WJEC CBAC

GCE AS/A level

0985/01

MATHEMATICS – S3
Statistics

A.M. MONDAY, 23 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

• a 12 page answer book;
• a Formula Booklet;
• a calculator;
• statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The weights of the plums grown in a certain orchard may be assumed to be normally distributed with mean weight μ grams. In order to estimate μ, a random sample of 60 plums was weighed, in grams, and the following sample statistics were calculated.

    $$\sum x = 3120, \ \sum x^2 = 162480$$

    Calculate an approximate 90% confidence interval for μ. [6]

2. The mean weight of a certain breed of bird is believed to be 4·5 kg. In order to test this belief, a random sample of 10 birds of the breed was obtained and weighed, with the following results in kg.

    4·38   4·18   4·46   4·59   4·16   4·57   4·16   4·26   4·49   4·35

    You may assume that the weights of this breed of bird are normally distributed.

    (a) State suitable hypotheses for testing the above belief using a two-sided test. [1]

    (b) Calculate unbiased estimates of the mean and the variance of the weights of this breed of bird. [5]

    (c) Carry out an appropriate test using a 1% significance level and state your conclusion in context, justifying your answer. [7]

3. *(a)* In an opinion poll of 1500 people, 654 said that they prefer Belgian chocolate to any other sort. Calculate a 95% confidence interval for the proportion of people in the population who prefer Belgian chocolate to any other sort. [6]

    (b) In another opinion poll of 1200 people on the same subject, the following confidence interval was calculated.

    $$[0·4348, \ 0·4852]$$

    Determine

    (i)   the number of people in the sample who stated that they prefer Belgian chocolate to any other sort,

    (ii) the confidence level of the confidence interval. [7]

4. A market gardener grows a large number of tomato plants, all of which are Variety A or Variety B. He wishes to investigate whether or not there is a difference in the mean weights of tomatoes obtained from these two varieties over the whole season. He therefore selects random samples of 80 plants of Variety A and 70 plants of Variety B and he records the total yield, in kg, from each plant. At the end of the season, he produces the following summary statistics.

	Variety A	Variety B
Sample size	80	70
Sample mean	3·52	3·65
Unbiased variance estimate	0·115	0·096

(a) State suitable hypotheses for the investigation. [1]

(b) Calculate the approximate *p*-value of the above results and state your conclusion in context. [8]

(c) Give two reasons why the *p*-value is approximate and not exact. [2]

5. The variables x and y are related by an equation of the form $y = \alpha + \beta x$. In order to estimate the unknown constants α and β, the following measurements were made.

x	2	4	6	8	10	12
y	19·8	33·9	49·9	64·1	77·9	95·0

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·5.

(i) Calculate an unbiased estimate of the value of y when $x = 5$.

(ii) Determine a 95% confidence interval for the value of y when $x = 5$.

(iii) It was thought beforehand that the value of β was 7·6. Determine whether or not, at the 5% significance level, the values in the table above are consistent with this value of β. [10]

6. The continuous random variable X is uniformly distributed on the interval $[0, \theta]$, where θ is unknown. In order to estimate θ, a random sample of n observations on X is obtained and $\overline{X}$ denotes the mean of this sample. An unbiased estimator for θ is given by $Y = k\overline{X}$.

(a) (i) Find the value of k.

(ii) Find the standard error of Y. [8]

(b) (i) Show that Y^2 is not an unbiased estimator for θ^2.

(ii) Find an unbiased estimator for θ^2. [6]

END OF PAPER