GCE AS/A level

0973/01

MATHEMATICS – C1
Pure Mathematics

A.M. MONDAY, 14 January 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
• a 12 page answer book;
• a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A and B have coordinates $(2, -3)$ and $(4, 1)$ respectively.
   The line L has equation $x + 2y - 11 = 0$.

   (a) Find the equation of AB and simplify your answer. [5]

   (b) Show that AB and L are perpendicular. [3]

   (c) The lines AB and L intersect at the point C. Show that C has coordinates $(5, 3)$. [2]

   (d) Find the lengths of AB and AC. Hence find the value of the constant k such that $AB = kAC$, giving your answer in its simplest form. [4]

2. Simplify

   (a) $\dfrac{6\sqrt{7} - 11\sqrt{2}}{\sqrt{7} - \sqrt{2}}$, [4]

   (b) $\dfrac{3}{2\sqrt{6}} + \left(\dfrac{\sqrt{6}}{2}\right)^3$. [3]

3. The curve C has equation $y = 3x^2 - 14x + 13$. The point P, whose x-coordinate is 3, lies on the curve C. Find the equation of the **tangent** to C at P. [5]

4. *(a)* (i) Express $x^2 + 8x + 5$ in the form $(x + a)^2 + b$, where the values of a, b are to be determined.

   (ii) **Use your answers to part (i)** to find the least value of $3x^2 + 24x + 15$ and the corresponding value of x. [4]

   (b) Solve the simultaneous equations $y = x^2 - x - 9$ and $y = 2x - 5$ algebraically.
   Write down a geometrical interpretation of your results. [5]

5. *(a)* Find the range of values of k for which the quadratic equation
   $$5x^2 + 6x - 3k = 0$$
   has two distinct real roots. [4]

   (b) Solve the inequality $2x^2 - 11x + 15 \leqslant 0$. [3]

6. *(a)* Given that $y = -x^2 + 4x - 6$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ from first principles. [5]

   (b) Differentiate $5x^{\frac{4}{3}} - \dfrac{9}{\sqrt{x}}$ with respect to x. [2]

7. In the binomial expansion of $(a + 4x)^6$, where $a \neq 0$, the coefficient of the term in x^2 is twice the coefficient of the term in x. Find the value of a. [4]

8. *(a)* Given that $x + 2$ is a factor of $px^3 + 18x^2 - 4x - 8$, write down an equation satisfied by p. Hence show that $p = 9$. [2]

   (b) Solve the equation $9x^3 + 18x^2 - 4x - 8 = 0$. [4]

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-5, 0)$ and $(7, 0)$ and has a minimum point at $(1, -2)$.
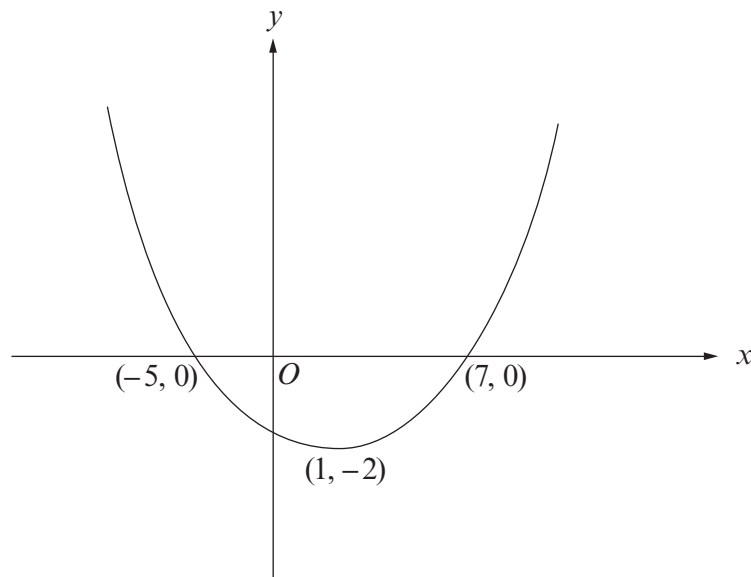
Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

   (a) $y = 3f(x)$ [3]

   (b) $y = f(-x)$ [3]

10. The curve C has equation

$$y = x^3 - 3x^2 - 9x + 14.$$

   (a) Find the coordinates and the nature of each of the stationary points of C. [6]

   (b) Sketch C, indicating the coordinates of each of the stationary points. [2]

   (c) Given that the equation

$$x^3 - 3x^2 - 9x + 14 = k$$

   has only one real root, find the range of possible values for k. [2]