GCE AS/A Level

0975/01

S17-0975-01

MATHEMATICS – C3
Pure Mathematics

WEDNESDAY, 7 JUNE 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_5^7 \ln\left(1+x^2\right)\mathrm{d}x.$$

Show your working and give your answer correct to one decimal place. [4]

(b) **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_5^7 \ln\left(\frac{3}{\sqrt{1+x^2}}\right)\mathrm{d}x.$$

[3]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$6\tan^2\theta - 6 = 4\sec^2\theta + 5\sec\theta.$$

[6]

(b) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$3\sec\phi + 5\tan\phi = 0.$$

[3]

3. *(a)* Given that

$$x^4 - 3x^2y + 2y^3 - 4x = 7,$$

find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(b) Given that $x = 7t + 2t^2$, $y = \dfrac{4+3t}{7+4t}$,

(i) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k}{(7+4t)^n}$,

where the values of the constants k and n are to be found,

(ii) find a similar expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [8]

4. A large tank in the form of a cuboid is used to store water. The width of the tank is denoted by x m. The length of the tank is 4 m **greater** than its width, whilst the height of the tank is 2 m **less** than its width. The volume of the tank is 150 m^3.

(a) (i) Show that $x^3 + 2x^2 - 8x - 150 = 0$.

(ii) Show that $5 < x < 6$. [4]

(b) The recurrence relation

$$x_{n+1} = \left(150 + 8x_n - 2x_n^2\right)^{\frac{1}{3}},$$

with x_0 = 6, can be used to find the value of x. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to two decimal places and prove that this is the value of x correct to two decimal places. [5]

5. *(a)* Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\sqrt{3x^2 + 5x}$ (ii) $\sin^{-1} 3x$ [4]

(b) By first writing $y = \cot^{-1} x$ as $x = \cot y$ and then assuming the derivative of cot y, find $\dfrac{dy}{dx}$ in terms of x. [4]

6. *(a)* Find each of the following integrals, simplifying your answer wherever possible.

(i) $\displaystyle\int 8e^{2-5x}dx$ (ii) $\displaystyle\int \frac{6}{\sqrt[3]{4x-7}}dx$

(iii) $\displaystyle\int \cos\left(\frac{7x-9}{3}\right)dx$ [6]

(b) (i) Differentiate $\ln(3x^2 - 8)$ with respect to x.

(ii) **Use your answer to *(b)*(i)** to evaluate

$$\int_2^6 \frac{3x}{3x^2 - 8}dx.$$

Give your answer in the form $\ln k$, where k is an integer whose value is to be found. [6]

TURN OVER

7. *(a)* Show, by counter-example, that the following statement is false.

'If $\dfrac{7x-200}{x} > 5$, then $x > 100$.' [2]

(b) The graph of $y = f(x)$ has a single maximum which is situated at the point $(-2, 4)$. The graph of $y = af(x + b)$ has a single minimum which is situated at the point $(4, -2)$. Find the values of the constants a and b. [2]

8. The function f has domain $[8, \infty)$ and is defined by

$$f(x) = 2 + \frac{3}{\sqrt{5x-4}} .$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

9. The function f has domain $[2, \infty)$ and is defined by

$$f(x) = 4x + k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[-3, \infty)$ and is defined by

$$g(x) = x^2 - 9.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k, for $gf(x)$.

(ii) Given that $gf(2) = 7$, find the value of k.

[5]

END OF PAPER