WELSH JOINT EDUCATION COMMITTEE
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MATHEMATICS S3

STATISTICS 3

P.M. THURSDAY, 15 June 2006

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A random sample of 3 numbers is chosen without replacement from the 5 numbers (1, 1, 2, 3, 4).

(a) List all the possible samples. [2]

(b) Determine the sampling distributions of

(i) the sample mean,

(ii) the sample median. [6]

2. In order to estimate the proportion p of a certain population who are bilingual, a random sample of 1200 members of the population is questioned. It is found that 498 of them are bilingual.

(a) Calculate an unbiased estimate of p. [1]

(b) Estimate the standard error of your estimate. [2]

(c) Calculate an approximate 90% confidence interval for p. [3]

(d) Give **two** reasons why your interval is approximate. [2]

3. Two different varieties of wine, A and B, are sold in 1 litre bottles. A consumer organisation measured the amount of wine, x litres, in each of 100 randomly selected bottles of variety A. The results are summarised below.

$$\sum x = 103{\cdot}4; \sum x^2 = 106{\cdot}95$$

The organisation also measured the amount of wine, y litres, in each of 150 randomly selected bottles of variety B. The results are summarised below.

$$\sum y = 152{\cdot}4; \sum y^2 = 154{\cdot}86$$

Determine an approximate 95% confidence interval for the difference in the population means of the amounts of wine in 1 litre bottles of the two varieties. [9]

4. The weights (in kg) of ten randomly chosen turkeys on a farm were as follows.

8·3 9·6 7·8 7·9 9·4 8·0 8·8 8·9 9·2 7·5

Assume that these weights can be regarded as a random sample from a $N(\mu, \sigma^2)$ distribution.

(a) Calculate unbiased estimates of μ, and σ^2. [4]

(b) Investigate, at the 5% significance level, a claim by the farmer that the mean weight of turkeys on the farm is 9 kg. Use a two-sided test, stating your hypotheses. [7]

5. Alan is investigating the relationship between the resistance, y ohms, of a new type of electrical component and the temperature, x°C. He obtains the following results.

x	0	5	10	15	20	25
y	10·3	11·8	14·2	16·6	17·4	18·9

(a) Evaluate $\sum x$, $\sum y$, $\sum xy$ and $\sum x^2$. [2]

(b) Assuming a linear relationship $y = \alpha + \beta x$, calculate a and b, the least squares estimates of α and β. [6]

The values of x are exact whereas the values of y are subject to independent normally distributed errors with zero mean and standard deviation 0·4.

(c) (i) Use your values of a and b to estimate the true value of the resistance at 20°C. Determine the standard error of your estimate.

(ii) Hence find a 95% confidence interval for the true value of the resistance at 20°C.

(d) Alan predicted beforehand that the value of β would be 0·4. Determine, at the 1% significance level, whether or not his results are consistent with this prediction. [12]

6. The random variable X has mean μ and variance σ^2. Independently, the random variable Y has mean 2μ and variance $3\sigma^2$. Consider the following estimators for μ.

$$U = a(X + 2Y); \quad V = b(2X + Y)$$

(a) Find the values of the constants a and b for which U and V are unbiased estimators for μ. [5]

(b) Find the variances of these unbiased estimators and hence determine which of the two is the better estimator. [5]

(c) Now consider the estimator

$$W = \frac{X + kY}{1 + 2k} \quad (\text{where } k \neq -\frac{1}{2}).$$

(i) Show that W is an unbiased estimator for μ for all possible values of k.

(ii) Find the variance of W in terms of k and σ.

(iii) Hence find the value of k which gives the best estimator of this form. [9]

[You may assume that any stationary value that you find is a minimum.]