WJEC CBAC

GCE AS/A level

976/01

MATHEMATICS C4
Pure Mathematics

A.M. MONDAY, 20 June 2011

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S11-0976-01)

1. Given that $f(x)=\dfrac{x^2+x+13}{(x+2)^2(x-3)}$,

(a) express $f(x)$ in terms of partial fractions, [4]

(b) evaluate

$$\int_6^7 f(x)\mathrm{d}x,$$

giving your answer correct to three decimal places. [3]

2. Find the equation of the normal to the curve

$$x^4 - 2x^2y + y^2 = 4$$

at the point (1, 3). [5]

3. *(a)* Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan 2x = 4\tan x.$$ [5]

(b) Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta-\alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.
Hence, find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$7\cos\theta + 24\sin\theta = 16.$$ [6]

4. The curve C has the parametric equations

$$x = 3\cos t,\ y = 4\sin t.$$

The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

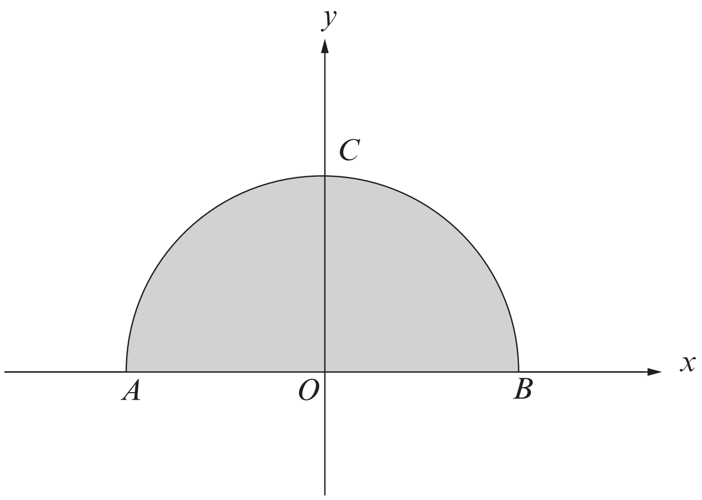
$$(3\sin p)y + (4\cos p)x - 12 = 0.$$ [5]

(b) The tangent to C at the point P meets the x-axis at the point A and the y-axis at the point B. Given that $p = \dfrac{\pi}{6}$,

(i) find the coordinates of A and B,

(ii) show that the exact length of AB is $2\sqrt{19}$. [4]

5. The region shaded in the diagram below is bounded by the x-axis and that part of the curve with equation $x^2 + y^2 = 9$ lying above the x-axis. The points of intersection of the curve with the coordinate axes are denoted by A, B and C.



(a) Write down the coordinates of A, B and C. [1]

(b) (i) By carrying out an appropriate integration, find the volume generated when the region shaded in the diagram is rotated through four right-angles about the x-axis.

(ii) Give a geometrical interpretation of your answer. [4]

6. Expand $4(1+2x)^{\frac{1}{2}} - \frac{1}{(1+3x)^2}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which your expansion is valid. [7]

7. *(a)* Find $\int x \sin 2x \, \mathrm{d}x$. [4]

(b) Use the substitution $u = 5 - x^2$ to evaluate

$$\int_0^2 \frac{x}{(5-x^2)^3} \, \mathrm{d}x \, .$$ [4]

TURN OVER

8. The size N of the population of a small island may be modelled as a continuous variable. At time t, the rate of increase of N is directly proportional to the value of N.

(a) Write down the differential equation that is satisfied by N. [1]

(b) Show that $N = Ae^{kt}$, where A and k are constants. [3]

(c) Given that $N = 100$ when $t = 2$ and that $N = 160$ when $t = 12$,

(i) show that $k = 0{\cdot}047$, correct to three decimal places,

(ii) find the size of the population when $t = 20$. [7]

9. *(a)* Given that the vectors $5\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$ and $4\mathbf{i} + 6\mathbf{j} + a\mathbf{k}$ are perpendicular, find the value of the constant a. [3]

(b) The line L_1 passes through the point with position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(i) Write down the vector equation of the line L_1.

(ii) The line L_2 has vector equation

$$\mathbf{r} = 4\mathbf{i} + 7\mathbf{j} + 5\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).$$

Show that L_1 and L_2 do not intersect. [6]

10. Prove by contradiction the following proposition.

When x is real and positive,

$$4x + \frac{9}{x} \geqslant 12.$$

The first line of the proof is given below.

Assume that there is a positive and real value of x such that

$$4x + \frac{9}{x} < 12.$$ [3]