WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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983/01

MATHEMATICS S1

Statistics

A.M. THURSDAY, 9 June 2005

($1\frac{1}{2}$ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. An A-level Mathematics class contains 5 boys and 6 girls. The teacher is told that 3 members of the class can go to a special lecture and she decides to select these 3 members at random. Find the probability that

(a) 2 boys and 1 girl are selected, [3]

(b) the 3 members selected are all of the same gender. [4]

2. A tennis club has 150 members in the following categories.

	Junior	Senior	Social
Male	20	30	30
Female	15	20	35

A new tennis court has been built and it is decided to select one of the members at random to perform the opening ceremony. Let A denote the event that the selected member is male and let B denote the event that the selected member is a junior.

(a) Evaluate

(i) $P(A)$,

(ii) $P(B \mid A)$,

(iii) $P(A \cup B)$. [6]

(b) Determine whether or not A and B are independent. [3]

3. Ann and Ben play the following game. They toss a fair coin alternately, starting with Ann, and the winner is the first to toss a 'head'.

(a) Write down the probability that Ann wins with her first toss. [1]

(b) Find the probability that Ann wins with her second toss. [2]

(c) Write down the first three terms of the infinite geometric series for the probability that Ann wins the game. [2]

(d) Hence find the probability that Ann wins the game. [2]

4. Mrs. Jones sells jars of home made jam at a Sunday market. She knows from previous experience that the demand each Sunday for these jars can be modelled by a Poisson distribution with mean 15.

(a) Find the probability that the demand on a randomly chosen Sunday is

(i) exactly 10 jars,

(ii) fewer than 12 jars. [4]

(b) She takes 20 jars to the market every Sunday. Find the probability that, on a randomly chosen Sunday, she is unable to satisfy the demand. [2]

(c) Mrs. Jones wants the probability of being able to satisfy the demand to be at least 0·99. Find the minimum number of jars that she needs to take to the market. [2]

5. A new test to determine whether or not newly born babies have a certain disease is being trialled. The test gives a positive result with probability 0·9 when applied to a baby with the disease and it gives a positive result with probability 0·05 when applied to a baby who does not have the disease. It is known that 1% of babies have the disease. The test is applied to a randomly chosen baby.

(a) Calculate the probability that

(i) the test gives a positive result,

(ii) the baby does not have the disease given that a positive result is obtained. [6]

(b) Comment on the weakness of the test in the light of your answer to *(a)* (ii). [1]

6. *(a)* A fair die is tossed 10 times and X denotes the number of times a '6' is obtained.

(i) State the distribution of X.

(ii) Find the mean and variance of X.

(iii) Calculate $P\,(X \leqslant 2)$. [6]

(b) Two fair dice are tossed 81 times and Y denotes the number of times a total of 12 is obtained. Use a Poisson approximation to evaluate $P(Y = 4)$. [4]

7. The discrete random variable X has probability distribution given by

$$\begin{aligned} P(X = x) &= k(1 + x) \qquad && \text{for } x = 1, 2, 3, 4, 5, \\ P(X = x) &= 0 && \text{otherwise.} \end{aligned}$$

(a) Show that $k = \dfrac{1}{20}$. [2]

(b) Find the mean and variance of X. [5]

(c) Given that X_1, X_2 are two independent observations of X, evaluate

$$P(X_1 + X_2 = 4).$$ [4]

(d) The random variable Y is defined by

$$Y = 2X + 3.$$

Find the mean and variance of Y. [4]

8. The continuous random variable X has cumulative distribution function F given by

$$\begin{aligned} F(x) &= 0 \qquad && \text{for } x < 0, \\ F(x) &= 4x^3 - 3x^4 && \text{for } 0 \leqslant x \leqslant 1, \\ F(x) &= 1 && \text{for } x > 1. \end{aligned}$$

(a) Evaluate $P(0{\cdot}2 \leqslant X \leqslant 0{\cdot}8)$. [3]

(b) Show that the lower quartile of X lies between 0·45 and 0·46. [3]

(c) Find an expression for $f(x)$, valid for $0 \leqslant x \leqslant 1$, where f denotes the probability density function of X. [2]

(d) Evaluate $E(X)$. [4]