

GCE AS/A level

0976/01

MATHEMATICS C4 Pure Mathematics

A.M. THURSDAY, 14 June 2012 1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer all questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{11 + x - x^2}{(x+1)(x-2)^2}.$$

- (a) Express f(x) in terms of partial fractions.
- -
- (b) Use your result to part (a) to find the value of f'(0).
- 2. Find the equation of the tangent to the curve

$$v^3 - 4x^2 - 3xy + 25 = 0$$

at the point (2, -3).

[4]

[4]

[3]

3. (a) Find all values of θ in the range $0^{\circ} \le \theta \le 360^{\circ}$ satisfying

$$4\cos 2\theta = 1 - 2\sin \theta. \tag{6}$$

- (b) (i) Express $8 \sin x + 15 \cos x$ in the form $R \sin (x + \alpha)$, where R and α are constants with R > 0 and $0^{\circ} < \alpha < 90^{\circ}$.
 - (ii) Find all values of x in the range $0^{\circ} \le x \le 360^{\circ}$ satisfying $8 \sin x + 15 \cos x = 11$.
 - (iii) Find the greatest possible value for k so that

$$8\sin x + 15\cos x = k$$

has solutions. Give a reason for your answer.

[7]

4. The region R is bounded by the curve $y = \sqrt{x} + \frac{5}{\sqrt{x}}$, the x-axis and the lines x = 3, x = 4.

Find the volume generated when *R* is rotated through four right-angles about the *x*-axis. Give your answer correct to the nearest integer. [5]

5. Expand $\left(1+\frac{x}{3}\right)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2 .

State the range of values of x for which your expansion is valid.

Hence, by writing $x = \frac{1}{5}$ in your expansion, find an approximate value for $\sqrt{15}$ in the form $\frac{a}{b}$, where a and b are integers whose values are to be found. [5]

- **6.** The parametric equations of the curve C are $x = t^2$, y = 2t.
 - (a) Show that the normal to C at the point P with parameter p has equation

$$y + px = p^3 + 2p. ag{5}$$

- (b) The normal to C at the point P intersects C again at the point with parameter 3.
 - (i) Show that $p^3 7p 6 = 0$.
 - (ii) Hence show that *P* can be one of two points. Find the coordinates of each of these two points. [6]

7. (a) Find
$$\int xe^{-2x} dx$$
. [4]

(b) Use the substitution $u = 1 + 3 \ln x$ to evaluate

$$\int_1^e \frac{1}{x(1+3\ln x)} dx.$$

Give your answer correct to four decimal places.

- **8.** Water is leaking from a hole at the bottom of a large tank. The volume of the water in the tank at time t hours is $V \text{m}^3$. The rate of decrease of V is directly proportional to V^3 .
 - (a) Write down a differential equation satisfied by V.
 - (b) Given that V = 60 when t = 0, show that

$$V^2 = \frac{3600}{at+1}$$

where *a* is a constant.

(c) When t = 2, the volume of the water in the tank is $50 \,\mathrm{m}^3$. Find the value of t when the volume of the water in the tank is $27 \,\mathrm{m}^3$. Give your answer correct to one decimal place.

[4

[4]

[4]

[1]

TURN OVER

9. The position vectors of the points A and B are given by

$$a = 4i + j - 6k,$$

 $b = 6i + 2j - 4k,$

respectively.

- Determine whether or not the vectors **a** and **b** are perpendicular, giving a reason for your (a)
- Write down the vector AB. (b) (i)
 - Find the vector equation of the line AB.

[3]

The vector equation of the line L is given by (c)

$$r = 2i + 6j + pk + \mu(-2i + j + 3k),$$

where p is a constant.

Given that the lines AB and L intersect, find the value of p.

[5]

10. Complete the following proof by contradiction to show that $\sqrt{5}$ is irrational.

Assume that $\sqrt{5}$ is rational. Then $\sqrt{5}$ may be written in the form $\frac{a}{b}$, where a, b are integers having no common factors.

- $\therefore a^2 = 5b^2.$ $\therefore a^2 \text{ has a factor 5.}$
- \therefore a has a factor 5 so that a = 5k, where k is an integer.

[3]