2)(\sqrt{11}-2)}$$

Numerator:  $22 - 4\sqrt{11} - 3\sqrt{11} + 6$  A1

Denominator:  $11 - 4$  A1

$$\frac{2\sqrt{11}-3}{\sqrt{11}+2} = 4 - \sqrt{11}$$
 (c.a.o.) A1

**Special case**

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by  $\sqrt{11} + 2$

(b) 
$$\frac{22}{\sqrt{2}} = 11\sqrt{2}$$
 B1

$$\sqrt{50} = 5\sqrt{2}$$
 B1
 
$$(\sqrt{2})^5 = 4\sqrt{2}$$
 B1
 
$$\frac{22}{\sqrt{2}} - \sqrt{50} - (\sqrt{2})^5 = 2\sqrt{2}$$
 (c.a.o.) B1

3. An attempt to differentiate, at least one non-zero term correct M1
- $$\frac{dy}{dx} = 6 \times -2 \times x^{-3} + \frac{7}{4}$$
- A1

An attempt to substitute  $x = 2$  in candidate's derived expression for  $\frac{dy}{dx}$  M1

Value of  $\frac{dy}{dx}$  at  $P = \frac{1}{4}$  (c.a.o.) A1

Gradient of normal =  $\frac{-1}{\text{candidate's derived value for } \frac{dy}{dx}}$  M1

Equation of normal to  $C$  at  $P$ :  $y - 3 = -4(x - 2)$  (or equivalent)  
 (f.t. candidate's value for  $\frac{dy}{dx}$  provided all three M1's are awarded) A1

4. (a)  $a = 4$  B1  
 $b = -1$  B1  
 $c = 3$  B1

(b)  $\frac{1}{c}$  on its own or greatest value =  $\frac{1}{c}$ , with correct explanation or no explanation B2

**If B2 not awarded**

$\frac{1}{c}$  on its own or greatest value =  $\frac{1}{c}$ , with incorrect explanation B1

least value =  $\frac{1}{c}$  with no explanation B1

least value =  $\frac{1}{c}$  with incorrect explanation B0

5. (a) An expression for  $b^2 - 4ac$ , with at least two of  $a$ ,  $b$  or  $c$  correct M1  
 $b^2 - 4ac = 3^2 - 4 \times k \times (-5)$  A1  
 $b^2 - 4ac < 0$  m1  
 $k < -\frac{9}{20}$   
(f.t. only for  $k > -\frac{9}{20}$  from  $b^2 - 4ac = 3^2 - 4 \times k \times 5$ ) A1
- (b) Finding critical values  $x = -1.5, x = 2$  B1  
A statement (mathematical or otherwise) to the effect that  
 $x < -1.5$  or  $2 < x$  (or equivalent)  
(f.t. critical values  $\pm 1.5, \pm 2$  only) B2  
Deduct 1 mark for each of the following errors  
the use of  $\leq$  rather than  $<$   
the use of the word ‘and’ instead of the word ‘or’
6. (a)  $y + \delta y = 3(x + \delta x)^2 - 7(x + \delta x) - 5$  B1  
Subtracting  $y$  from above to find  $\delta y$  M1  
 $\delta y = 6x\delta x + 3(\delta x)^2 - 7\delta x$  A1  
Dividing by  $\delta x$  and letting  $\delta x \rightarrow 0$  M1  
 $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 6x - 7$  (c.a.o.) A1
- (b)  $\frac{dy}{dx} = a \times \frac{5}{2} \times x^{3/2}$  B1  
Substituting  $x = 4$  in candidate’s expression for  $\frac{dy}{dx}$  and putting  
expression equal to  $-2$  M1  
 $a = -\frac{1}{10}$  (c.a.o.) A1
7. Coefficient of  $x = {}^5C_1 \times a^4 \times 3(x)$  B1  
Coefficient of  $x^2 = {}^5C_2 \times a^3 \times 3^2(x^2)$  B1  
 $10 \times a^3 \times m = k \times 5 \times a^4 \times 3$  (o.e.)  $(m = 9 \text{ or } 3, k = 8 \text{ or } \frac{1}{8})$  M1  
 $a = \frac{3}{4}$  (c.a.o.) A1

8. (a)  $f(-2) = 15$  B1

**Either:** When  $f(x)$  is divided by  $x + 2$ , the remainder is 15

**Or:**  $x + 2$  is not a factor of  $f(x)$

[f.t. candidate's value for  $f(-2)$ ] E1

(b) Attempting to find  $f(r) = 0$  for some value of  $r$  M1

$f(-1) = 0 \Rightarrow x + 1$  is a factor A1

$f(x) = (x + 1)(2x^2 + ax + b)$  with one of  $a, b$  correct M1

$f(x) = (x + 1)(2x^2 + 9x - 5)$  A1

$f(x) = (x + 1)(x + 5)(2x - 1)$  (f.t. only  $2x^2 - 9x - 5$  in above line) A1

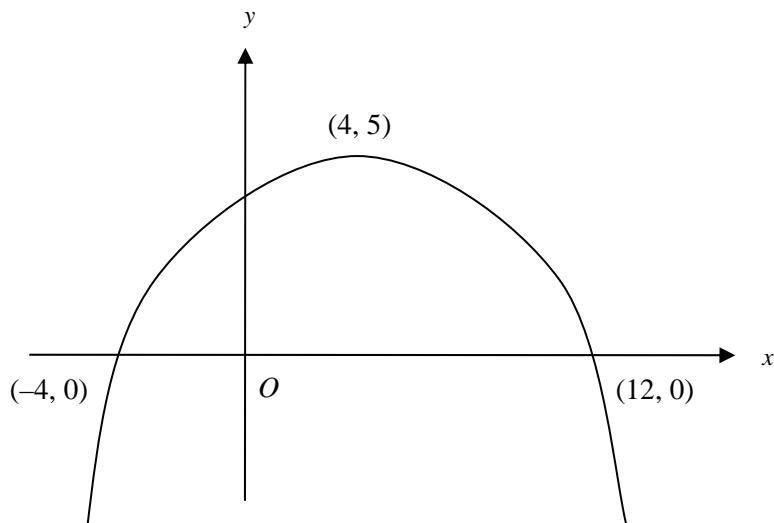
Roots are  $x = -1, -5, 1/2$

(f.t. only from  $(x + 1)(x - 5)(2x + 1)$  in above line) A1

**Special case**

Candidates who, after having found  $x + 1$  as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1

9. (a)



Concave down curve and y-coordinate of maximum = 5 B1

$x$ -coordinate of maximum = 4 B1

Both points of intersection with  $x$ -axis B1

(b)  $y = f(x - 4)$  B2

**If B2 not awarded**

$y = f(x + 4)$  B1

**10.** (a)  $\frac{dy}{dx} = 3x^2 - 12x$  B1  
 $\frac{dy}{dx}$

Putting derived  $\frac{dy}{dx} = 0$  M1  
 $\frac{dy}{dx}$

$x = 0, 4$  (both correct) (f.t. candidate's  $\frac{dy}{dx}$ ) A1  
 $\frac{dy}{dx}$

Stationary points are  $(0, 20)$  and  $(4, -12)$  (both correct) (c.a.o) A1  
A correct method for finding nature of stationary points yielding

