WJEC
CBAC

GCE AS/A level

982/01

MATHEMATICS M3
Mechanics 3

P.M. TUESDAY, 22 June 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

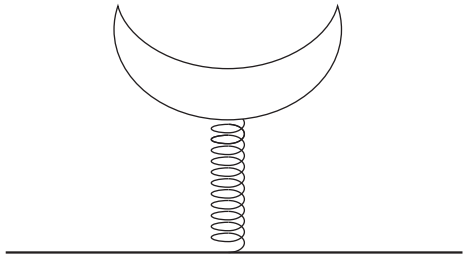
1. A car, of mass 720 kg, moves along a straight horizontal road. The engine of the car exerts a constant power of 81 kW. The car experiences a resistance to motion which has magnitude $90v$ N, where v ms^{-1} is the speed of the car at time t s.

(a) Show that v satisfies the differential equation

$$900 - v^2 = 8v\frac{\mathrm{d}v}{\mathrm{d}t}.$$ [5]

(b) Find a general expression for t in terms of v, and hence evaluate the time it takes for the car to accelerate from a speed of 5 ms^{-1} to 20 ms^{-1} . [7]

2. The diagram shows a playground ride consisting of a seat, of mass 12 kg, attached to a vertical spring, which is fixed to a horizontal board. When the ride is at rest with nobody on it, the compression of the spring is 0·05 m.



The seat is modelled as a particle P and the spring is modelled as a light spring of natural length 0·75 m and modulus of elasticity λ.

(a) Find the value of λ. [2]

The seat is now pushed vertically downwards a further 0·05 m and is then released from rest.

(b) Show that P makes Simple Harmonic oscillations of period $\frac{\pi}{7}$ and write down the amplitude of the motion. [5]

(c) Find the maximum speed of P. [2]

(d) Calculate the speed of P when it is at a distance 0·03 m from the equilibrium position. [3]

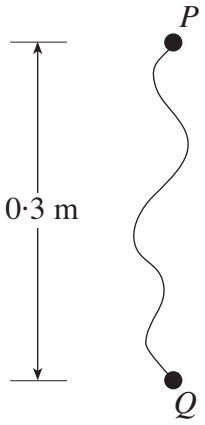
(e) Find the distance of P from the equilibrium position 1·6 s after it is released. [3]

3. Find the solution of the differential equation

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 18t - 87,$$

such that $x = 5$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 10$ when $t = 0$. [12]

4. Two particles P and Q, of mass 3 kg and 5 kg respectively, are attached one to each end of a light inextensible string of length 0·6 m. Initially, the particles are at rest on a smooth horizontal surface a distance 0·3 m apart, as shown in the diagram.



The particle Q is projected across the surface with speed 8 ms^{-1} in a direction at 90° to the line joining the initial positions of P and Q. Determine the impulsive tension in the string during the jerk, stating your unit clearly. Find the speed with which each particle begins to move immediately after the jerk. [11]

5. An object, of mass 150 kg, descends vertically and experiences a total resistance to motion of $10v^2$ N, where v is the speed of the object at time t seconds. At time $t = 0$ it passes point A with speed 30 ms^{-1}. The distance from point A of the object at time t seconds is s metres.

(a) Show that s satisfies the differential equation

$$15v\frac{dv}{ds} = 15g - v^2.$$ [3]

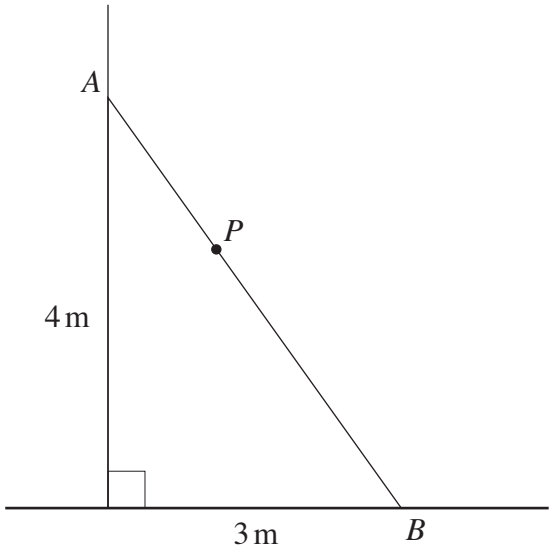
(b) Find an expression for s in terms of v. [6]

(c) Given that the object hits the ground with a speed of 14 ms^{-1}, calculate the height of the point A. [2]

(d) Find an expression for v^2 in terms of s. [3]

TURN OVER

6. A uniform ladder *AB,* of length 5 m and mass 20 kg, rests with end *A* against a rough vertical wall and end *B* on rough horizontal ground. The vertical distance of *A* from the ground is 4 m, and the horizontal distance of *B* from the wall is 3 m. When a man *P,* of mass 80 kg, stands on the ladder 3 m from the lower end, the frictional force at *A* is limiting. The coefficient of friction between the ladder and the wall is 0·3.



(a) Find the normal reaction at *A*. [6]

(b) Find the least value of the coefficient of friction between the ladder and the ground. Give your answer correct to three significant figures. [5]