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MATHEMATICS FP1

Further Pure Mathematics

P.M. THURSDAY, 14 June 2007

(1 $\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Differentiate x^4 from first principles. [6]

2. Solve the following equation for the complex number z.

$$2z + \bar{z} = \frac{1+7\mathrm{i}}{3+\mathrm{i}}$$

Give your answer in its simplest form. [6]

3. In the cubic equation

$$x^3 + px^2 + qx + 26 = 0,$$

the constants p and q are real. Given that $2 + 3\mathrm{i}$ is a root of this equation,

(a) find the other two roots of the equation, [4]

(b) determine the values of p and q. [4]

4. The sum of the first n terms of a series is $3n^2 + 2n$.

(a) Show that the nth term is given by

$$T_n = 6n - 1 \ .$$ [2]

(b) Show that

$$\sum_{r=1}^{n} T_r^2 = an^3 + bn^2 + cn$$

where a, b, c are constants to be determined. [5]

5. Use mathematical induction to show that

$$\sum_{r=1}^{n}\left[r \times \left(\frac{1}{2}\right)^r \right] = 2 - (n+2)\left(\frac{1}{2}\right)^n$$

for all positive integers n. [8]

6. Given that

$$y = x^x \text{ for } x > 0,$$

show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = x^x \left(1 + \ln x\right)^2 + x^{x-1}.$$ [7]

7. *(a)* Show that the matrix **A** defined below is singular.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 4 & 1 \\ 1 & 8 & -5 \end{bmatrix}$$ [3]

(b) (i) Find the value of k for which the following equations are consistent.

$$\begin{aligned} 2x + y + 2z &= 3 \\ 3x + 4y + z &= 1 \\ x + 8y - 5z &= k \end{aligned}$$

(ii) For this value of k, find the general solution of these equations. [9]

8. *(a)* The transformation T_1 in the plane transforms the point (x, y) to the point (x', y') and is defined by

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}.$$

Write down the images under T_1 of the points (1, 0) and (0, 1). Given that T_1 is a reflection in the line $x + y = 0$, use your results to find the values of a, b, c and d. [4]

(b) The transformation T_2 is a translation in which the point (x, y) is transformed to the point $(x + 2, y + 2)$. The transformation T is defined as T_1 followed by T_2.

(i) Find the 3×3 matrix representing T.

(ii) Show that the fixed points of T lie on a straight line and state the equation of this line.

(iii) Describe in words the transformation T. [8]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2.$$

(a) Obtain expressions for u and v in terms of x and y. [3]

(b) The point P moves along the curve with equation $y^2 = 2x^2 - 1$. Find the Cartesian equation of the locus of Q. [6]