GCE AS/A level

0976/01

MATHEMATICS – C4
Pure Mathematics

A.M. THURSDAY, 13 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{6 + x - 9x^2}{x^2(x+2)}.$$

(a) Express $f(x)$ in terms of partial fractions. [4]

(b) Using your result to part *(a)*,

(i) find an expression for $f'(x)$,

(ii) verify that $f(x)$ has a stationary value when $x = 2$. [3]

2. Find the equation of the normal to the curve

$$x^3 - 2xy^2 + y^3 = 5$$

at the point (2, 1). [5]

3. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$8\cos 2\theta + 6 = \cos^2\theta + \cos\theta.$$ [6]

(b) Express $\sqrt{15}\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.
Hence find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sqrt{15}\cos\theta - \sin\theta = 3.$$ [6]

4. The region R is bounded by the curve $y = \sin 2x$, the x-axis and the lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{2}$.
Find the volume generated when R is rotated through four right angles about the x-axis. Give your answer correct to three decimal places. [5]

5. *(a)* (i) Expand $(1 + 6x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2.

(ii) State the range of values of x for which your expansion is valid. [3]

(b) Use your expansion in part *(a)* to find an approximate value for one root of the equation

$$2(1 + 6x)^{\frac{1}{3}} = 2x^2 - 15x.$$ [2]

6. The curve C has the parametric equations

$$x = at,\ y = \frac{b}{t},$$

where a, b are positive constants.
The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$ap^2y + bx - 2abp = 0.$$ [5]

(b) The tangent to C at the point P meets the x-axis at the point A and the y-axis at the point B. Find the area of triangle AOB, where O denotes the origin. Give your answer in its simplest form. [3]

(c) The point D has coordinates $(2a, b)$. Show that there is no point P on C such that the tangent to C at the point P passes through D. [3]

7. *(a)* Find $\int (3x - 1)\cos 2x \, \mathrm{d}x$. [4]

(b) Use the substitution $u = 2x + 1$ to evaluate

$$\int_0^1 \frac{x}{(2x+1)^3} \, \mathrm{d}x.$$ [5]

8. Part of the surface of a small lake is covered by green algae. The area of the lake covered by the algae at time t years is $A\,\mathrm{m}^2$. The rate of increase of A is directly proportional to $\sqrt{A}$.

(a) Write down a differential equation satisfied by A. [1]

(b) The area of the lake covered by the algae at time $t = 3$ is $64\,\mathrm{m}^2$ and the area covered at time $t = 5{\cdot}5$ is $196\,\mathrm{m}^2$. Find an expression for A in terms of t. [6]

TURN OVER

9. The position vectors of the points A and B are given by

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k},$$
$$\mathbf{b} = 7\mathbf{i} - \mathbf{j} + 5\mathbf{k},$$

respectively.

(a) Write down the vector **AB**. [1]

(b) The point C lies on the line AB and is such that $AC : CB = 3 : 1$.
Find the position vector of C. [2]

(c) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} + \mathbf{j} + 11\mathbf{k} + \lambda\,(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}).$$

(i) Find the vector equation of the line parallel to L which passes through A.

(ii) **Verify** that B is in fact the foot of the perpendicular from A to L. [8]

10. Prove by contradiction the following proposition.

When x is real,

$$(5x - 3)^2 + 1 \geqslant (3x - 1)^2.$$

The first line of the proof is given below.

Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2.$$ [3]