WJEC
CBAC

GCE AS/A level

976/01

MATHEMATICS C4
Pure Mathematics

P.M. MONDAY, 15 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$f(x) = \frac{3x}{(1+x)^2(2+x)} \quad ,$$

(a) express $f(x)$ in terms of partial fractions, [4]

(b) evaluate

$$\int_0^1 f(x)\mathrm{d}x,$$

giving your answer correct to three decimal places. [4]

2. Find all the values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying $3\sin 2\theta = 2\sin\theta$. [5]

3. *(a)* Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\cos\theta + \sqrt{3}\sin\theta = 1.$$ [4]

4. The region bounded by the curve $y = \cos 2x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{8}$, is rotated about the x-axis through four right-angles. Find the volume of the solid generated. [6]

5. The parametric equations of the curve C are $x = t^2$, $y = t^3$. The point P has parameter p.

(a) Show that the equation of the tangent to C at the point P is $3px - 2y = p^3$. [4]

(b) The tangent to C at the point P intersects C again at the point $Q(q^2, q^3)$. Given that $p = 2$, show that q satisfies the equation $q^3 - 3q^2 + 4 = 0$ and determine the value of q. [5]

6. *(a)* Find $\int (x+3)\mathrm{e}^{2x}\,\mathrm{d}x$. [4]

(b) Use the substitution $u = 2\cos x + 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\sqrt{(2\cos x + 1)}}\,\mathrm{d}x.$$

[5]

7. The value of an electronic component may be modelled as a continuous variable. The value of the component at time t years is £P. The rate of decrease of P is directly proportional to P^3.

(a) Write down a differential equation that is satisfied by P. [1]

(b) The value of the component when $t = 0$ is £20. Show that

$$\frac{1}{P^2} = \frac{1}{400} + At,$$

where A is a positive constant. [5]

(c) Given that the value of the component when $t = 1$ is £10, find the time when the value is £5. [4]

8. *(a)* The position vectors of the points A and B are given by

$$\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}, \quad \mathbf{b} = 4\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}.$$

(i) Find the vector equation of the line AB.

(ii) The vector equation of the line L is

$$\mathbf{r} = 5\mathbf{i} + 6\mathbf{j} + \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}).$$

Show that AB and L intersect and find the position vector of the point of intersection. [9]

(b) Show that the vectors $3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ are perpendicular. [2]

9. Expand $(1+4x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^2. State the range of values of x for which your expansion is valid.

Expand $(1+4k+16k^2)^{\frac{1}{2}}$ in ascending powers of k as far as the term in k^2. [6]

10. Complete the following proof by contradiction to show that $\sqrt{3}$ is irrational.

Assume that $\sqrt{3}$ is rational. Then $\sqrt{3}$ may be written in the form $\frac{a}{b}$ where a and b are integers having no common factors.

$\therefore$ $a^2 = 3b^2$.
$\therefore$ a^2 has a factor 3.
$\therefore$ a has a factor 3 so that $a = 3k$, where k is an integer. [4]

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