WJEC
CBAC

GCE AS/A level

981/01

MATHEMATICS M2
Mechanics 2

P.M. FRIDAY, 5 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

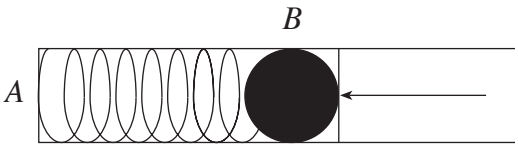
1. A particle moves along the x-axis and its velocity v ms^{-1} at time t s is given by

$$v = \cos 2t - 3\sin t.$$

(a) Find the acceleration of the body when $t = \pi$. [4]

(b) Given that $x = 4$ when $t = 0$, calculate the distance of the particle from the origin O when $t = \frac{\pi}{4}$. [6]

2. The diagram shows a spring of natural length of 0·25 m in a smooth horizontal tube with one end A fixed and a small ball bearing B of mass 0·36 kg held in equilibrium by a force of magnitude 80 N compressing it against the free end of the spring. The length of the compressed spring is 0·2 m.



(a) Find the modulus of elasticity of the spring. [3]

(b) The ball bearing is released by removing the force. Find, by using energy considerations, the speed of the ball bearing just as the spring attains its natural length. [5]

3. A point A is situated at the bottom of a rough plane inclined at an angle α to the horizontal where $\tan \alpha = \frac{3}{4}$. An object, of mass 3·5 kg, is projected from A with speed of u ms^{-1} up the plane along a line of greatest slope of the plane. The object comes to rest at point B where $AB = 2$ m. The coefficient of friction between the object and the plane is $\frac{1}{4}$.

(a) Calculate the work done against friction as the object travels from A to B. [5]

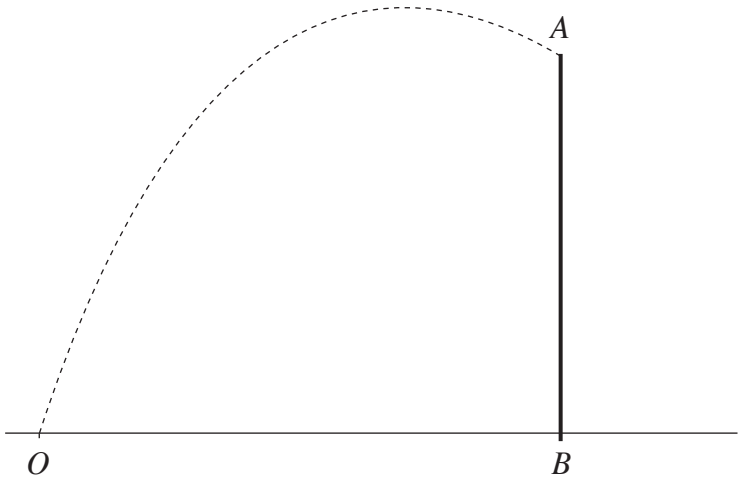
(b) By using energy considerations, find the value of u. [6]

4. A vehicle of mass 5000 kg travels along a straight horizontal road. The resistance to motion is modelled as a constant force of 1500 N.

(a) Find the power which is developed at the instant when the speed of the vehicle is 12 ms^{-1} and the acceleration is 0·2 ms^{-2}. [4]

(b) The maximum power of the vehicle's engine is 45 kW. Calculate the maximum speed of the vehicle along the road. [4]

5. The diagram shows a vertical wall AB and a point O on the same horizontal level as B where $OB = 25{\cdot}2$ m. At time $t = 0$, a ball is projected from O with speed $17{\cdot}5$ ms^{-1} in a direction inclined at an angle α above the horizontal, where $\tan \alpha = \frac{4}{3}$. The ball just clears the top of the wall at A.



(a) Find the time at which the ball passes over the wall. [3]

(b) Calculate the height of the wall AB. [4]

(c) Find the time when the ball reaches its greatest height. [3]

6. A particle, of mass 2 kg, moves in a horizontal plane such that its position vector $\mathbf{r}$ m at time t s is given by

$$\mathbf{r} = (1 - 4t^2)\,\mathbf{i} + (3t^2 - 5t)\mathbf{j}.$$

(a) Find, in terms of t, an expression for the momentum of the particle at time t s. [3]

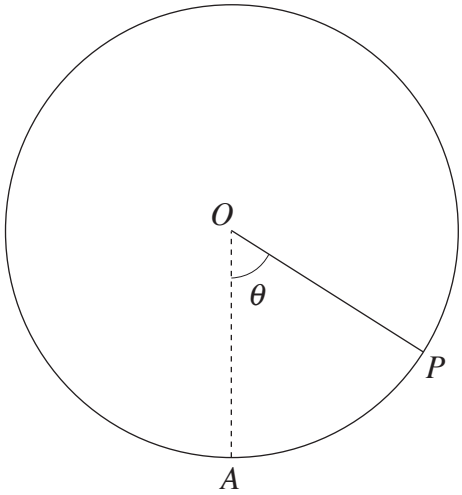
(b) Show that the acceleration of the particle is constant and find its magnitude. [4]

(c) Find the time when the velocity of the particle is perpendicular to its acceleration. [4]

7. A car, of mass 1000 kg, is travelling in a horizontal circle of radius 250 m on a track which is banked at an angle α to the horizontal. When the car is travelling at 28 ms^{-1}, it has no tendency to slip sideways. Calculate the value of α. [7]

TURN OVER

8. In the diagram below, A is the lowest point on the smooth inside surface of a sphere, with centre O and radius 2m. The point P is on the inside surface of the sphere such that $\hat{AOP} = \theta$. A particle, of mass 5 kg, is projected horizontally from A with speed 9 ms^{-1} so that it moves in the vertical circle with centre O which passes through P.



(a) Calculate, in terms of θ, the speed of the particle at P. [4]

(b) Find, in terms of θ, the reaction between the particle and the sphere at P. [4]

(c) Will the particle move in complete circles? Give a reason for your answer. [2]