WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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975/01

MATHEMATICS C3

Pure Mathematics

P.M. TUESDAY, 5 June 2007

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_{1}^{1\cdot4} \frac{1}{2+\ln x}\,\mathrm{d}x\ .$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\cos 2\theta \equiv 1 - 2\cos^2\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\cot^2\theta = 7 - 2\operatorname{cosec}\theta.$$ [6]

3. *(a)* A function is defined parametrically by $x = 5t^2,\ y = t^5 + \dfrac{20t^3}{3}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(ii) Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$, show that $t^3 + 4t - 2 = 0$. [5]

(b) Show that the equation

$$t^3 + 4t - 2 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$t_{n+1} = \frac{2 - t_n^3}{4}$$

with $t_0 = 0\cdot5$ can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

4. *(a)* Sketch the graphs of $y = x^2 - 4$ and $y = |x^2 - 4|$, indicating the points where the graphs meet the x-axis and the y-axis. [4]

(b) Solve the inequality

$$|5x - 3| > 4.$$ [3]

5. Given that

$$3y^2 + x^2y^3 + x^4 - x^2 - 11 = 0,$$

find the value of $\frac{dy}{dx}$ when $x = 2$, $y = -1$. [4]

6. *(a)* Differentiate each of the following with respect to x and simplify your answers, wherever possible.

(i) $x^2 \sin x$ (ii) $\ln(x^2 + 3)$ (iii) e^{9-2x} (iv) $\dfrac{4}{(3x+7)^2}$

(v) $\sin^{-1} 3x$ [10]

(b) Given $y = \dfrac{1 + \tan x}{1 - \tan x}$ $(\tan x \neq 1)$, show that $\frac{dy}{dx}$ is always positive. [4]

7. *(a)* Find (i) $\displaystyle\int \frac{1}{(5-2x)}\,dx$, (ii) $\displaystyle\int (3x+2)^{20}\,dx$,

(iii) $\displaystyle\int e^{7x}\,dx$. [7]

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \cos\left(3x + \frac{\pi}{3}\right)\,dx$. [4]

8. The functions f and g have domains $[0, \infty)$ and $(-\infty, \infty)$ respectively, and are defined by

$$f(x) = e^x,$$

$$g(x) = x^2 + 1.$$

(a) Find the range of f and the range of g. [2]

(b) Find an expression for $gf(x)$, simplifying your expression as much as possible. [2]

(c) Write down the domain and range of gf. [2]

(d) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = gf(x)$ indicating where the graphs meet the y-axis. [5]

9. The function f has domain $x \geqslant 0$ and is defined by

$$f(x) = \frac{8}{x+2}.$$

Find an expression for $f^{-1}(x)$ and write down the domain of f^{-1}. [4]