GCE AS/A level

973/01

MATHEMATICS C1
Pure Mathematics

A.M. FRIDAY, 9 January 2009

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points *A, B, C* have coordinates $(2, -1)$, $(-7, 1)$, $(5, 4)$, respectively. The line through *A* perpendicular to the line *BC* intersects *BC* at the point *D*.

   (a) Show that the equation of *BC* is

   $$x - 4y + 11 = 0,$$

   and find the equation of *AD*. [7]

   (b) Show that the coordinates of *D* are $(1, 3)$. [2]

   (c) Find the length of *CD*. [2]

   (d) The line *AD* is extended to *E* so that *D* is the mid-point of *AE*. Find the coordinates of *E*. [2]

2. Simplify

   (a) $\dfrac{10\sqrt{3} - 1}{4 - \sqrt{3}}$, [4]

   (b) $\left(2 + \sqrt{5}\right)\left(5 - \sqrt{20}\right)$. [4]

3. The curve *C* has equation $y = x^2 - 9x + 13$.

   (a) The point *P* has coordinates $(6, -5)$ and lies on *C*. Find the equation of the **tangent** to *C* at *P*. [4]

   (b) The point *Q* lies on *C* and is such that the gradient of the **normal** to *C* at *Q* is $\dfrac{1}{7}$.

   Find the *x*-coordinate of *Q*. [3]

4. Express $3x^2 - 12x + 17$ in the form $a(x + b)^2 + c$, where the values of the constants *a*, *b* and *c* are to be found.
   Hence, sketch the graph of $y = 3x^2 - 12x + 17$, indicating the coordinates of its stationary point. [5]
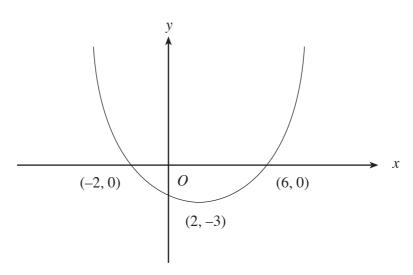
5. Given that the quadratic equation

   $$(3k - 2)x^2 + 8x + k = 0$$

   has no real roots, show that

   $$3k^2 - 2k - 16 > 0.$$

   Find the range of values of *k* satisfying this inequality. [7]

6. *(a)* Expand $(a + b)^5$. [2]

(b) Use your answer to part *(a)* to find the coefficient of x^3 in the expansion of $\left(\dfrac{1}{4} + 2x\right)^5$.

Simplify your answer. [2]

7. *(a)* Find the remainder when $x^3 - 17$ is divided by $x - 3$. [2]

(b) Solve the equation $6x^3 - 7x^2 - 14x + 8 = 0$. [6]

8. *(a)* Given that $y = 7x^2 + 5x - 2$, find $\dfrac{dy}{dx}$ from first principles. [5]

(b) Differentiate $\dfrac{2}{x^3} + 5x^{\frac{2}{3}}$ with respect to x. [2]

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-2, 0)$ and $(6, 0)$ and has a minimum point at $(2, -3)$.

Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the *x*-axis.

(a) $y = f(x - 3)$, [3]

(b) $y = -2f(x)$. [3]

TURN OVER.

10. The curve C has equation

$$y = x^3 + 3x^2 - 9x - 13.$$

(a) Find the stationary points of C and determine the nature of each of these points. [6]

(b) Sketch C, indicating the coordinates of the stationary points. [2]

(c) State, giving a reason, the number of real roots of the equation

$$x^3 + 3x^2 - 9x - 13 = 0.$$ [2]