WJEC CBAC

GCE AS/A level

975/01

MATHEMATICS C3

Pure Mathematics

A.M. THURSDAY, 15 January 2009

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_0^{\frac{2\pi}{9}} \ln(\cos x)\,\mathrm{d}x.$$

Show your working and give your answer correct to four decimal places.

Deduce an approximate value for

$$\int_0^{\frac{2\pi}{9}} \ln(\cos^2 x)\,\mathrm{d}x.$$

[5]

2. *(a)* Show, by counter-example, that the statement

$$\cos 2\theta \equiv 2\cos^2\theta - \sin^2\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$3\tan^2\theta = 7 + \sec\theta.$$ [6]

3. *(a)* Given that $x^2 + 3xy + 2y^2 - 2x = 13$, find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point (1, 2). [4]

(b) Given that $x = 2e^t + 6$, $y = 4e^{2t} + 3e^t + 1$, find the value of t when $\frac{\mathrm{d}y}{\mathrm{d}x} = 6$, giving your answer correct to three decimal places. [7]

4. *(a)* By sketching the graphs of $y = x^3$ and $y = 4 - x$, determine the number of real roots of the equation $x^3 + x - 4 = 0$. [3]

(b) **You may assume** that the equation $x^3 + x - 4 = 0$ has a root α between 1 and 2. The recurrence relation

$$x_{n+1} = (4 - x_n)^{\frac{1}{3}}$$

with $x_0 = 1{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [5]

5. *(a)* Differentiate **each** of the following with respect to x and simplify your answers, wherever possible.

(i) $\ln(\sin x)$ (ii) $\sin^{-1}(4x)$ (iii) $\dfrac{3x^2+2}{x^2+5}$

[8]

(b) By first writing $y = \tan^{-1}x$ as $x = \tan y$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. [4]

6. Solve the following.

(a) $\dfrac{2\,|\,x\,|+9}{|\,x\,|+1}=5$ [2]

(b) $|\,5x+7\,|\leqslant 4$ [3]

7. *(a)* Find (i) $\displaystyle\int \frac{7}{6x+5}\,\mathrm{d}x$, (ii) $\displaystyle\int \cos 5x\,\mathrm{d}x$. [4]

(b) Evaluate $\displaystyle\int_0^1 \frac{9}{(2x+1)^2}\,\mathrm{d}x$. [4]

8. Given that $f(x)=\ln x$, sketch the graphs of $y=f(x)$ and $y=-f(x+1)$ on the same diagram. Label the coordinates of the points of intersection with the x-axis and indicate the behaviour of the graphs for large positive and negative values of y. [5]

9. The function f has domain $x\leqslant 0$ and is defined by $f(x)=5x^2+4$.

(a) Find an expression for $f^{-1}(x)$. [5]

(b) Write down the domain and range of f^{-1}. [1]

10. The function f has domain $[1,\infty)$ and is defined by

$$f(x)=2x-k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[0,\infty)$ and is defined by

$$g(x)=3x^2+4.$$

(b) Find the largest value of k that allows the function gf to be formed. [2]

(c) Given that $gf(2)=31$, find the value of k. [4]