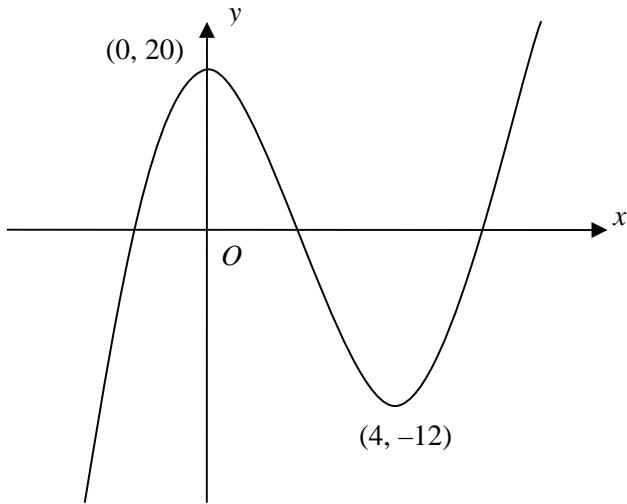
**either**  $(0, 20)$  is a maximum point

**or**  $(4, -12)$  is a minimum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(b)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(c) Use of both  $k = -12$ ,  $k = 20$  to find the range of values for  $k$  M1

(f.t. candidate's y-values at stationary points) A1

$-12 < k < 20$  (f.t. candidate's y-values at stationary points) A1

## C2

### Solutions and Mark Scheme

### Final Version

1.	1	1.414213562		
	1.1	1.337908816		
	1.2	1.2489996		
	1.3	1.144552314	(5 values correct)	B2
	1.4	1.019803903	(3 or 4 values correct)	B1

Correct formula with  $h = 0.1$

$$I \approx \frac{0.1}{2} \times \{1.414213562 + 1.019803903 + 2(1.337908816 + 1.2489996 + 1.144552314)\}$$

$$I \approx 0.494846946$$

$$I \approx 0.495 \quad (\text{f.t. one slip}) \quad \text{A1}$$

**Special case** for candidates who put  $h = 0.8$

1	1.414213562		
1.08	1.35410487		
1.16	1.286234815		
1.24	1.209297317		
1.32	1.121427662		
1.4	1.019803903	(all values correct)	B1

Correct formula with  $h = 0.08$

$$I \approx \frac{0.08}{2} \times \{1.414213562 + 1.019803903 + 2(1.35410487 + 1.286234815 + 1.209297317 + 1.121427662)\}$$

$$I \approx 0.495045871$$

$$I \approx 0.495 \quad (\text{f.t. one slip}) \quad \text{A1}$$

2. (a)  $3 - 7 \cos \theta = 6(1 - \cos^2 \theta)$  (correct use of  $\sin^2 \theta = 1 - \cos^2 \theta$ ) M1  
 An attempt to collect terms, form and solve quadratic equation in  $\cos \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \cos \theta + b)(c \cos \theta + d)$ ,  
 with  $a \times c = \text{coefficient of } \cos^2 \theta$  and  $b \times d = \text{constant}$  m1  
 $6 \cos^2 \theta - 7 \cos \theta - 3 = 0 \Rightarrow (3 \cos \theta + 1)(2 \cos \theta - 3) = 0$   
 $\Rightarrow \cos \theta = -\frac{1}{3}, \quad (\cos \theta = \frac{3}{2}) \quad (\text{c.a.o.}) \quad \text{A1}$   
 $\theta = 109.47^\circ, 250.53^\circ \quad \text{B1 B1}$   
 Note: Subtract (from final two marks) 1 mark for each additional root in range from  $3 \cos \theta + 1 = 0$ , ignore roots outside range.  
 $\cos \theta = -, \text{ f.t. for 2 marks}, \quad \cos \theta = +, \text{ f.t. for 1 mark}$
- (b)  $2x + 45^\circ = 35^\circ, 215^\circ, 395^\circ \quad (\text{one value}) \quad \text{B1}$   
 $x = 85^\circ, 175^\circ \quad \text{B1, B1}$   
 Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.
- (c) Correct use of  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  (o.e.) M1  
 $\theta = 194.48^\circ \quad \text{A1}$   
 $\theta = 345.52^\circ \quad \text{A1}$   
 Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.
3. (a)  $x^2 = 8^2 + (x + 2)^2 - 2 \times 8 \times (x + 2) \times \cos 60^\circ \quad (\text{correct use of cos rule}) \quad \text{M1}$   
 $x^2 = 64 + x^2 + 4x + 4 - 8x - 16 \quad \text{A1}$   
 $x = 13 \quad (\text{f.t. only } x = 21 \text{ from } + 16 \text{ in the line above}) \quad \text{A1}$
- (b)  $\frac{\sin A}{8} = \frac{\sin 60^\circ}{13} \quad (\text{substituting correct values in the correct places in the sin rule, f.t. candidate's derived value for } x) \quad \text{M1}$   
 $ACB = 32.2^\circ \quad (\text{f.t. candidate's derived value for } x) \quad \text{A1}$
4. (a) At least one correct use of the sum formula M1  
 $\frac{8}{2} \times [2a + 7d] = 124$   
 $\underline{2} \times [2a + 19d] = 910 \quad (\text{both correct}) \quad \text{A1}$   
 $\underline{2}$   
 An attempt to solve the candidate's two equations simultaneously by eliminating one unknown M1  
 $d = 5 \quad (\text{c.a.o.}) \quad \text{A1}$   
 $a = -2 \quad (\text{f.t. candidate's value for } d) \quad \text{A1}$
- (b)  $-2 + 5(n - 1) = 183 \quad (\text{f.t. candidate's values for } a \text{ and } d) \quad \text{M1}$   
 $n = 38 \quad (\text{c.a.o.}) \quad \text{A1}$

5. (a)  $S_n = a + ar + \dots + ar^{n-1}$  (at least 3 terms, one at each end) B1

$$rS_n = ar + \dots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - ar^n \quad (\text{multiply first line by } r \text{ and subtract}) \text{ M1}$$

$$(1 - r)S_n = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad (\text{convincing}) \text{ A1}$$

(b) **Either:**  $\frac{a(1 - r^4)}{1 - r} = 73.8$

**Or:**  $a + ar + ar^2 + ar^3 = 73.8$

B1

$$\frac{a}{1 - r} = 125$$

B1

An attempt to solve these equations simultaneously by eliminating one of the variables M1

$$r^4 = 0.4096$$

A1

$$r = 0.8$$

(c.a.o.) A1

$$a = 25$$

(f.t. candidate's value for  $r$ ) A1

6. (a)  $\frac{x^{4/3}}{4/3} - 2 \times \frac{x^{3/4}}{3/4} + c$  B1, B1  
 (-1 if no constant term present)

(b) (i)  $5 + 4x - x^2 = 8$  M1

An attempt to rewrite and solve quadratic equation  
 in  $x$ , either by using the quadratic formula or by getting the  
 expression into the form  $(x + a)(x + b)$ , with  $a \times b = 3$  m1  
 $(x - 1)(x - 3) = 0 \Rightarrow x = 1, 3$  (c.a.o.) A1

