GCE AS/A level

0977/01

MATHEMATICS – FP1
Further Pure Mathematics

P.M. WEDNESDAY, 18 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Differentiate $\frac{1}{x^2}$ from first principles. [6]

(b) The function f is defined on the domain $\left(0, \frac{\pi}{2}\right)$ by

$$f(x) = (\sec x)^x.$$

Obtain an expression for $f'(x)$, simplifying your answer. [4]

2. *(a)* Find an expression in its simplest form for

$$\sum_{r=1}^{n} r(r+3).$$ [4]

(b) Given that the sum of the first n terms of another series is $n(n + 3)$, obtain an expression for the nth term of the series. [3]

3. Consider the following equations.

$$\begin{aligned} x \quad &+ 2y \quad + 4z \quad = 3, \\ x \quad &- y \quad + 2z \quad = 4, \\ 4x \quad &- y \quad + 10z = k. \end{aligned}$$

Given that the equations are consistent,

(a) find the value of k, [5]

(b) determine the general solution of the equations. [3]

4. The complex number z is given by

$$z = \frac{1 + 2\mathrm{i}}{1 - \mathrm{i}}.$$

Find the modulus and the argument of z. [6]

5. The roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0$$

are denoted by α, β, γ.

(a) Find the cubic equation whose roots are $\beta\gamma$, $\gamma\alpha$, $\alpha\beta$. [6]

(b) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = 0.$$

Deduce the number of real roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0,$$

justifying your answer. [4]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 2 & 3 \\ -1 & 1 & 1 \\ 2 & \lambda & 2 \end{bmatrix}.$$

(a) Find the values of λ for which **A** is singular. [4]

(b) Given that $\lambda = -1$,

(i) find the adjugate matrix of **A**,
(ii) find the inverse of **A**. [5]

7. The transformation *T* in the plane consists of a clockwise rotation through 90° about the origin, followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$, followed by a reflection in the y-axis.

(a) Show that the matrix representing *T* is

$$\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [5]

(b) Find the equation of the image under *T* of the line $y = 2x + 1$. [4]

8. Using mathematical induction, prove that

$$\sum_{r=1}^{n} \left(r \times 2^{r-1} \right) = 1 + 2^n (n - 1),$$

for all positive integers n. [7]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z(z - 1).$$

(a) Obtain expressions for u and v in terms of x and y. [4]

(b) The point *P* moves along the line $x + y = 0$. Find the equation of the locus of *Q*. [5]

END OF PAPER

