GCE AS/A level

974/01

MATHEMATICS C2
Pure Mathematics

P.M. FRIDAY, 15 January 2010

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_{1}^{1\cdot4} \sqrt{3 - x^2}\, dx \quad.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$3 - 7\cos\theta = 6\sin^2\theta.$$ [5]

(b) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying
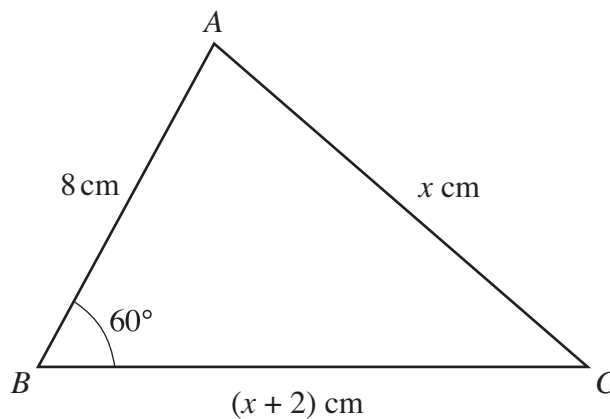
$$\tan(2x + 45°) = 0\cdot7.$$ [3]

(c) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4\tan\theta\cos\theta + 1 = 0.$$ [3]

3. The diagram below shows a sketch of the triangle ABC with $AB = 8\,\text{cm}$, $AC = x\,\text{cm}$, $BC = (x + 2)\,\text{cm}$ and $A\widehat{B}C = 60°$.

(a) Write down and simplify an equation satisfied by x. Hence evaluate x. [3]

(b) Find the size of $A\widehat{C}B$. [2]

4. The sum of the first eight terms of an arithmetic series is 124 and the sum of the first twenty terms of the series is 910.

  (a) Find the first term and common difference of the series. [5]

 *(b)* The nth term of the series is 183. Find the value of n. [2]

5. *(a)* A geometric series has first term a and common ratio r. Prove that the sum of the first n terms is given by

$$S_n = \frac{a(1 - r^n)}{1 - r} \; .$$

[3]

  (b) The common ratio of a geometric series is positive. The sum of the first four terms of the series is 73·8. The sum to infinity of the series is 125. Find the common ratio and the first term of the geometric series. [6]

6. *(a)* Find $\displaystyle\int\left(x^{\frac{1}{3}} - \frac{2}{x^{\frac{1}{4}}} \right)\mathrm{d}x$. [2]

  (b)
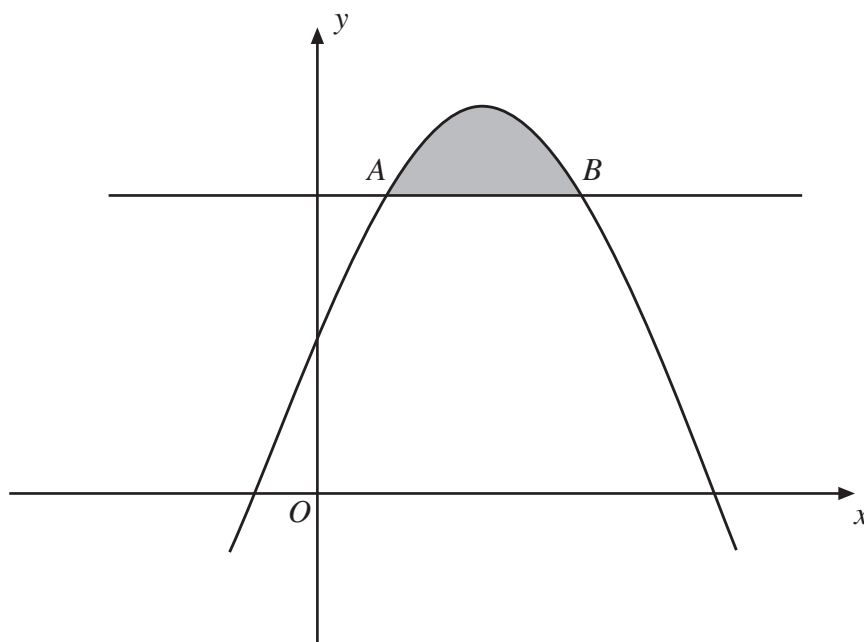
The diagram shows a sketch of the curve $y = 5 + 4x - x^2$ and the line $y = 8$. The curve and the line intersect at the points A and B.

  (i) Showing your working, find the x-coordinates of A and B.

  (ii) Find the area of the shaded region. [10]

TURN OVER

7. *(a)* Given that $x > 0$, show that

$$\log_a x^n = n \log_a x.$$ [3]

(b) Express $\frac{1}{2} \log_a 324 + \log_a 56 - 2 \log_a 12$ in the form $\log_a b$, where b is a constant whose value is to be found. [4]

(c) (i) Rewrite the equation

$$3^x = 2^{x+1}$$

in the form

$$c^x = d,$$

where the values of the constants c and d are to be found.

(ii) Hence or otherwise, solve the equation

$$3^x = 2^{x+1},$$

giving your answer correct to two decimal places. [4]

8. The circle C has centre A and equation
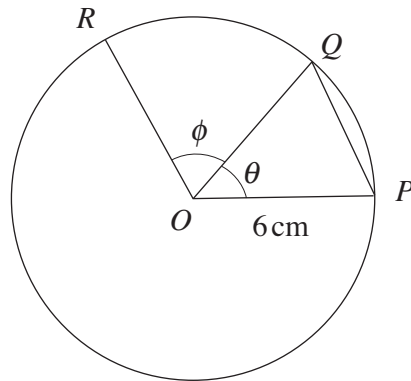
$$x^2 + y^2 + 4x - 8y + 10 = 0.$$

(a) Find the coordinates of A and the radius of C. [3]

(b) The line L has equation

$$x - 3y + 4 = 0.$$

Show that L is a tangent to the circle C. [4]

9.

The diagram shows three points P, Q and R on a circle with centre O. The circle has radius $6\,\text{cm}$, $P\hat{O}Q = \theta$ radians and $Q\hat{O}R = \phi$ radians.

(a) The area of the **triangle** POQ is $9 \cdot 1\,\text{cm}^2$. Find the value of θ, giving your answer correct to two decimal places. [2]

(b) Find the area of the **sector** POQ. Give your answer correct to two decimal places. [2]

(c) The **perimeter** of the **sector** QOR is equal to half the circumference of the circle. Find the value of ϕ, giving your answer correct to two decimal places. [2]

10. The nth term of a number sequence is denoted by t_n. The $(n + 1)$th term of the sequence satisfies

$$t_{n+1} = 2t_n + 1,$$

for all positive integers n. Given that $t_4 = 63$,

(a) evaluate t_1, [2]

(b) without carrying out any further calculations, explain why 6043582 cannot be one of the terms of this number sequence. [1]