976/01

MATHEMATICS C4

Pure Mathematics

A.M. MONDAY, 20 June 2005

$(1\frac{1}{2}$ hours$)$

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Express $\dfrac{8x^2 + x - 5}{(2x-1)^2(x+2)}$ in partial fractions. [4]

    (b) Find $\displaystyle\int \dfrac{8x^2 + x - 5}{(2x-1)^2(x+2)}\ dx$. [3]

2. Expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

    Hence, by writing $x = \dfrac{1}{8}$ in your expansion, find an approximate value for $\sqrt{3}$ in the form $\dfrac{a}{b}$, where a and b are integers. [5]

3. Find the equation of the tangent to the curve

    $$4x^2 + 3xy - y^2 = 21$$

    at the point (2, 1). [4]

4. *(a)* Find all values of θ in the range $0 \leqslant \theta \leqslant 360°$ satisfying

    $$\sin 2\theta = \cos\theta.$$ [4]

    (b) Find all values of θ in the range $0 \leqslant \theta \leqslant 360°$ satisfying

    $$4\sin\theta + \cos\theta = 2,$$

    giving your answers in degrees correct to one decimal place. [6]

5. The region bounded by the curve $y = \sqrt{x} + \dfrac{4}{\sqrt{x}}$, the x-axis and the lines $x = 1$, $x = 4$ is rotated through four right-angles about the x-axis.
    Find, correct to one decimal place, the volume of the solid formed. [5]

6. The parametric equations of the curve C are

$$x = 2t + 1, \quad y = t^2 + 3.$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$px - y = p^2 + p - 3. \hspace{4cm} [4]$$

(b) The tangent to C at the point P passes through the point $(2, -3)$. Given that the point P is in the second quadrant, find the equation of the tangent. [4]

7. (a) Use the substitution $u = 2x - 1$ to evaluate

$$\int_0^1 x(2x-1)^9 \, dx \ . \hspace{4cm} [5]$$

(b) (i) Find $\displaystyle\int x \cos 2x \, dx$. [4]

(ii) Use the result of (b)(i) to find

$$\int x \cos^2 x \ dx \ . \hspace{4cm} [3]$$

8. The size P of a population of bacteria at time t days is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P.

(a) Write down a differential equation that is satisfied by P. [1]

(b) Given that the initial size of the population is P_0, show that $P = P_0 \, e^{kt}$, where k is a positive constant. [5]

(c) Two days after the start, the population is $1{\cdot}2P_0$. Find when the population will be $2P_0$. [4]

9. (a) The position vectors of the points A and B are given by $\quad \mathbf{a} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k},$
$\mathbf{b} = -7\mathbf{i} + 4\mathbf{j} - \mathbf{k}.$

(i) Find the vector equation of the line AB. [3]

(ii) The vector equation of the line L is

$$\mathbf{r} = -\mathbf{i} + 7\mathbf{j} + 8\mathbf{k} + \mu(2\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}).$$

Given that AB and L intersect, find the position vector of the point of intersection. [5]

(b) Show that the vectors $\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ are perpendicular. [2]

TURN OVER

10. Complete the following proof by contradiction to show that $x + \dfrac{25}{x} \geqslant 10$ when x is real and positive.

Assume that $x + \dfrac{25}{x} < 10$ *, when x is real and positive.*

Since x is positive, multiplication of both sides of the inequality by x gives $x^2 + 25 < 10x$. **[4]**