GCE AS/A Level

0982/01

S17-0982-01

MATHEMATICS – M3
Mechanics

FRIDAY, 23 JUNE 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle moves along the x-axis such that its displacement x metres at time t seconds satisfies the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} + x = 2.$$

The particle passes through the origin when $t = 0$.

(a) Find the time when the particle reaches the point $x = 1$, and determine an expression for x at time t. [7]

(b) Hence find an expression for the acceleration of the particle at time t. [3]

2. Two particles *P* and *Q*, of mass 3 kg and 7 kg respectively, are attached one to each end of a light inextensible string. Initially, the string is slack and the particles are at rest on a smooth horizontal surface. The particle *Q* is then projected across the surface with speed 8 ms^{-1} away from *P* along the straight line passing through the initial positions of *P* and *Q*. Find the speed with which the particles begin to move immediately after the jerk and determine the impulsive tension in the string during the jerk. [6]

3. The function x satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 6\frac{\mathrm{d}x}{\mathrm{d}t} + (10-k)x = \frac{1}{50}k(k-5)(12t-26),$$

where k is a constant. When $t = 0$, $x = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 16$. Find x in each of the following cases.

(a) $k = 5$. [5]

(b) $k = 0$. [5]

(c) $k = 10$. [8]

4. An object *P*, of mass 0·5 kg, moves along a horizontal straight line. The object experiences a resistive force of magnitude $3v^2$ N, where v ms^{-1} is the speed of *P* at time t seconds. When $t = 0$, *P* is at a point *O* and moving with speed 2 ms^{-1}.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -6v^2.$$ [2]

(b) Find an expression for v in terms of t. [4]

(c) Obtain an expression for v in terms of x, where x metres is the distance of *P* from *O* at time t seconds. [5]

(d) Determine, in terms of x, the rate at which work is being done against the resistance when *P* is at a distance x metres from *O*. [3]

5. The speed v ms^{-1} of a particle moving along the x-axis is given by

$$v^2 = -4x^2 + 8x + 21.$$

(a) Show that the motion is simple harmonic and write down the centre of the motion. [5]

(b) Show that the period of the motion is π seconds and determine the amplitude. [4]

(c) Given that when $t = 0$, the particle is at the centre of the motion and moving with positive velocity, write down an expression for x in terms of t and calculate the time taken for the particle to reach $x = 3$ for the first time. [4]

6. A ladder AB, of length 8 m and weight WN, rests with one end A against a vertical wall and the other end B on horizontal ground. The ladder makes an angle α with the horizontal where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the ladder and the wall is λ and the coefficient of friction between the ladder and the ground is μ.

(a) Consider the case when the ladder is **uniform**. Given that $\lambda = 0$ and the ladder is on the point of slipping, determine the value of μ in this case. [4]

(b) Consider the case when the ladder is **non-uniform** and its centre of mass is x m from A. Given that $\lambda = \mu = 0{\cdot}6$ and the ladder is on the point of slipping, calculate the value of x in this case. [10]

END OF PAPER