**Note:** Answer only with no working earns 0 marks

(ii) **Either:**

$$\text{Total area} = \int_1^3 (5 + 4x - x^2) dx - \int_1^3 8 dx \quad (\text{use of integration}) \quad \text{M1}$$

$$\text{Either: } \int 5 dx = 5x \text{ and } \int 8 dx = 8x \text{ or: } \int 3 dx = 3x \quad \text{B1}$$

$$\int 4x dx = 2x^2, \quad \int x^2 dx = \frac{x^3}{3} \quad \text{B1 B1}$$

$$\text{Total area} = [-3x + 2x^2 - (1/3)x^3]_1^3 \quad (\text{o.e})$$

$$= (-9 + 18 - 9) - (-3 + 2 - 1/3)$$

(substitution of candidate's limits in at least one integral) m1

Subtraction of integrals with correct use of candidate's

$x_A, x_B$  as limits m1

$$\text{Total area} = \frac{4}{3} \quad (\text{c.a.o.}) \quad \text{A1}$$

**Or:**

Area of rectangle = 16

(f.t. candidate's  $x$ -coordinates for  $A, B$ ) B1

$$\text{Area under curve} = \int_1^3 (5 + 4x - x^2) dx \quad (\text{use of integration}) \quad \text{M1}$$

$$= [5x + 2x^2 - (1/3)x^3]_1^3$$

(correct integration) B2

$$= (15 + 18 - 9) - (5 + 2 - 1/3)$$

(substitution of candidate's limits) m1

$$= \frac{52}{3}$$

Use of candidate's,  $x_A, x_B$  as limits and trying to find total area  
 by subtracting area of rectangle from area under curve m1

$$\text{Total area} = \frac{52}{3} - 16 = \frac{4}{3} \quad (\text{c.a.o.}) \quad \text{A1}$$

7. (a) Let  $p = \log_a x$   
 Then  $x = a^p$   
 $x^n = a^{pn}$   
 $\therefore \log_a x^n = pn$   
 $\therefore \log_a x^n = pn = n \log_a x$

(relationship between log and power) B1  
 (the laws of indices) B1  
 (relationship between log and power) B1  
 (convincing) B1

(b)  $\frac{1}{2} \log_a 324 = \log_a 324^{1/2}$   
 $2 \log_a 12 = \log_a 12^2$  (at least one use of power law) B1  
 $\frac{1}{2} \log_a 324 + \log_a 56 - 2 \log_a 12 = \log_a \frac{324^{1/2} \times 56}{12^2}$   
 (use of addition law) B1  
 (use of subtraction law) B1

$\frac{1}{2} \log_a 324 + \log_a 56 - 2 \log_a 12 = \log_a 7$  (c.a.o) B1

**Note: Answer only of  $\log_a 7$  without any working earns 0 marks**

(c) (i)  $2^{x+1} = 2^x \times 2$  B1  
 $3^x = 2^{x+1} \Rightarrow (1.5)^x = 2$  B1

(ii) **Hence:**  $x \log_{10} 1.5 = \log_{10} 2$   
 (taking logs on both sides and using the power law) M1  
 (f.t. candidate's values for  $c$  and  $d$ )  
 $x = 1.71$  (c.a.o.) A1

**Otherwise:**  
 $x \log_{10} 3 = (x + 1) \log_{10} 2$   
 (taking logs on both sides and using the power law) M1  
 $x = 1.71$  (c.a.o.) A1

8. (a)  $A(-2, 4)$  B1  
 A correct method for finding radius M1  
 $\text{Radius} = \sqrt{10}$  A1

(b) An attempt to substitute  $(3y - 4)$  for  $x$  in the equation of the circle M1  
 $10y^2 - 20y + 10 = 0$  A1  
**Either:** Use of  $b^2 - 4ac$  m1  
 Determinant = 0  $\Rightarrow x - 3y + 4 = 0$  is a tangent to the circle A1  
**Or:** An attempt to factorise candidate's quadratic m1  
 Repeated (single) root  $\Rightarrow x - 3y + 4 = 0$  is a tangent to the circle A1

9. (a)  $\frac{1}{2} \times 6^2 \times \sin \theta = 9.1$  M1  
 $\theta = 0.53$  A1

(b) Substitution of values in formula for area of sector M1  
Area  $= \frac{1}{2} \times 6^2 \times 0.53 = 9.54 \text{ cm}^2$  (f.t. candidate's value for  $\theta$ ) A1

(c)  $6 + 6 + 6\varphi = \pi \times 6$  M1  
 $\varphi = 1.14$  A1

10. (a)  $t_3 = 31$  B1  
 $t_1 = 7$  (f.t. candidate's value for  $t_3$ ) B1

(b) All terms of the sequence are odd numbers E1

# C3

## Solutions and Mark Scheme

### Final Version

1.	0	0.69314718		
	0.25	0.825939419		
	0.5	0.974076984		
	0.75	1.136871006	(5 values correct)	B2
	1	1.313261688	(3 or 4 values correct)	B1

Correct formula with  $h = 0.25$

$$I \approx \frac{0.25}{3} \times \{0.69314718 + 1.313261688 + 4(0.825939419 + 1.136871006) \\ + 2(0.974076984)\}$$

$$I \approx 11.80580453 \div 12$$

$$I \approx 0.983817044$$

$$I \approx 0.984$$

(f.t. one slip) A1

2.	(a)	e.g. $\theta = \frac{\pi}{2}$		
		$\sin 4\theta = 0$	(choice of $\theta$ and one correct evaluation)	B1
		$4 \sin^3 \theta - 3 \sin \theta = 1$	(both evaluations correct but different)	B1

$$(b) \quad 3(1 + \tan^2 \theta) = 7 - 11 \tan \theta. \quad (\text{correct use of } \sec^2 \theta = 1 + \tan^2 \theta) \quad M1$$

An attempt to collect terms, form and solve quadratic equation in  $\tan \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \tan \theta + b)(c \tan \theta + d)$ ,

with  $a \times c = \text{coefficient of } \tan^2 \theta$  and  $b \times d = \text{constant}$  m1

$$3 \tan^2 \theta + 11 \tan \theta - 4 = 0 \Rightarrow (3 \tan \theta - 1)(\tan \theta + 4) = 0$$

$$\Rightarrow \tan \theta = \frac{1}{3}, \tan \theta = -4 \quad (\text{c.a.o.}) \quad A1$$

$$\theta = 18.4^\circ, 198.4^\circ \quad B1$$

$$\theta = 104.0^\circ, 284.0^\circ \quad B1 \quad B1$$

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\tan \theta = +, -$ , f.t. for 3 marks,  $\tan \theta = -, -$ , f.t. for 2 marks

$\tan \theta = +, +$ , f.t. for 1 mark

3. (a)  $\frac{d(y^3)}{dx} = 3y^2 \frac{dy}{dx}$  B1  
 $\frac{d(2x^3y)}{dx} = 2x^3 \frac{dy}{dx} + 6x^2y$  B1  
 $\frac{d(3x^2 + 4x - 3)}{dx} = 6x + 4$  B1  
 $x = 2, y = 1 \Rightarrow \frac{dy}{dx} = -\frac{8}{19}$  (c.a.o.) B1

