WELSH JOINT EDUCATION COMMITTEE CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education **Tystysgrif Addysg Gyffredinol**

Advanced Level/Advanced Subsidiary **Safon Uwch/Uwch Gyfrannol**

MATHEMATICS C4

Pure Mathematics

Specimen Paper 2005/2006

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Write down and simplify the binomial expansion of $(1+2x)^{-\frac{1}{2}}$ up to and including the term in x^3.

Find the expansion of $\dfrac{(1-x)^2}{(1+2x)^{\frac{1}{2}}}$ in ascending powers of x up to and including the term in x^3. State the range of values of x for which the expansion is valid. [7]

2. *(a)* Use a counter-example to show the statement $\cos 2\theta = 2\cos\theta$ is not always true. [2]

(b) Showing all your working, find the values of θ between 0° and 360° satisfying

$$3\cos 2\theta = 1 - \sin\theta.$$ [6]

3. Showing all your working, find the values of θ between 0° and 360° satisfying the equation

$$5\sin\theta + 4\cos\theta = 3.$$ [7]

4. *(a)* Express $\dfrac{3x^2+2x+1}{x^2(x-1)}$ in terms of partial fractions. [4]

(b) Find $\displaystyle\int \frac{3x^2+2x+1}{x^2(x-1)}\mathrm{d}x.$ [3]

5. A curve C has parametric equations $x = at^2$, $y = 2at$. Show that the equation of the normal to C at the point P, whose parameter is p, is

$$px + y - 2ap - ap^3 = 0.$$

The normal to C at P meets the x-axis at Q. The perpendicular from P to the x-axis meets the x-axis at R. Find the length of QR. [8]

6. Actinium is a radioactive substance which decays slowly.

Initially, 2 kg of actinium is present and the rate of decay of its mass is 64 g/year. Subsequently, t years later when the actinium has a mass x kg, the rate of decrease of mass is proportional to the value of x.

(a) Show that $\frac{dx}{dt} = -0{\cdot}032x$. [3]

(b) Deduce that $t = \frac{125}{4}\ln\left(\frac{2}{x}\right)$. [5]

(c) Find the value of t when half the actinium has decayed, giving your answer correct to two decimal places. [2]

7. Find the volume of the solid generated when the portion of the curve $y = \sqrt{x^3 \ln x}$ between $x = 1$ and $x = e$ is rotated about the x-axis. [6]

8. *(a)* Show that

$$\int_0^{\frac{\pi}{4}} \cos^2\theta \, d\theta = \frac{\pi}{8} + \frac{1}{4}.$$ [4]

(b) Use the substitution $x = 3\tan\theta$ to evaluate

$$\int_0^3 \frac{27}{(9+x^2)^2} dx.$$ [6]

9. The vector equations of two lines are

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k}),$$

$$\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + t\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k}),$$

where t is a constant.

(a) Given that the two lines intersect, show that $t = -1$ and find the position vector of the point of intersection. [6]

(b) Find the acute angle between the lines, giving your answer correct to the nearest degree. [6]