GCE AS/A level

0978/01

MATHEMATICS FP2
Further Pure Mathematics

P.M. MONDAY, 25 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The piecewise function f is defined by

$$f(x) = ax^2 - 8 \quad (x \leqslant 2),$$
$$f(x) = x^3 - bx \quad (x > 2),$$

   where a and b are constants.

   Given that f and its derivative f' are continuous when $x = 2$, find the values of a and b. [5]

2. Using the substitution $u = e^x$, evaluate the integral

$$\int_0^1 \frac{1}{\left(e^x + 4e^{-x}\right)}\,dx.$$

   Give your answer correct to three decimal places. [6]

3. By putting $t = \tan\left(\frac{x}{2}\right)$, find the general solution to the equation

$$3\sin x = \tan\left(\frac{x}{2}\right).$$

   [8]

4. The function f is given by

$$f(x) = \frac{3x^2 - 4x + 1}{(x-2)(x^2+1)}.$$

   (a) Express $f(x)$ in partial fractions. [4]

   (b) Hence evaluate

$$\int_3^4 f(x)\,dx,$$

   giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a, b are positive integers. [5]

5. *(a)* The function f is defined by

$$f(x) = x^2 \sin x.$$

   Determine whether f is an even function or an odd function. [3]

   (b) The function g is defined by

$$g(x) = x^n \sin x,$$

   where n is a positive integer. Determine the set of values of n for which g is

   (i) an even function,
   (ii) an odd function. [3]

6. The function f is defined by
$$f(x) = \frac{2}{x-3} + x - 6.$$

(a) Determine the coordinates of the points where the graph of f intersects the coordinate axes. [5]

(b) Find the coordinates of the stationary points on the graph of f. [5]

(c) State the equation of each of the asymptotes on the graph of f. [2]

(d) Sketch the graph of f. [2]

7. A parabola has equation
$$y^2 - 2y - 8x + 25 = 0.$$

(a) Find

    (i)   the coordinates of the vertex,
    (ii)   the coordinates of the focus,
    (iii) the equation of the directrix. [6]

(b) The line $y = mx$ cuts the parabola at the points P_1 and P_2.

    (i)   Obtain a quadratic equation whose roots are the x-coordinates of P_1 and P_2.
    (ii)   Hence find the gradients of the two tangents from the origin to the parabola. [7]

8. *(a)* Using mathematical induction, prove that
$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

for positive integral values of n. [7]

(b) (i) The complex number w is a cube root of the complex number z. Show that

$w\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ is another cube root of z.

    (ii)   Write down the real cube root of -8. Using the result in (i), or otherwise, find the two complex cube roots of -8, giving your answers in the form $x + iy$. [7]