(b)	(i)	$\frac{dx}{dt} = 6t$ , $\frac{dy}{dt} = 12t^2 + 6t^5$	(all three terms correct)	B2
			(one term correct)	B1
		Use of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		M1
		$\frac{dy}{dx} = 2t + t^4$	(c.a.o.)	A1
	(ii)	$\frac{d}{dt} \left[ \frac{dy}{dx} \right] = 2 + 4t^3$	(f.t. candidate's expression for $\frac{dy}{dx}$ )	B1
		$\frac{d^2y}{dx^2} = \frac{d}{dt} \left[ \frac{dy}{dx} \right] \div \frac{dx}{dt}$		M1
		$\frac{d^2y}{dx^2} = \frac{1 + 2t^3}{3t}$	(c.a.o.)	A1

4.  $f(x) = 2 - 10x + \sin x$

An attempt to check values or signs of  $f(x)$  at  $x = 0, x = \pi/8$  M1

$f(0) = 2 > 0, f(\pi/8) = -1.54 < 0$

Change of sign  $\Rightarrow f(x) = 0$  has root in  $(0, \pi/8)$  A1

$x_0 = 0.2$

$x_1 = 0.219866933$  ( $x_1$  correct, at least 5 places after the point) B1

$x_2 = 0.221809976$

$x_3 = 0.221999561$

$x_4 = 0.222018055 = 0.22202$  ( $x_4$  correct to 5 decimal places) B1

An attempt to check values or signs of  $f(x)$  at  $x = 0.222015, x = 0.222025$  M1

$f(0.222015) = 4.56 \times 10^{-5} > 0, f(0.222025) = -4.46 \times 10^{-5} < 0$  A1

Change of sign  $\Rightarrow \alpha = 0.22202$  correct to five decimal places A1

**Note:** ‘change of sign’ must appear at least once

5. (a)  $\frac{dy}{dx} = \frac{3}{1 + (3x)^2}$  or  $\frac{1}{1 + (3x)^2}$  or  $\frac{3}{1 + 3x^2}$  M1  
 $\frac{dy}{dx} = \frac{3}{1 + 9x^2}$  A1
- (b)  $\frac{dy}{dx} = \frac{ax + b}{2x^2 - 3x + 4}$  (including  $a = 0, b = 1$ ) M1  
 $\frac{dy}{dx} = \frac{4x - 3}{2x^2 - 3x + 4}$  A1
- (c)  $\frac{dy}{dx} = e^{2x} \times m \cos x + k e^{2x} \times \sin x$  ( $m = \pm 1, k = 1, 2$ ) M1  
 $\frac{dy}{dx} = e^{2x} \times m \cos x + k e^{2x} \times \sin x$  (either  $m = 1$  or  $k = 2$ ) A1  
 $\frac{dy}{dx} = e^{2x} \times \cos x + 2e^{2x} \times \sin x$  (c.a.o.) A1
- (d)  $\frac{dy}{dx} = \frac{(1 + \cos x) \times m \sin x - (1 - \cos x) \times k \sin x}{(1 + \cos x)^2}$  ( $m = \pm 1, k = \pm 1$ ) M1  
 $\frac{dy}{dx} = \frac{(1 + \cos x) \times -(-\sin x) - (1 - \cos x) \times (-\sin x)}{(1 + \cos x)^2}$  A1  
 $\frac{dy}{dx} = \frac{2 \sin x}{(1 + \cos x)^2}$  A1

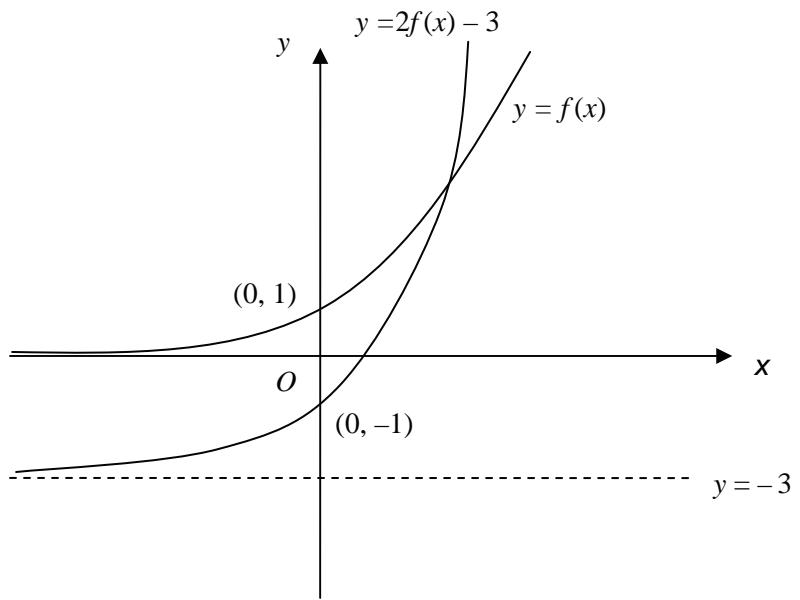
6. (a) (i)  $\int \frac{1}{4x-7} dx = k \times \ln|4x-7| + c \quad (k=1, 4, \frac{1}{4})$  M1  
 $\int \frac{1}{4x-7} dx = \frac{1}{4} \times \ln|4x-7| + c$  A1
- (ii)  $\int e^{3x-1} dx = k \times e^{3x-1} + c \quad (k=1, 3, \frac{1}{3})$  M1  
 $\int e^{3x-1} dx = \frac{1}{3} \times e^{3x-1} + c$  A1
- (iii)  $\int \frac{5}{(2x+3)^4} dx = -\frac{5}{3k} \times (2x+3)^{-3} + c \quad (k=1, 2, \frac{1}{2})$  M1  
 $\int \frac{5}{(2x+3)^4} dx = -\frac{5}{6} \times (2x+3)^{-3} + c$  A1
- (b)  $\int \sin\left[2x + \frac{\pi}{4}\right] dx = \left[ k \times \cos\left[2x + \frac{\pi}{4}\right] \right] \quad (k=-1, -2, \pm \frac{1}{2})$  M1  
 $\int \sin\left[2x + \frac{\pi}{4}\right] dx = \left[ -\frac{1}{2} \times \cos\left[2x + \frac{\pi}{4}\right] \right]$  A1  
 $\int_0^{\frac{\pi}{4}} \sin\left[2x + \frac{\pi}{4}\right] dx = k \times \left[ \cos\left[\frac{3\pi}{4}\right] - \cos\left[\frac{\pi}{4}\right] \right]$   
(f.t. candidate's value for  $k$ ) A1  
 $\int_0^{\frac{\pi}{4}} \sin\left[2x + \frac{\pi}{4}\right] dx = \frac{\sqrt{2}}{2}$  (c.a.o.) A1

7. (a)  $2|x+1|-3=7 \Rightarrow |x+1|=5$  B1  
 $x=4, -6$  B1
- (b) Trying to solve either  $5x-8 \geq 3$  or  $5x-8 \leq -3$  M1  
 $5x-8 \geq 3 \Rightarrow x \geq 2.2$   
 $5x-8 \leq -3 \Rightarrow x \leq 1$  (both inequalities) A1  
Required range:  $x \leq 1$  or  $x \geq 2.2$  (f.t. one slip) A1

#### Alternative mark scheme

- $(5x-8)^2 \geq 9$  (forming and trying to solve quadratic) M1  
Critical points  $x=1$  and  $x=2.2$  A1  
Required range:  $x \leq 1$  or  $x \geq 2.2$  (f.t. one slip in critical points) A1

8.



