WELSH JOINT EDUCATION COMMITTEE **CYD-BWYLLGOR ADDYSG CYMRU**

General Certificate of Education **Tystysgrif Addysg Gyffredinol**

Advanced Level/Advanced Subsidiary **Safon Uwch/Uwch Gyfrannol**

MATHEMATICS C3

Pure Mathematics

Specimen Paper 2005/2006

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Show that the equation

$$x^3 + 10x - 4 = 0$$

has a root α between 0 and 1.

The iterative formula

$$x_{n+1} = \frac{4 - x_n^3}{10}$$

with $x_0 = 0{\cdot}3$ may be used to find α.

Calculate and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and show that it is the value of α correct to five decimal places. [7]

2. Use Simpson's Rule with five ordinates to evaluate the integral

$$\int_1^2 \sqrt{1 + x^4}\,\mathrm{d}x\,.$$

Show your working and give your answers correct to two decimal places. [4]

3. Solve the inequality

$$|2x - 5| < 9.$$ [4]

4. *(a)* Given that

$$y^3 - x^2y^2 = x^2 + 3x + 1,$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(b) Given that $x = t^3 + 2$, $y = t^2 + 3$,

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{2}{9t^4}.$$ [5]

5. Showing all your working, find the values of θ between 0º and 360º satisfying

$$\cot^2\theta = 7 - 2\,\mathrm{cosec}\,\theta.$$ [7]

6. Differentiate the following with respect to x, simplifying your answers as much as possible.

(a) $e^{2x}\sin x$

(b) $\dfrac{2x^2-4}{x^2+3}$

(c) $\tan(4x^2+3)$ [4], [3], [2]

7. *(a)* Find

(i) $\displaystyle\int e^{-4x+1}\,dx,$

(ii) $\displaystyle\int\left(\frac{1}{2x+1}+\frac{1}{(3x+7)^3}\right)dx.$ [7]

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}}\sin 2x\,dx.$ [3]

8. *(a)* Given that $y = \tan^{-1}x$, show that

$$\frac{dy}{dx}=\frac{1}{x^2+1}.$$ [3]

(b) Differentiate $\ln(x^2+1)$ with respect to x. [2]

(c) Use the results derived in *(a)* and *(b)* to find

$$\int\frac{3+x}{1+x^2}dx.$$ [4]

9. Given that $f(x) = e^x$, sketch, on the same diagram, the graphs of $y = f(x)$ and $y = 2f(x) + 3$. Label any points of intersection of the graphs with the y-axis. Indicate the behaviour of the graphs for large positive and negative values of x. [5]

10. *(a)* The function f has domain $x \geq 2$ and is defined by

$$f(x) = \ln(2x - 3) + 4.$$

(i) Find an expression for $f^{-1}(x)$.

(ii) State the domain and range of $f^{-1}(x)$. [6]

(b) The functions g and h are defined for all x by

$$g(x) = x^2 + 3,$$

$$h(x) = 2x + 2.$$

Solve the equation

$$gh(x) = 2\,hg(x) + 15.$$ [5]