WJEC CBAC

GCE AS/A level

0975/01

MATHEMATICS C3
Pure Mathematics

P.M. FRIDAY, 20 January 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \cos^2 x \, \mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

(b) **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \sin^2 x \, \mathrm{d}x.$$ [2]

2. *(a)* Show, by counter-example, that the statement

$$\sin(\theta + \phi) \equiv \sin\theta + \sin\phi$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sec^2\theta + 8 = 4\tan^2\theta + 5\sec\theta.$$ [6]

3. *(a)* A function is defined parametrically by

$$x = 3t^2, \; y = t^6 - 4t^3.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(ii) Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{7}{2}$, show that $2t^4 - 4t - 7 = 0$. [5]

(b) Show that the equation

$$2t^4 - 4t - 7 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$t_{n+1} = \left(\frac{4t_n + 7}{2}\right)^{\frac{1}{4}}$$

with $t_0 = 1{\cdot}6$ can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

4. Given that $x^2y^2 + x^4 + 6 = 2y^3 + 2x$, find the value of $\frac{dy}{dx}$ at the point (2, 3). [4]

5. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $\tan^{-1}4x$ *(b)* e^{x^3} [2], [2]

(c) $x^5\ln x$ *(d)* $\frac{3-2x^2}{5-4x^2}$ [3], [3]

6. *(a)* Find each of the following, simplifying your answer wherever possible.

(i) $\int \sin\left(\frac{x}{4}\right)dx$, (ii) $\int e^{\frac{2x}{3}}\,dx$, (iii) $\int \frac{7}{8x-2}\,dx$. [6]

(b) Evaluate $\int_1^9 \frac{3}{\sqrt{5x+4}}\,dx$. [4]

7. Solve the following.

(a) $|4x-5| \geqslant 3$, [3]

(b) $(3|x|+1)^{\frac{1}{3}} = 4$. [2]

8. The function f is defined by $f(x) = e^x$.

(a) Sketch the graph of $y = f(x)$. Write down the coordinates of the point of intersection of the graph with the y-axis. [2]

(b) Using a separate set of axes,

(i) sketch the graph of $y = f(3x) - 4$, indicating the behaviour of your graph for large negative values of x,

(ii) write down the coordinates of the point of intersection of the graph with the y-axis,

(iii) find the x-coordinate of the point of intersection of the graph with the x-axis. Give your answer correct to three decimal places. [4]

9. The function f has domain $[6, \infty)$ and is defined by

$$f(x) = 3 - \frac{1}{\sqrt{x-2}}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

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10. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = 3x + k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[-2, \infty)$ and is defined by

$$g(x) = x^2 - 6.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k, for $gf(x)$.

(ii) Given that $gf(2) = 3$, find the value of k. [5]