The  $x$ -axis is an asymptote for  $f(x)$  at  $-\infty$ , correct behaviour at  $+\infty$  M1

$y = f(x)$  cuts  $y$ -axis at  $(0, 1)$  A1

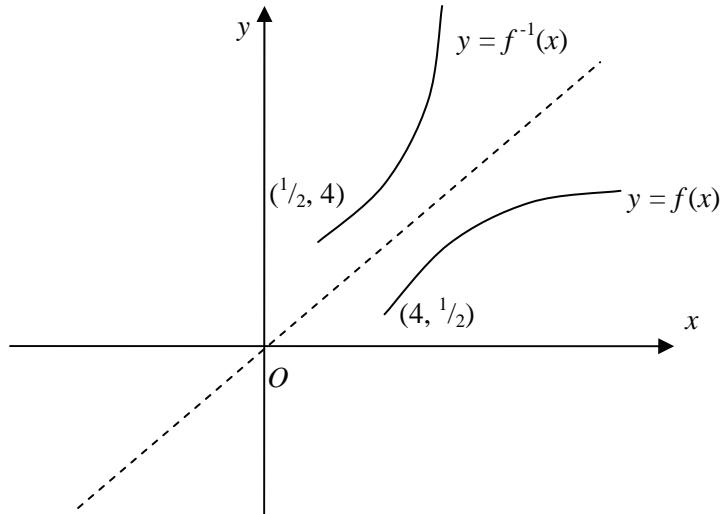
$y = 2f(x) - 3$  cuts  $y$ -axis at  $(0, -1)$  (f.t. candidate's  $y$ -intercept for  $f(x)$ ) B1

$y = -3$  is an asymptote for  $2f(x) - 3$  at  $-\infty$ , with graph above  $y = -3$  B1

The diagram shows that the graph of  $y = 2f(x) - 3$  is steeper than the graph of  $y = f(x)$  in the first quadrant B1

9. (a)  $y = \frac{1}{2}\sqrt{x-3}$  and an attempt to isolate  $x$  M1  
 $2y = \sqrt{x-3} \Rightarrow x = 4y^2 + 3$  A1  
 $f^{-1}(x) = 4x^2 + 3$  (f.t. one slip in candidate's expression for  $x$ ) A1  
 $R(f^{-1}) = [4, \infty)$  B1  
 $D(f^{-1}) = [\frac{1}{2}, \infty)$  B1

(b)



- $y = f^{-1}(x)$  a parabola  
 starting at  $(\frac{1}{2}, 4)$  (f.t. candidate's  $D(f^{-1})$ ) B1  
 $y = f(x)$  as in diagram (c.a.o.) B1

10. (a)  $R(f) = (-1, \infty)$  B1  
 $R(g) = (3, \infty)$  B1
- (b)  $f(1) = 0$  is not in the domain of  $g$  E1
- (c) (i)  $fg(x) = (2x-1)^2 - 1$  M1  
 $fg(x) = 4x(x-1)$  or  $4x^2 - 4x$  A1  
(ii)  $D(fg) = (2, \infty)$  B1  
 $R(fg) = (8, \infty)$  B1

# FP1

## Solutions and Mark Scheme

### Final Version

- 1.** Let  $f(x) = x^3 + x + 10$
- Then,  $f(1 + 2i) = (1 + 2i)^3 + 1 + 2i + 10$  M1  
 $= 1 + 6i + 12i^2 + 8i^3 + 1 + 2i + 10$  A1  
 $= 1 + 6i - 12 - 8i + 1 + 2i + 10$  A1  
 $= 0$  so  $1 + 2i$  is a root
- Another root is  $1 - 2i$ . B1
- Let the third root be  $\alpha$ . M1
- Then, sum of roots =  $1 + 2i + 1 - 2i + \alpha = 0$  A1
- So  $\alpha = -2$  A1
- Accept the following alternative solutions.
- EITHER
- The quadratic equation with roots  $1 \pm 2i$  is M1A1
- $$x^2 - 2x + 5 = 0$$
- Using long division or otherwise
- $$x^3 + x + 10 = (x + 2)(x^2 - 2x + 5)$$
- M1A2
- The roots are therefore  $1 \pm 2i, -2$ . M1A1
- OR
- By inspection,  $x = -2$  is a root. M1A1
- Using long division or otherwise
- $$x^3 + x + 10 = (x + 2)(x^2 - 2x + 5)$$
- M1A2
- The other two roots are therefore  $1 \pm 2i$ . M1A1
- 2.** (a)  $\det \mathbf{A} = 7 \times 2 - 3 \times 5 = -1$  B1
- Cofactor matrix = 
$$\begin{bmatrix} 2 & -3 \\ -5 & 7 \end{bmatrix}$$
 B1
- Inverse matrix = 
$$\begin{bmatrix} -2 & 5 \\ 3 & -7 \end{bmatrix}$$
 B1
- (b)  $\mathbf{AX} = \mathbf{B} \Rightarrow \mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$  M1
- $$= \left[ \begin{array}{cc|cc} -2 & 5 & 1 & 2 \\ 3 & -7 & 3 & 4 \end{array} \right]$$
- $$= \left[ \begin{array}{cc} 13 & 16 \\ -18 & -22 \end{array} \right]$$
- A1
- [FT their inverse matrix]
- [Award M1 for letting the 4 elements of  $\mathbf{X}$  be  $a, b, c, d$  and making a reasonable attempt at finding their values and A1 for finding 2 or 3 values correctly]

3. (a) 
$$\begin{aligned} z &= \frac{(1+8i)(1+2i)}{(1-2i)(1+2i)} \\ &= \frac{1+8i+2i+16i^2}{1-2i+2i-4i^2} \\ &= -3+2i \end{aligned}$$

M1  
A1  
A1

(b)  $\text{Mod}(z) = \sqrt{13}$   
 $\text{Arg}(z) = \tan^{-1}(-2/3) (+\pi)$   
 $= 146^\circ (2.55 \text{ rads})$

B1  
M1  
A1

[FT from (a); award M1 for  $-34^\circ (-0.59 \text{ rads})$ ]

4. (a) Determinant  $= 1 \times (7-15) + 2 \times (12-14) + 2 \times (10-4) = 0$  M1A1

(b)(i) Using row operations,  
 $x + 2y + 2z = 1$   
 $3y + z = -1$   
 $3y + z = 4 - \lambda$

M1  
A1  
A1

For consistency,  $4 - \lambda = -1$  so  $\lambda = 5$ . A1

(ii) Let  $z = \alpha$  M1  
Then,  $y = -\frac{(1+\alpha)}{3}$  A1  
 $x = \frac{(5-4\alpha)}{3}$  A1

