GCE AS/A level

0976/01

S15-0976-01

MATHEMATICS – C4
Pure Mathematics

A.M. FRIDAY, 12 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that $f(x) = \dfrac{2x^2 + 5x + 25}{(x+3)^2(x-1)}$,

(a) express $f(x)$ in terms of partial fractions, [4]

(b) evaluate

$$\int_3^{10} f(x)\,\mathrm{d}x,$$

giving your answer correct to two decimal places. [3]

2. The curve *C* has equation

$$x^4 + 3x^2y - 2y^2 = 34.$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4x^3 + 6xy}{4y - 3x^2}$. [3]

(b) Find the coordinates of each of the points on *C* where the tangent is parallel to the y-axis. [4]

3. *(a)* Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan(x + 45°) = 8\tan x.$$ [5]

(b) (i) Express $\sqrt{13}\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sqrt{13}\sin\theta - 6\cos\theta = -4.$$ [6]

4. The line *L* has equation $y = mx$, where $m > 0$. The region *R* is bounded by *L*, the x-axis and the line $x = a$, where $a > 0$.

(a) Using integration, find an expression, in terms of a and m, for the volume *V* generated when *R* is rotated through four right angles about the x-axis. [3]

(b) The point with coordinates (a, b) lies on *L*.

(i) Rewrite the expression for the volume *V* found in part *(a)* in terms of a and b.

(ii) Give a geometrical interpretation of your answer. [3]

5. Expand $\left(1+\frac{x}{8}\right)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2.

State the range of values of x for which your expansion is valid.
Hence, by writing $x = 1$ in your expansion, find an approximate value for $\sqrt{2}$ in the form $\frac{a}{b}$, where a and b are integers whose values are to be found. [5]

6. The parametric equations of the curve *C* are $x = at^2$, $y = 2at$, where a is a positive constant. The points *P* and *Q* lie on *C* and have parameters p and q respectively.

(a) Simplifying your answer in each case, find

(i) the gradient of the tangent to *C* at the point *P*,

(ii) the equation of the tangent to *C* at the point *P*. [4]

(b) (i) Find an expression, in its simplest form, for the gradient of the line *PQ*.

(ii) Explain how you could use the answer of *(b)*(i) to derive the gradient of the tangent to *C* at the point *P*. [4]

7. *(a)* Use the substitution $u = 12 - x^3$ to evaluate

$$\int_0^2 \frac{x^2}{\left(12-x^3\right)^2}\,\mathrm{d}x\,.$$ [4]

(b) (i) Find $\int x\cos 2x\,\mathrm{d}x$.

(ii) Use the result of (*b*)(i) to find

$$\int x\sin^2 x\,\mathrm{d}x.$$ [7]

8. The position vectors of the points A and B are given by

$$\mathbf{a} = 5\mathbf{i} - \mathbf{j} - \mathbf{k},$$
$$\mathbf{b} = 4\mathbf{i} - 3\mathbf{j} + 6\mathbf{k},$$

respectively.

(a) (i) Write down the vector $\mathbf{AB}$.

(ii) Find the vector equation of the line AB. [3]

(b) The vector equation of the line L is given by

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6]

TURN OVER

9. A bookseller values a rare book at £A on August 1st 2010. The value, £P, of the book t years after this date may be modelled as a continuous variable. The rate of increase of P may be assumed to be directly proportional to P^2.

(a) Write down a differential equation satisfied by P. [1]

(b) Show that

$$\frac{1}{k}\left(\frac{P-A}{PA}\right) = t,$$

where k is a constant. [4]

(c) The value of the book is £800 on August 1st 2013 and £900 on August 1st 2014. Find the value of A. [3]

10. Prove by contradiction the following proposition.

If a and b are odd integers such that 4 is a factor of $a - b$,
then 4 is **not** a factor of $a + b$.

The first lines of the proof are given below.

Assume that 4 is a factor of $a + b$.
Then there exists an integer c *such that* $a + b = 4c$. [3]

END OF PAPER

