WELSH JOINT EDUCATION COMMITTEE
**General Certificate of Education**
**Advanced Subsidiary/Advanced**

CYD-BWYLLGOR ADDYSG CYMRU
**Tystysgrif Addysg Gyffredinol**
**Uwch Gyfrannol/Uwch**

978/01

**MATHEMATICS FP2**

**Further Pure Mathematics**

A.M. FRIDAY, 22 June 2007

($1\frac{1}{2}$ hours)

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

## INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** Use the substitution $x = y^2$ to evaluate the integral

$$\int_1^4 \frac{\mathrm{d}x}{\sqrt{x(9-x)}},$$

giving your answer correct to two significant figures. [6]

**2.** Find the two square roots of the complex number $1 + \sqrt{3}\,\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$. [6]

**3.** Let

$$f(x) = \frac{(x+1)(x+2)}{(x-1)(x^2+1)}.$$

*(a)* Express $f(x)$ in partial fractions. [5]

*(b)* Find $\int f(x)\mathrm{d}x$. [4]

**4.** Find the general solution of the equation

$$\sin 2\theta + \sin 4\theta = \cos\theta.$$ [9]

**5.** The ellipse $E$ has equation

$$16x^2 + 25y^2 = 400.$$

*(a)* Find the coordinates of the foci of $E$. [4]

*(b)* Show that the point $P$ with coordinates $(5\cos\theta, 4\sin\theta)$ lies on $E$. [1]

*(c)* (i) Show that the equation of the normal to $E$ at $P$ is

$$4y\cos\theta - 5x\sin\theta + 9\sin\theta\cos\theta = 0.$$

(ii) This normal intersects the $x$-axis at $Q$ and the $y$-axis at $R$. Show that the locus of $M$, the mid-point of $QR$, is an ellipse. [10]

**6.** The function $f$ is defined by

$$f(x) = \frac{x^2 + 4}{x}.$$

*(a)* Find the coordinates of the stationary points on the graph of $f$. [4]

*(b)* Find the equation of each of the two asymptotes. [2]

*(c)* Sketch the graph of $f$. [2]

*(d)* Find $f(A)$ where $A$ is the interval $[1, 5]$. [4]

**7.** *(a)* Given that

$$z = \cos\theta + \mathrm{i}\sin\theta,$$

use de Moivre's Theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

for all positive integers $n$. [3]

*(b)* Hence by expanding $\left(z + \frac{1}{z}\right)^5$, show that

$$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos\theta$$

where $a$, $b$ and $c$ are constants to be determined. [5]

**8.** The function $f$ is defined on the domain $(0, 2)$ by

$$f(x) = 4x^2 \qquad \text{for } 0 < x < 1,$$

$$f(x) = (x+1)^2 \qquad \text{for } 1 \leqslant x < 2.$$

*(a)* Determine whether or not $f$ is continuous when $x = 1$. [2]

*(b)* Show that $f$ is a strictly increasing function. [2]

*(c)* Obtain an expression for $f^{-1}(x)$ on each part of its domain. [6]