]FT their value of  $\lambda$  from (a)]

5. METHOD 1  
Let the roots be  $\alpha, \alpha, \beta$ . M1

Then  $2\alpha + \beta = 0$   
 $\alpha^2 + 2\alpha\beta = -q$   
 $\alpha^2\beta = -r$

A1

Substituting from the 1<sup>st</sup> equation into the 2<sup>nd</sup> and 3<sup>rd</sup> equations, M1

$$\begin{aligned} 3\alpha^2 &= q \\ 2\alpha^3 &= r \end{aligned}$$

A1

Eliminating  $\alpha$ ,

$$\left(\frac{q}{3}\right)^3 = \left(\frac{r}{2}\right)^2$$

M1A1

whence  $4q^3 = 27r^2$

**METHOD 2**

Let  $f(x) = x^3 - qx + r$  so  $f'(x) = 3x^2 - q$  B1

If  $f(x) = 0$  has equal roots then this is also a root of  $f'(x) = 0$  M1

The root of  $f'(x) = 0$  is given by

$$x^2 = \frac{q}{3} \quad \text{A1}$$

Substitute into

$$(x^3 - qx)^2 = r^2, \text{ ie } x^6 - 2qx^4 + q^2x^2 = r^2 \quad \text{M1}$$

giving

$$\frac{q^3}{27} - 2q \times \frac{q^2}{9} + q^2 \times \frac{q}{3} = r^2 \quad \text{A1}$$

$$\frac{q^3}{27}(1 - 6 + 9) = r^2 \quad \text{A1}$$

leading to  $4q^3 = 27r^2$

6. (a) The statement is true for  $n = 1$  since  $1 \times 1! = 1$  and  $2! - 1 = 1$

Let the statement be true for  $n = k$ , ie

$$S_k = (k+1)! - 1 \quad \text{M1}$$

Consider

$$S_{k+1} = (k+1)! - 1 + (k+1) \times (k+1)! \quad \text{M1}$$

$$= (k+2)(k+1)! - 1 \quad \text{A1}$$

$$= (k+2)! - 1 \quad \text{A1}$$

True for  $n = k \Rightarrow$  true for  $n = k + 1$ , hence proved by induction.

A1

$$\begin{aligned}
 (b) \quad \sum_{r=1}^n r(3r+1) &= 3 \sum_{r=1}^n r^2 + \sum_{r=1}^n r && \text{M1} \\
 &= \frac{3n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} && \text{A1A1} \\
 &= \frac{n(n+1)}{2}(2n+1+1) && \text{A1} \\
 &= n(n+1)^2 \text{ cao} && \text{A1}
 \end{aligned}$$

7.	(a)	$\ln f(x) = x \ln \cosecx$	M1
		$\frac{f'(x)}{f(x)} = \ln \cosecx + \frac{x}{\cosecx} \times -\cosecx \cot x$	A1A1
		$f'(x) = (\cosecx)^x (\ln \cosecx - x \cot x)$ cao	A1
	(b)(i)	At a stationary point, $f'(\alpha) = 0$ so	M1
		$\ln \cosec \alpha = \alpha \cot \alpha$	A1
		or $\alpha = \tan \alpha \ln \cosec \alpha$	
	(ii)	$\alpha_0 = 0.5$	
		$\alpha_1 = 0.401623391$	M1A1
		$\alpha_2 = 0.398915619$	
		$\alpha_3 = 0.398614546$	
		$\alpha_4 = 0.398580233$	
		$\alpha_5 = 0.398576311$	
		$\alpha = 0.3986$	A1

8. (a) Reflection matrix =  $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  B1

Translation matrix =  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$  B1

Rotation matrix =  $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  B1

$$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 M1

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 A1

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Fixed points satisfy

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
 M1

or  $x - 1 = x$

$-y - 1 = y$

The first equation has no solutions therefore no fixed points. A1

(ii) METHOD I

A general point on the line is  $(\lambda, 2\lambda + 1)$ . M1

Consider

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda \\ 2\lambda + 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda - 1 \\ -2 - 2\lambda \\ 1 \end{bmatrix}$$
 M1

$x = \lambda - 1; y = -2 - 2\lambda$

Eliminating  $\lambda$ ,

$y = -2x - 4$

M1A1

METHOD II

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \quad M1$$

$$x' = x - 1, y' = -y - 1 \quad A1$$

$$x = x' + 1, y = -y' - 1 \quad A1$$

$$y = 2x + 1 \text{ gives } y' = -2x' - 4 \quad M1A1$$

[Accept solution without primes]

9. (a)  $u + iv = 1 + (x + iy)^2 = 1 + x^2 + 2ixy - y^2 \quad M1A1$

$$u = 1 + x^2 - y^2 \quad A1$$

$$v = 2xy \quad A1$$

(b) Putting  $y = 2x, \quad M1$

$$u = 1 - 3x^2 \quad A1$$

$$v = 4x^2 \quad M1$$

Eliminating  $x,$

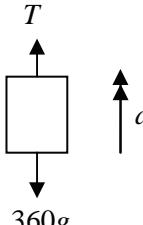
$$u = 1 - \frac{3}{4}v \quad A1$$

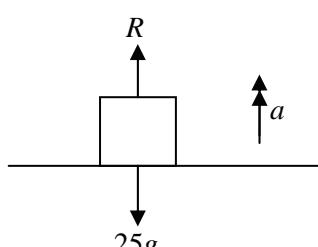
# M1

## Solutions and Mark Scheme

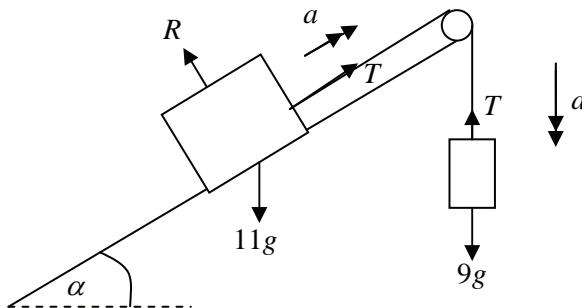
### Final Version

1. (a) Using  $v^2 = u^2 + 2as$  with  $u = 18.2$ ,  $a = (-)9.8$ ,  $v = 0$  o.e. M1  
 $0 = 18.2^2 + 2(-9.8)s$  A1  
 $s = \underline{16.9}$  (m) cao A1
- (b) Using  $s = ut + at^2$  with  $s = 0$ ,  $u = 18.2$ ,  $a = (-)9.8$  M1  
 $0 = 18.2t - 4.9t^2$  A1  
 $t = 0, \frac{26}{7}$   
 Ball returns to point A after  $\frac{26}{7}$  s. cao A1
- (c) Using  $v = u + at$  with  $u = 18.2$ ,  $t = 2.5$ ,  $a = (-)9.8$  M1  
 $v = 18.2 + (-9.8) \times 2.5$  A1  
 $= -6.3$   
 Ball is moving downwards with speed  $\underline{6.3} \text{ ms}^{-1}$ . A1

2. (a) (i)
- 
- Apply Newton's second law to lift dim. correct. M1  
 $T - 360g = 360a$  A1  
 When  $a = -3$ ,  $T = 360 \times 9.8 - 360 \times 3$  cao A1  
 $= \underline{2448}$  (N)
- (ii)  $T = 360g = (3528 \text{ N})$  B1

- (b)
- 
- N2L dim. correct M1  
 $R - 25g = 25a$  A1  
 $a = \frac{1}{25}(280 - 25 \times 9.8)$   
 $a = \underline{1.4} (\text{ms}^{-2})$  cao A1

3.



N2L applied to B.  
 $9g - T = 9a$

dim. correct, all forces

M1  
A1

N2L applied to A.  
 $T - 11g \sin\alpha = 11a$

dim. correct, all forces

M1  
A1

Attempt to eliminate one variable

dep. on both M's

m1

Adding  $9g - 11g \sin\alpha = 20a$

$$a = \underline{2.254} \text{ (ms}^{-2}\text{)}$$

cao

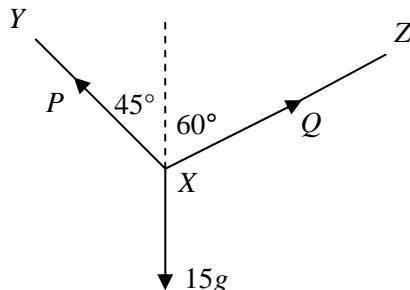
A1

$$T = \underline{67.914} \text{ (N)}$$

cao

A1

4.



Resolve vertically attempt at equation with P, Q resolved

M1

$$P \cos 45^\circ + Q \cos 60^\circ = 15g$$

A1

$$\frac{P}{\sqrt{2}} + \frac{1}{2}Q = 15g$$

Resolve horizontally attempt at equation with P, Q resolved

M1

$$P \cos 45^\circ - Q \cos 30^\circ = 0$$

A1

$$\frac{P}{\sqrt{2}} - \frac{Q\sqrt{3}}{2} = 0$$

Attempt to eliminate one variable

m1

Subtract  $Q\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 15g$

cao

A1

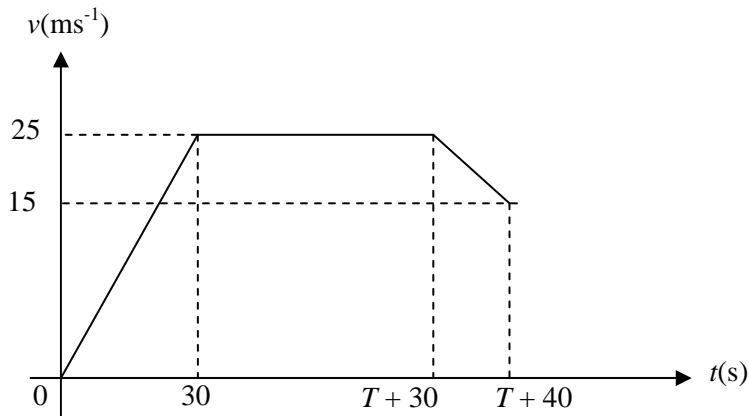
$$Q = \underline{107.6} \text{ (N)}$$

$$P = \underline{131.8} \text{ (N)}$$

cao

A1

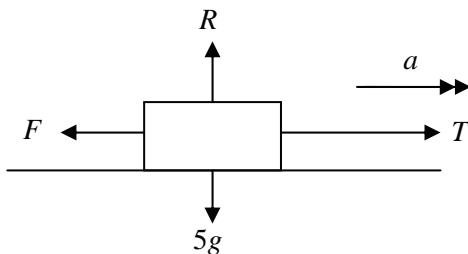
5. (a)



- Line segment (0,0) to (30, 25) B1  
 Line segment (30, 25) to ((T + 30), 25) B1  
 Line segment ((T + 30), 25) to ((T + 40), 15) time interval required B1  
 Correct labelling + 2 previous B marks gained. B1

- (b) An attempt at area under graph = 8000 o.e. M1  
 Any correct distance B1  
 $0.5 \times 25 \times 30 + 25(T + 30) + 0.5(25 + 15) \times 10 = 8000$  A1  
 $375 + 25T + 200 = 8000$   
 $T = \underline{297}$  s cao A1  
 Total time =  $297 + 30 + 10$   
 $= \underline{337}$  s ft A1

6.



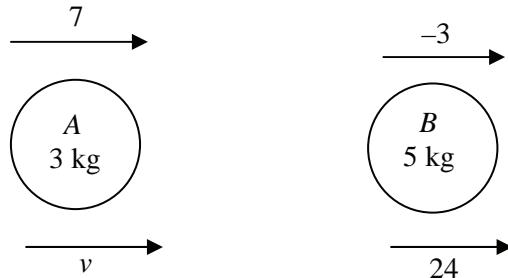
- (a)  $R = 5g$  B1  
 Limiting friction =  $5g \times 0.6$  B1  
 $= 3g = 29.4$  N

N2L applied to particle  
 $40 - 29.4 = 5a$   
 $a = \underline{2.12} \text{ ms}^{-2}$

dim correct, all forces M1  
 ft friction A1  
 cao A1

- (b) Particle will not start moving. B1  
 Since  $T = 20$  N,  $T$  is smaller than limiting friction. So friction will be equal to  $T$ . Since resultant is 0, there is no motion. E1

7.



(a) Conservation of momentum

$$3v + 5 \times 2.4 = 7 \times 3 - 3 \times 5 \\ v = -2 \text{ (ms}^{-1}\text{)}$$

attempted

any correct form  
cao

M1

A1  
A1

Restitution

$$2.4 - v = e(-3 - 7) \\ e = \underline{0.44}$$

attempted

any correct form  
ft  $v$ 

M1

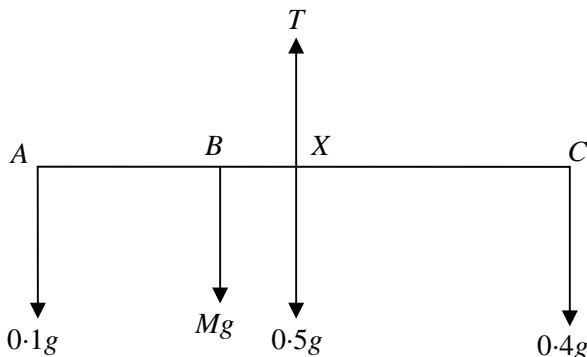
A1  
A1(b) Speed of B after collision with the wall =  $v'$ 

$$v' = 0.6 \times (\pm)2.4 \\ v' = (\pm)\underline{1.44} \text{ (ms}^{-1}\text{)}$$

cao

M1  
A1

8.



(a) Moments about X to obtain equation.

At least one correct moment

$$0.1g \times 10 + Mg \times 2 = 0.4g \times 10 \\ M = \underline{1.5} \text{ (kg)}$$

M1

B1  
A1any correct equation  
cao

(b) Resolve vertically

$$T = (0.1 + 1.5 + 0.5 + 0.4)g \\ T = \underline{24.5} \text{ (N)}$$

ft  $M$   
ft  $M$ M1  
A1  
A1

9.	(a)	Area	from AC	from AB	
		$ABC$	4	2	B1
		$PQRS$	3	3	B1
		Lamina	$x$	$y$	B1

Moments about  $AC$  M1

$$32x + 4 \times 3 = 36 \times 4 \quad \text{ft} \quad \text{A1}$$

$$x = \frac{33}{8} = \underline{4.125 \text{ cm}} \quad \text{cao} \quad \text{A1}$$

Moments about  $AB$  M1

$$32y + 4 \times 3 = 36 \times 2 \quad \text{ft} \quad \text{A1}$$

$$y = \frac{15}{8} = \underline{1.875 \text{ cm}} \quad \text{cao} \quad \text{A1}$$

(b)	Mass	$x$	$y$	
	10	4	0	
	5	3	8	
	2	-5	6	
	3	-1	2	

Moments about  $y$ -axis (or  $x$ -axis) M1

$$20x = 10 \times 4 + 5 \times 3 + 2 \times (-5) + 3 \times (-1) \quad \text{A1}$$

$$x = \underline{2.1} \quad \text{cao} \quad \text{A1}$$

Moments about  $x$ -axis

$$20y = 10 \times 0 + 5 \times 8 + 2 \times 6 + 3 \times 2 \quad \text{A1}$$

$$y = \underline{2.9} \quad \text{cao} \quad \text{A1}$$

# S1

## Solutions and Mark Scheme

### Final Version

1. (a)  $P(2Y) = \frac{3}{10} \times \frac{2}{9}$  or  $\frac{\binom{3}{2}}{\binom{10}{2}} = \frac{1}{15}$  M1A1

(b)  $P(2B) = \frac{2}{10} \times \frac{1}{9}$  or  $\frac{\binom{2}{2}}{\binom{10}{2}} = \left(\frac{1}{45}\right)$  A1

$P(2G) = \frac{4}{10} \times \frac{3}{9}$  or  $\frac{\binom{4}{2}}{\binom{10}{2}} = \left(\frac{2}{15}\right)$  A1

$P(\text{Same colour}) = \text{Sum of above probabilities}$  M1  
 $= \frac{2}{9}$  A1  
[FT one arithmetic slip]

(c)  $P(0G) = \frac{6}{10} \times \frac{5}{9}$  or  $\frac{\binom{6}{2}}{\binom{10}{2}} = \frac{1}{3}$  M1A1

2. (a)  $P(A \cap B) = P(A) + P(B) - P(A \cup B)$  M1  
 $= 0.2 + 0.4 - 0.52$  A1  
 $= 0.08$  A1  
 $P(A)P(B) = 0.08$  A1  
Independent because  $P(A \cap B) = P(A)P(B)$  A1

(b)  $P(\text{Exactly one event}) = P(A \cup B) - P(A \cap B)$  or  $P(A')P(B) + P(A)P(B')$  M1  
 $= 0.44$  A1

(c) Req'd prob =  $\frac{0.2 \times 0.6}{0.44}$  B1B1  
 $= 3/11 (0.273)$  B1  
[FT their answer to (b)]

3. (a) Mean =  $np = 10$ , Variance =  $npq = 9$  B1B1  
Dividing, M1  
 $q = 0.9$  so  $p = 0.1$  A1  
 $n = 10/0.1 = 100$  A1

[Sp case : Award B1B0M1A1A0 for taking variance equal to 3 and getting  $p = 0.7$ ]

(b)  $Y$  is  $B(380, 0.016)$  which is approx  $P(6.08)$  si B1  
 $P(Y < 3) = e^{-6.08} (1 + 6.08 + 6.08^2 / 2)$  M1A1  
 $= 0.058$  A1  
[Award just M1 for  $P(Y \leq 3)$ ; award M0 for using tables]

4. (a) [0,0.4] [Accept (0,0.4)] B1B1

(b) (i)  $E(X) = 0.1 \times 2 + 0.2 \times 3 + 0.3 \times 4 + 5\lambda + 6(0.4 - \lambda)$  M1  
 $= 4.4 - \lambda$  A1  
Putting this equal to 4.25 gives  $\lambda = 0.15$  A1  
[FT from their expression for  $E(X)$  if sensible value]

(ii)  $E(X^2) = 0.1 \times 4 + 0.2 \times 9 + 0.3 \times 16 + 0.15 \times 25 + 0.25 \times 36$  (19.75)  
M1A1  
 $\text{Var}(X) = 19.75 - 4.25^2 = 1.6875$  A1  
[FT their value of  $\lambda$  if sensible answer]

5. (a) Number of seeds germinating,  $X$ , is  $B(20,0.8)$  si B1

$$(i) \text{ Prob} = \binom{20}{15} \times 0.8^{15} \times 0.2^5 = 0.1746 \quad \text{M1A1}$$

- (ii) Number of seeds failing to germinate,  $Y$ , is  $B(20,0.2)$  si B1  
 We require  $P(X \geq 15) = P(Y \leq 5) = 0.8042$  or  $1 - 0.1958$  M1A1

- (b) Prob that they all germinate =  $0.8^n$  B1  
 Solving  $0.8^n = 0.10737$  by any valid method M1  
 $n = 10$  A1  
 [Award 3 marks for  $n = 10$  using tables]

6. (a)  $P(\text{No heads}) = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{8}$  M1A1A1A1  
 $= \frac{7}{24}$  A1

(b)  $P(2 | \text{no heads}) = \frac{1/12}{7/24}$  B1B1  
 [FT their denominator from (a)]  
 $= \frac{2}{7}$  cao B1

7. (a) (i) Prob = 0.1205 or  $1 - 0.8795$  M1A1  
 [Award M1A0 for use of adjacent row or column]

- (ii) Prob =  $e^{-1.2} = 0.301$  M1A1  
 [For candidates using tables award M0 for wrong row, M1A0 if adjacent column used]

(b) Required prob =  $\frac{0.1205}{1 - 0.301}$  B1B1  
 $= 0.172$  cao B1  
 [FT numerator and denominator from (a)]

- (c) Reqd prob =  $0.301 \times 0.301 \times (1 - 0.301) = 0.063$  M1A1  
 [FT from (a)(ii); Award M1A0 for  $0.301 \times 0.301 \times 1.2e^{-1.2}(0.361)$ ]

8.	(a)	(i)	$\text{Prob} = F(2.5) - F(2)$	M1
			$= \frac{1}{10}(2.5^2 + 2.5 - 2 - 2^2 - 2 + 2)$	A1
			$= 0.275$	A1
	(ii)		$F(m) = 0.5$ leading to	M1
			$m^2 + m - 7 = 0$	A1
			$m = \frac{-1 \pm \sqrt{29}}{2}$	m1
			$= 2.19$	A1
	(b)	(i)	$f(x) = F'(x)$	M1
			$= \frac{1}{10}(2x + 1)$	A1
	(ii)		$f(4) = 0$	B1
	(iii)		$E(X) = \frac{1}{10} \int_1^3 x(2x + 1) dx$	M1
			$= \frac{1}{10} \int_1^3 (2x^2 + x) dx$	A1
			$= \frac{1}{10} \left[ \frac{2x^3}{3} + \frac{x^2}{2} \right]_1^3$	A1
			[Limits need not be seen until line 3 ; FT their $f(x)$ as far as possible]	
			$= 2.13$ cao	A1



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